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The shape of aggregate production functions: evidence from estimates of the World Technology Frontier

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Abstract

The article provides multifaceted evidence on the shape of the aggregate country-level production function, derived from the World Technology Frontier, estimated on the basis of annual data on inputs and output in 19 highly developed OECD countries in the period 1970–2004. A comparison of its estimates based on Data Envelopment Analysis and Bayesian Stochastic Frontier Analysis uncovers a number of significant discrepancies between the nonparametric estimates of the frontier and the Cobb–Douglas and translog production functions in terms of implied efficiency levels, partial elasticities, and returns-to-scale properties. Furthermore, the two latter characteristics as well as elasticities of substitution are found to differ markedly across countries and time, providing strong evidence against the constant-returns-to-scale (CRS) Cobb–Douglas specification, frequently used in related literature. We also find notable departures from perfect substitutability between unskilled and skilled labor, consistent with the hypotheses of skill-biased technical change and capital–skill complementarity. In the Appendix, as a corollary from our results, we have also conducted a series of development accounting and growth accounting exercises.

Keywords and Phrases: world technology frontier, aggregate production function, Data Envelopment Analysis, Stochastic Frontier Analysis, partial elasticity, returns to scale, substitutability

JEL Classification Numbers: E23, O11, O14, O33, O47

1 Introduction

It is paradigmatic in the contemporary macroeconomics literature to assume that the production process can be summarized by an aggregate production function, mapping the stocks of appropriately specified aggregate inputs onto the unique aggregate output. When the precise shape of this function is not the central question of the economic problem at hand, it is also frequently assumed to take the constant-returns-to-scale Cobb–Douglas form, valued for its simplicity, analytical tractability, and agreement with a few broad patterns observed in the data.

Seen from an empirical viewpoint, there is however no consensus on the preferred functional form of the aggregate (country-level) production function.[1] Estimation of aggregate production functions is notoriously difficult due to multiple empirical issues: measurement uncertainty of input and output aggregates such as GDP, physical capital and human capital, problems with comparability across countries and time, endogeneity of input variables, just to name a few. Yet another important issue, and one that we carefully address in the current paper is that even though the production function is a *technological* concept, one of a technical relationship between inputs and outputs, in reality, country-level productivity may also be affected by *non-technological* variables such as taxation, presence of various barriers to doing business (corruption, crime, complicated bureaucratic procedures, etc.), sectoral composition of production, labor market institutions, or financial constraints. To obtain reliable estimates of the technological production function itself, one ought to control for differences in these institutional conditions across countries and time. We achieve this goal by taking the World Technology Frontier approach.[2]

The objective of the current paper is then to estimate the aggregate, country-level production function as a relationship between countries' aggregate inputs and their

[1]Taking aggregation issues seriously, it is even dubious if such an aggregate production function exists at all (see e.g., Felipe and Fisher, 2003). The ability to aggregate local input–output relationships into an aggregate function where total output depends on total stocks of inputs only and not on their distribution across plants, requires strong homogeneity assumptions imposed on the individual production processes – which are very unlikely to hold. Keeping this caveat in mind, the "aggregate production function", which we refer to, can then be viewed only as an *approximate* relationship between aggregate inputs and output, which could be altered due to shifts in factor distribution. See Temple (2006) for a discussion of this interpretation.

[2]It should be kept in mind that although the World Technology Frontier approach enables us to filter out non-technological productivity differences across countries and time *given the allocation of inputs*, there still remains the possibility that the aforementioned non-technological variables may affect input allocation as well. For example, corruption creates both inefficiency in factor use (part of output is diverted away instead of being included in GDP) and suboptimal investment decisions, resulting in suboptimal capital stocks. In our approach, the potential product of a hypothetical "corrupt" country, computed at the frontier, would then be corrected for the first source of inefficiency but not the second, which we are unable to address. In fact, throughout the paper, we will be taking the allocation of inputs as given, without considering its optimality.

maximum attainable output, computed on the basis of the World Technology Frontier (WTF hereafter) – where the WTF is the best-practice frontier at each moment in time. By doing so, we are able to single out technological aspects of the production processes from their institutional background, at least up to a multiplicative constant. Such estimates of the aggregate production function will be then used as a convenient starting point for further analyses, aimed at deriving this function's crucial characteristics, and discussing which parametric form agrees most with the available empirical evidence. As crucial features of the estimated aggregate production function, we shall investigate its implications for the cross-country distribution of technical inefficiency, the pattern of dependence of its (variable) partial elasticities on factor endowments, (variable) returns-to-scale properties, and its implied (Morishima and Allen–Uzawa) elasticities of substitution.

We estimate the aggregate production function with two alternative methods. First, we apply the nonparametric Data Envelopment Analysis (DEA) approach,[3] augmented with the Simar and Wilson (1998, 2000) bootstrap procedure which enables us to adjust for the bias of DEA efficiency estimates as well as to compute standard errors and confidence intervals for these estimates. The advantage of this first approach is that it does not require one to make *a priori* assumptions on the functional form of the aggregate production function – and yields testable predictions instead. Unfortunately, since the DEA approach is based on piecewise linear approximations of the true aggregate production function, it is not suited to providing predictions on the function's curvature features such as the elasticities of substitution.

Second, we also apply the Stochastic Frontier Analysis (SFA) methodology[4] which allows us to estimate the production function directly, under certain predefined (parametric) functional specifications. Such parametric models are estimated with Bayesian techniques, particularly well-suited to production function estimation due to their relative robustness under collinearity and measurement error. The advantage of the SFA approach is that it allows to test several parametric specifications directly. It is also useful for drawing precise conclusions on the aggregate production function's elasticities of substitution.[5]

[3]For applications in macroeconomics, see e.g., Färe et al. (1994), Kumar and Russell (2002), Henderson and Russell (2005), Jerzmanowski (2007), Badunenko, Henderson and Zelenyuk (2008), and Growiec (2012).

[4]For applications in macroeconomics, see e.g., Koop, Osiewalski and Steel (1999, 2000) and Bos et al. (2010). See Kumbhakar and Knox Lovell (2000) for general reference.

[5]SFA allows us to estimate the production function directly, but even if the estimated parametric function is *misspecified* when taken at face value, sometimes it can still be considered as a reasonable *approximation* of the true aggregate production function, sufficiently good within some range of input combinations. The translog production function is indeed frequently viewed this way, i.e., as a local second-order Taylor approximation of an arbitrary function.

Based on these methods, we obtain the following principal results:

- the CRS Cobb–Douglas production function fails to reproduce the important properties of our data (regarding the distribution of inefficiency levels, partial elasticities and elasticities of substitution),
- the (non-parametric) bootstrap-augmented DEA frontier is not only markedly different from the CRS Cobb–Douglas production function specification, but also from the unrestricted Cobb–Douglas and the translog, even though the latter offers much more flexibility and can be fitted to the data relatively well,
- regardless of the approach taken, the ranking of countries with respect to their technical efficiency is relatively stable (although individual distances to the frontier may vary),
- partial elasticities of the aggregate production function are correlated with inputs both in the DEA and in the translog case, and they vary substiantially across countries and time, providing evidence against the Cobb–Douglas specification and lending support to the skill-biased technical change hypothesis,
- tests of returns to scale based on the DEA, Cobb–Douglas and translog representations of the frontier provide mixed evidence on this property, although DRS seems more prevalent in smaller economies, and IRS – in larger economies,
- unskilled and skilled labor are not perfectly substitutable,
- (Morishima and Allen–Uzawa) elasticities of substitution vary largely across countries and time, staying in broad agreement with the hypothesis of capital–skill complementarity.

Based on our WTF production function estimates, we have also conducted a series of development accounting and growth accounting exercises. The discussion of their results has been delegated to the Appendix, so that they do not interrupt our main line of reasoning. They are nevertheless an important corollary of our estimations.[6] In the Appendix, we have found that:

- according to DEA, differences in GDP per worker between the USA and most Western European countries in 1980 have been mostly due to differences in efficiency and skilled labor endowments, whereas in 2004 they have been mostly due to differences in efficiency and physical capital endowments. Average efficiency differences have grown visibly between 1980 and 2004;

[6]Included in the Appendix is also one specific robustness check.

- according to the Cobb–Douglas production function specification, the differences in GDP per worker between the USA and other countries in the sample have been predominantly Total Factor Productivity (TFP)-driven, with a few exceptions where physical capital differences played an equally important role;

- according to DEA, factor accumulation and technological progress have provided significant positive contributions to GDP growth in 1980–2004, with technological progress being particularly powerful in 1990–2004. Average efficiency levels have been declining, providing negative contributions to GDP growth;

- according to the Cobb–Douglas production function specification, TFP growth, physical capital accumulation, and human capital accumulation have all provided positive contributions to GDP growth throughout 1980–2004. The variance of their relative strength across countries and time was large.

The remainder of the paper is structured as follows. Section 2 presents the dataset and methodology. Section 3 offers an overview of our basic results, and a characterization of the World Technology Frontier viewed through the lens of the DEA approach. Section 4 discusses the properties of the aggregate, country-level production function, inferred from the WTF. Section 5 concludes.

2 Data and methodology

2.1 Data sources and the construction of variables

The macroeconomic dataset used in the current study covers 19 highly developed OECD economies in the period 1970–2004. The output variable is GDP and the input variables are the aggregate stocks of physical capital, human capital, subdivided into unskilled and skilled labor, and (for auxiliary purposes only) the "raw" number of employees.

International, annual data on GDP and GDP per worker as well as the total number of workers in 1970–2004 have been taken from the Total Economy Database, developed by the Conference Board and Groningen Growth and Development Data Centre (GGDC). The unit of measurement is the US dollar, converted to constant prices as of year 2008 using updated 2005 EKS PPPs.[7]

Physical capital stocks have been constructed using the perpetual inventory method (cf. Caselli, 2005). We have used country-level investment shares from the Penn World Table 6.2 (cf. Heston, Summers and Aten, 2006). Following Caselli (2005), we also assumed an annual depreciation rate of 6%.

Country-level human capital data have been taken from de la Fuente and Doménech (2006). The raw variables provided in this contribution are shares of population aged 25 or above having completed primary, some secondary, secondary, some tertiary, tertiary, or postgraduate education. The considered dataset is of 5-year frequency only and ends in 1995. Nevertheless, the de la Fuente–Doménech dataset has been given priority among all possible education attainment databases due to its presumed superior quality. The original de la Fuente–Doménech data have then been extrapolated forward in the time-series dimension until the year 2000 using Cohen and Soto (2007) schooling data as a predictor for the trends. Neither Barro and Lee (2001) nor Cohen and Soto (2007) data could be used directly for this purpose because neither of them is (even roughly) in agreement with the de la Fuente–Doménech dataset – nor with each other – in the period where all datasets offer data points. Furthermore, the human capital data have been extrapolated to all intermediate years as well, for human capital variables are, in general, very persistent and not susceptible to business cycle variations.

Human capital aggregates have been constructed from these educational attainment data using the Mincerian exponential formula with a concave exponent, following Hall and Jones (1999), and more directly, Caselli (2005) and Growiec (2012):

$$H^U = \left(\sum_{i \in S_U} \psi_i e^{\phi(s_i)} \right) L, \qquad H^S = \left(\sum_{i \in S_S} \psi_i e^{\phi(s_i)} \right) L, \tag{1}$$

[7] The Conference Board and Groningen Growth and Development Centre, Total Economy Database, January 2009. http://www.conference-board.org/data/economydatabase/

where S_U is the set of groups of people who completed less than 12 years of education (less than elementary, elementary, less than secondary), S_S is the set of groups of people who completed 12 years of education or more (secondary, less than college, college or more), ψ_i captures the share of i-th education group in total working-age population of the given country, s_i represents years of schooling in i-th education group (cf. de la Fuente and Doménech, 2006), L is the total number of workers, and $\phi(s)$ is a concave piecewise linear function:

2

$$\phi(s) = \begin{cases} 0.134s & s < 4, \\ 0.134 \cdot 4 + 0.101(s-4) & s \in [4, 8), \\ 0.134 \cdot 4 + 0.101 \cdot 4 + 0.068(s-8) & s \geq 8. \end{cases} \tag{2}$$

The overall human capital index may be computed as the sum of unskilled and skilled labor: $H = H^U + H^S$.[8] We have however allowed these two types of labor to be imperfectly substitutable and thus enter the production function separately. The "perfect substitution" case where only total human capital matters for production (and its distribution between unskilled and skilled labor has no impact whatsoever) is an interesting special case of our generalized formulation. The data do not support this assumption.

All data used in DEA and bootstrap-augmented DEA analyses are at annual frequency, and the WTF is estimated *sequentially*, so that for computing the WTF in each period t, data from periods $\tau = 1, 2, ..., t$ are used.

Should significant outliers be found within our sample, the final results are likely to be biased. The same problem could also appear due to business-cycle fluctuations, especially that we only measure the total *stocks* of physical and human capital in the considered countries, without taking account of their *utilization rates* which vary significantly across the cycle. Escaping short- and medium-term disturbances appears extremely important in an aggregate production function analysis such as ours. Thus, the Hodrick and Prescott (1997) filter with the usual smoothing parameter ($\lambda = 6.25$ for annual data) has been applied to all our data to exclude the outliers and high-frequency cyclical variation present in the data.

Unfortunately, when employing the aforementioned panel dataset in parametric analyses such as the SFA, we face a critical problem. Namely, due to the strong multicollinearity present in the time domain of our smoothed time series, the parametric, Bayesian estimation procedures applied here might lead to uninformative, uninterpretable results. To avoid this unwelcome outcome, we have decided to narrow down the time dimension of the dataset used in our SFA estimations, limiting ourselves to data covering entire decades instead of single years. One further potential advantage

[8]The cutoff point of 12 years of schooling, delineating unskilled and skilled labor, seems adequate for the relatively highly developed OECD economies in our sample, though it might be set too high if developed economies were to be considered as well (cf. Caselli and Coleman, 2006).

of this approach is that original human capital data are readily available at decadal frequency.

The presentation of our results in the following sections takes into account the fact that our DEA results have been obtained for the whole dataset and the SFA results for its subset only. We concentrate on cross-sectional comparisons or on the inferred "time-less" characteristics such as the slope and curvature of the aggregate production function, and do not compare goodness-of-fit statistics if they are computed on the basis of different datasets.

2.2 Methodological issues

The objective of the current paper is to draw conclusions on the shape of the aggregate, country-level production function, based on two types of estimates of the World Technology Frontier: deterministic DEA-based ones, augmented with the stochastic, nonparametric Simar–Wilson bootstrap, and parametric SFA-based ones, computed using Bayesian procedures. Let us now provide a brief description of both approaches.

2.2.1 Data Envelopment Analysis

The idea behind DEA is to construct the best-practice production function as a convex hull of production techniques (input–output configurations) used in countries present in the data.

The production function is then inferred indirectly as a fragment of the boundary of this convex hull for which is output is maximized given inputs. More precisely, for each observation $i = 1, 2, ..., I$ and $t = 1, 2, ..., T$, output y_{it} is decomposed as:

$$y_{it} = E_{it} f_t(x_{it}) \tag{3}$$

i.e., into a product of the maximum attainable output given inputs $y^*_{it} \equiv f_t(x_{it})$ and the Shephard distance function $E_{it} \in (0, 1]$. In other words, the efficiency index E_{it} measures (vertical) distance to the technology frontier, while the frontier itself is computed nonparametrically as $y^*_{it} = f_t(x_{it})$. The vector of inputs, x_{it}, could in principle be of any length $n \in \mathbb{N}$, but if one distinguishes too many types of inputs then (i) the DEA could run into numerical problems due to the "curse of dimensionality" (cf. Färe et al., 1994), and (ii) the efficiency levels could be overestimated due to too small a sample size. Throughout most of our analysis, we will be assuming $x_{it} = (K_{it}, H^U_{it}, H^S_{it})$, however, making our study immune to both these criticisms.

Formally, the (output-based) deterministic DEA method is a linear programming technique allowing one find the Shephard distance function E_{jt} for each unit $j = 1, 2, ..., I$ and given $t \in \{1, 2, ..., T\}$ in the sample such that its reciprocal – the Debreu–Farrell efficiency index θ_{jt} is maximized subject to a series of feasibility constraints (cf. Fried, Knox Lovell and Schmidt, 1993):

2

$$
\begin{aligned}
& \max_{\{\theta_{jt},\lambda_{11},\ldots,\lambda_{It}\}} \theta_{jt} \\
\text{s.t.} \quad & \theta_{jt} y_{jt} \leq \sum_{\tau=1}^{t} \sum_{i=1}^{I} \lambda_{i\tau} y_{i\tau}, \\
& \sum_{\tau=1}^{t} \sum_{i=1}^{I} \lambda_{i\tau} x_{1,i\tau} \leq x_{1,jt}, \\
& \sum_{\tau=1}^{t} \sum_{i=1}^{I} \lambda_{i\tau} x_{2,i\tau} \leq x_{2,jt}, \\
& \vdots \\
& \sum_{\tau=1}^{t} \sum_{i=1}^{I} \lambda_{i\tau} x_{n,i\tau} \leq x_{n,jt}, \\
& \lambda_{i\tau} \geq 0, \qquad i = 1, 2, \ldots, I, \tau = 1, 2, \ldots, t,
\end{aligned}
\tag{4}
$$

It is also additionally assumed that $\sum_{\tau=1}^{t} \sum_{i=1}^{I} \lambda_{i\tau} = 1$ in the VRS case (variable returns to scale), or $\sum_{\tau=1}^{t} \sum_{i=1}^{I} \lambda_{i\tau} \leq 1$ in the NIRS case (non-increasing returns to scale). Under the CRS (constant returns to scale) assumption, no further restriction on $\lambda_{i\tau}$'s is necessary.

The Shephard distance function E_{jt} is computed as the reciprocal of the (output-oriented Debreu–Farell) efficiency index θ_{jt} (that is, $E_{jt} = 1/\theta_{jt}$).

Since the data contain a finite number of data points, one for each country and each year, by construction the DEA–based production function is piecewise linear and its vertices are the actually observed *efficient* input–output configurations (i.e., not dominated by any linear combination of other observed input–output configurations).

As a rule, the WTF is estimated *sequentially*, so that for computing the WTF in each period t, data from periods $\tau = 1, 2, ..., t$ are used. This corresponds to the assumption that technologies, once developed, remain available for use forever (see e.g., Henderson and Russell, 2005).

2.2.2 Advantages and limitations of the deterministic DEA approach

The deterministic DEA is a data-driven approach to deriving the production function from observed input–output pairs. Its unquestionable strength lies in the fact that it does not require any assumptions on the functional form of the aggregate production function (provided that it satisfies the free-disposal property), and provides testable predictions on its shape instead. Indeed, the usual assumption of a Cobb–Douglas aggregate production function may lead to marked biases within growth accounting or levels accounting exercises leading to an overestimation of the role of total factor productivity (TFP), as argued by Caselli (2005) and Jerzmanowski (2007), a feature which is avoided when the DEA approach is adopted. As for the predicted shape of the production function, DEA can only offer its finite-sample, piecewise linear approximation. With sufficiently large data samples, however, certain parametric forms could be tested formally against this approximate DEA-based nonparametric benchmark, such as the Cobb–Douglas or translog.

There are also limitations of the DEA approach. First, its deterministic character makes it silent on the estimation precision of the aggregate production function and of the predicted efficiency levels if inputs and outputs are subject to stochastic shocks. This weakness is however removed in the current study by using bootstrap techniques due to Simar and Wilson (1998, 2000b).

Second, the DEA provides a biased proxy of the actual technological frontier. In fact, even the most efficient units in the sample could possibly operate with some extra efficiency, since they are already aggregates of smaller economic units and must therefore have some internal heterogeneity. Taking account of that, the frontier would be shifted upwards; efficiency is nevertheless normalized to 100% for the most efficient units in the sample. Again, the bootstrap method due to Simar and Wilson (1998, 2000b) helps in this respect by allowing for corrections in the bias as well as for estimating confidence intervals for the actual efficiency levels and the technological frontier.

Third, the DEA constructs the aggregate production function basing on the (relatively few) efficient data points. This makes it naturally sensitive to outliers and measurement error. This problem cannot be fully neutralized by bootstrap techniques. In this light, it is important to emphasize that our data have been carefully filtered, so that outlying observations and cyclical fluctuations have been removed. We are confident that thanks to this step, the risk of errors in our DEA has been minimized.

2.2.3 Simar and Wilson's bootstraps

As mentioned above, our deterministic DEA results have been complemented with Simar and Wilson's (SW) bootstraps. These procedures approximate the sampling distribution of an estimator by repeatedly simulating the Data Generating Process (DGP) under the assumption that the true production function is unknown and consequently the true Shephard distance functions E_{it} (for $i = 1, 2, ..., I$ and $t = 1, 2, ..., T$) are unknown, too. Simar and Wilson's bootstraps are then used to formulate an approximation of the sampling distribution of the difference $\hat{E}_{it} - E_{it}$, where $\hat{E}_{it}$ is the DEA estimator of E_{it}.

The exact procedure applied here is the homogenous bootstrap described by Simar and Wilson (1998). The procedure is based on the *homogeneity* assumption (cf. Simar and Wilson, 2000a), that random variables $E_{1t}, ..., E_{It}$ are i.i.d. with an unknown density function g on the support $(0, 1]$ (the output-oriented case). In particular, it means that we assume E_{it} to be independent of the random variables generating observed inputs and output, $(\tilde{x}_i^t, \tilde{y}_i^t)$, where $x_i^t = [x'_{11} \; ... \; x'_{it}]'$, $y_i^t = [y_{11} \; ... \; y_{it}]'$.[9]

[9] Vectors $(\tilde{x}_i^t, \tilde{y}_i^t)$, for $i = 1, 2, ..., I$ and $t = 1, 2, ..., T$, are assumed to be i.i.d., too. Their realizations are the observed input-output pairs $\{(x_i^t, y_i^t), i = 1, 2, ..., I, t = 1, 2, ..., T\}$. We use the procedure *boot.sw98* contained in the free software package FEAR (written in R).

2

As the outcome of the homogenous SW bootstrap we receive, for each unit $i = 1, 2, ..., I$ and $t = 1, 2, ..., T$, the bootstrap estimate of the Shephard distance function $\hat{E}_{it}$ and a set of bootstrap realizations $E_{itb}, b = 1, 2, ..., B$, where $B = 2000$ is the number of bootstrap iterations.[10] Consequently, we also obtain estimates of the bootstrap bias, variance of $\hat{E}_{it}$, and respective confidence intervals. Estimates $\hat{E}_{it}$ may also be additionally bias-corrected. If the bootstrap procedure is consistent, then asymptotically, these estimates may be used for E_{it}. Some Monte Carlo experiments conducted by Simar and Wilson (1998, 2000a) suggest that this SW bootstrap is indeed consistent. However, no rigorous proof of its consistency exists in the literature so far (cf. Simar and Wilson, 2000a).

It should be emphasized that the homogeneity assumption may be relaxed. The inefficiency of a unit would then depend on the observed values of inputs and outputs, i.e., on the pairs (x_i^t, y_i^t) $(i = 1, 2, ..., I$ and $t = 1, 2, ..., T)$. Such procedures are called a *heterogeneous* bootstraps and were first proposed in the paper by Simar and Wilson (2000b), where the pairs (x_i^t, y_i^t) were expressed in cylindrical coordinates.[11] In the papers by Kneip, Simar and Wilson (2008, 2009) as well as Park, Jeong and Simar (2009), generalized procedures were proposed, allowing for:

- orthonormal coordinates, with one of them being connected with E_{it},
- constant returns to scale.

These authors have also proposed formal proofs of consistency of certain bootstrap procedures.

Unfortunately, these procedures generate a lot of additional computational burden which greatly limits their practical applicability (see the comments in Kneip, Simar and Wilson, 2008, 2009). For example, they depend on unknown constants whose values are established arbitrarily. Moreover, for large numbers of units in the sample, complexity of the algorithm blows up calculation times beyond acceptable limits. For these reasons, the software is still in its infancy (see Kneip, Simar and Wilson, 2009) and could not be used for the purposes of the current study.

2.2.4 Testing local and global returns to scale

In order to test the extent of returns to scale in the production technology on the basis of DEA-based estimates of the WTF, we have used the resampling algorithm due to Simar and Wilson (1998) and then carried out formal tests of local and global returns to scale, introduced by Löthgren and Tambour (1999) and by Simar and Wilson (2002), respectively.

[10] See Simar and Wilson (1998). Usually, $B = 2000$ is considered sufficient in the literature.

[11] In DEA, inefficiency has a radial character, so (x_i^t, y_i^t) is strictly connected with E_{it}.

As far as the test of *local* returns to scale is concerned, we use a procedure based on bootstrap confidence intervals proposed by Löthgren and Tambour (1999). This returns-to-scale test (for each unit $j = 1, 2, ..., I$, and $t = 1, 2, ..., T$) is performed using the following nested testing procedure:

Test 1:

$H_0 : S_{jt}^{C-NIRS} = 1$ (scale-efficient or increasing returns to scale),

$H_1 : S_{jt}^{C-NIRS} > 1$ (decreasing returns to scale).

If H_0 in Test 1 is not rejected, we proceed with the second test:

Test 2:

$H_0 : S_{jt}^{CRS} = 1$ (scale-efficient),

$H_1 : S_{jt}^{CRS} > 1$ (increasing returns to scale),

where

$$S_{jt}^{CRS} = \frac{\theta_{jt}^{CRS}(x_j^t, y_j^t)}{\theta_{jt}^{VRS}(x_j^t, y_j^t)}, \quad S_{jt}^{C-NIRS} = \frac{\theta_{jt}^{CRS}(x_j^t, y_j^t)}{\theta_{jt}^{NIRS}(x_j^t, y_j^t)},$$

and $\theta_{jt}^{CRS}(x_j^t, y_j^t), \theta_{jt}^{VRS}(x_j^t, y_j^t)$ and $\theta_{jt}^{NIRS}(x_j^t, y_j^t)$ are the output-oriented Debreu–Farrell distance functions under the assumption of constant, variable, and non-increasing returns to scale, respectively.

Let then $\hat{S}_{jt}^{*C-NIRS}(\alpha)$ and $\hat{S}_{jt}^{*CRS}(\alpha)$ denote the lower bound of the bootstrap (1-α)-confidence interval for S_{jt}^{C-NIRS} and S_{jt}^{CRS}, respectively. The test procedure is straightforward: (i) if $\hat{S}_{jt}^{*C-NIRS}(\alpha) > 1$, then H_0 in Test 1 is rejected and we conclude that the technology features decreasing returns to scale; (ii) if $\hat{S}_{jt}^{*C-NIRS}(\alpha) = 1$, then H_0 in Test 1 cannot be rejected and we perform Test 2. If $\hat{S}_{jt}^{*CRS}(\alpha) > 1$, then the hypothesis of scale efficiency is rejected by Test 2 and we conclude that the technology exhibits increasing returns to scale; (iii) finally, if $\hat{S}_{jt}^{*CRS}(\alpha) = 1$, we conclude that the technology is scale-efficient.

In turn, our statistical test of *global* returns to scale is based on two nested tests proposed by Simar and Wilson (2002). In Test 1, the null hypothesis is tested that the aggregate production function (WTF) exhibits globally constant returns to scale (CRS) against an alternative hypothesis that the technology is characterized by variable returns to scale (VRS). That is:

Test 1:

H_0: technology is globally CRS,

H_1: technology is VRS.

If H_0 is rejected, we shall perform Test 2 with H_0 stating that the technology exhibits globally non-increasing returns to scale (NIRS) against H_1 that the technology is VRS:

Test 2:

H_0 : technology is globally NIRS,

H_1 : technology is VRS.

Simar and Wilson (2002) discussed various statistics for testing these hypotheses; among these, we have selected the following ratios of means:

2

$$\hat{S}_t^{CRS} = \frac{\sum_{\tau=1}^{t}\sum_{j=1}^{I}\hat{\theta}_{j\tau}^{CRS}(x_j^\tau, y_j^\tau)}{\sum_{\tau=1}^{t}\sum_{j=1}^{I}\hat{\theta}_{j\tau}^{VRS}(x_j^\tau, y_j^\tau)} \text{ in Test 1,}$$

$$\text{and } \hat{S}_t^{C-NIRS} = \frac{\sum_{\tau=1}^{t}\sum_{j=1}^{I}\hat{\theta}_{j\tau}^{CRS}(x_j^\tau, y_j^\tau)}{\sum_{\tau=1}^{t}\sum_{j=1}^{I}\hat{\theta}_{j\tau}^{NIRS}(x_j^\tau, y_j^\tau)} \text{ in Test 2,}$$

where $\hat{\theta}_{j\tau}^{CRS}(x_j^\tau, y_j^\tau), \hat{\theta}_{j\tau}^{VRS}(x_j^\tau, y_j^\tau)$ and $\hat{\theta}_{j\tau}^{NIRS}(x_j^\tau, y_j^\tau)$ are estimators of the (output-oriented) Debreu–Farrell distance function under the assumption of constant, variable, and non-increasing returns to scale, respectively.

By construction $\hat{S}_t^{CRS} \geq 1$ because $\hat{\theta}_{j\tau}^{CRS}(x_j^\tau, y_j^\tau) \geq \hat{\theta}_{j\tau}^{VRS}(x_j^\tau, y_j^\tau)$. The null hypothesis in Test 1 is rejected when $\hat{S}_t^{CRS}$ is significantly greater than 1. The p-value of the null hypothesis is derived by bootstrapping (see Simar and Wilson, 2002):[12]

$$p - \text{value} = \sum_{b=1}^{B} \frac{I_{[0,+\infty)}(\hat{S}_t^{CRS,b} - \hat{S}_{obs,t}^{CRS})}{B},$$

where $B = 2000$ is the number of bootstrap replications, $I_{[0,+\infty)}$ is the indicator function, $\hat{S}_t^{CRS,b}$ is the b-th bootstrap sample, and $\hat{S}_{obs,t}^{CRS}$ is the original observed value. The same methodology is used in Test 2.

2.2.5 Stochastic Frontier Analysis

To take a broader picture of the (in)efficiency in aggregate production processes in highly developed OECD countries, the results obtained with the DEA approach have been compared against estimates resulting from stochastic frontier analysis (SFA). In this alternative approach, stochastic disturbances are explicitly taken into account, and the potential biases in efficiency estimates caused by stochastic variation, outliers and measurement error are thus minimized. Unfortunately, these advantages are only conditional on finding the appropriate parametric representation of the aggregate, WTF-based production function.

In its simplest, log-linear form, the stochastic frontier model for panel data, employed in the current paper, can be written as:

$$y_{it} = x_{it}'\beta + v_{it} - u_{it}, \tag{5}$$

where $y_{it} = \ln Y_{it}$ represents the logarithm of output in country $i = 1, ..., I$ and period $t = 1, ..., T$, β represents the vector of estimated parameters, and the vector x_{it} carries information about n factors of production expressed in logarithms, plus a constant term. Given this notation, the case $x_{it} = (1, \ln K_{it}, \ln H_{it}^U, \ln H_{it}^S)$ represents our benchmark Cobb–Douglas specification with physical capital, unskilled labor and skilled labor as inputs. However, we shall also extend this vector to accommodate cross-terms as in

[12] To test the hypotheses regarding global returns to scale of the technology we use suitably modified codes written by Oleg Badunenko (see http://sites.google.com/site/obadunenko/codes).

$$x_{it} = \Big(1, \ln K_{it}, \ln H^U_{it}, \ln H^S_{it}, \ln^2 K_{it}, \ln^2 H^U_{it}, \ln^2 H^S_{it}, ... \\ ... \ln K_{it} \ln H^U_{it}, \ln K_{it} \ln H^S_{it}, \ln H^U_{it} \ln H^S_{it}\Big)$$

in which case equation (5) becomes the translog production function. Constant returns to scale are either tested or directly imposed, wherever necessary, by writing down the production function in its intensive form. We shall do this in some of our estimated specifications, along with introducing certain regularity conditions which serve as a source of prior information and depend on the specification of the frontier. These regularity conditions enter the analysis via restricting average input elasticities to be non-negative: $EL_K = \frac{\partial y_i}{\partial \ln K_i} \geq 0$, $EL_{H^U} = \frac{\partial y_i}{\partial \ln H^U_i} \geq 0$, $EL_{H^S} = \frac{\partial y_i}{\partial \ln H^S_i} \geq 0$. The sum of these three partial elasticities represents the measure of average returns to scale (scale elasticity).

The basic theoretical framework of stochastic frontier (SF) models was originally proposed by Aigner, Lovell and Schmidt (1977).[13] In their seminal paper, these authors assumed the total, "composed" error of the production function regression to be a sum of two components: a symmetric, normally distributed variable (the idiosyncracy, v_{it}) and the absolute value of a normally distributed variable (the inefficiency, u_{it}). Ever since, the main stream of research on stochastic frontier models appears to have focused primarily upon various assumptions about the distribution of the inefficiency term. Single-parameter distributional specifications of v_{it} and u_{it} (for instance, normal and truncated normal, respectively) have produced some skepticism in the subsequent literature (cf. Ritter and Simar, 1997; Greene, 2003), but nevertheless remain an important tool in comtemporary applied SFA-based research.

Another issue is that applying the Stochastic Frontier methodology to panel data requires one to keep track of technological progress which can strongly affect production capabilities. Obviously, it is "unfair" to evaluate the efficiency of observations from the past against a frontier estimated with a dataset including more recent data as well, since at earlier times, production processes could not enjoy the possibilities offered by technologies developed later on. However, even when computing the WTF in a sequential manner, akin to the one used in our DEA analysis, in SFA one ought to control for technological progress within the frontier so as to obtain a fair evaluation of the evolution of technical inefficiency across countries and time.

To address this issue, we employ Battese and Coelli's (1992, 1995) decomposition of the inefficiency term u_{it}. It takes the following form:

$$u_{it} = u_i \cdot z_t,$$

[13] Another seminal paper in this field is due to Meeusen and van den Broeck (1977).

2

where the random variable u_i has either a truncated normal or an exponential distribution[14] and $z_t = \exp[-\eta(t-T)]$, where positive (or negative) η indicates decreasing (or increasing, respectively) inefficiency over time. Hence, the Battese–Coelli methodology urges the modeler to assume that the random part of u_{it} is time-invariant, and its temporal evolution is described by a deterministic function z_t with an estimated parameter η. This rigidity is however partly overcome when the WTF is estimated sequentially, so that for each period t, data from periods $\tau = 1, 2, ..., t$ are used. In such case, temporal shifts in u_{it} appear not only due to changes in z_t, but also due to the consecutive re-estimations of the WTF. The inefficiency term u_{it}, the Debreu–Farrell efficiency measure θ_{it} and the Shephard distance measure E_{it} are interrelated via the equality $\theta_{it} = 1/E_{it} = \exp(-u_{it})$.

2

Estimating the WTF sequentially allows the fixed effect u_i to be reassessed in every period. In result, we dispose of the uneasy assumption of a unique pattern of convergence to the WTF across all countries and years (e.g., Kumbhakar and Wang (2005) assume u_i to be a function of capital per worker in the initial period). On the other hand, we do not risk overparametrization of our model which would have likely happened, had we assumed the parameters in β to be time-dependent (e.g., following linear trends as in Koop, Osiewalski and Steel, 1999 and Makieła, 2009). Such an approach would be inadequate for a time horizon comparable to the one employed in our study.

An alternative approach allowing one to deal with stochastic frontiers with time-varying inefficiencies was offered by Cornwell, Schmidt and Sickles (1990). Regrettably, this approach is a based on a "deterministic" frontier model, akin to DEA, and is distribution-free in terms of u_{it}, so it was not used for the purposes of the current analysis.[15]

In sum, the crux of the SFA approach lies with the decomposition of the error term into two components: the country- and time-specific idiosyncratic shock (or measurement error) v_{it}, and the technical inefficiency component u_{it} which is assumed to be non-negative. Both components are assumed to be independent of one another. Needless to say, this assumption stands in sharp contrast to the DEA approach where the whole distance between actual and potential output is automatically attributed to inefficiency.

In our analysis, we shall employ several different assumptions concerning the distribution of u_{it}. This issue is discussed in more details in the following section, along with the description of the estimation procedure and assumptions made in the course of our SFA analysis.

[14] Robustness tests have been done upon these two different distributional assumptions, though in terms of our final results, choosing any of them makes little difference. The results are available from the authors upon request.

[15] Given the purposes of the current study, there are two major disadvantages of Cornwell, Schmidt and Sickles' (1990) model: (i) it labels all omitted time-invariant effects as inefficiency, and (ii) it can only measure countries relative to each other, not relative to the frontier, set up in absolute terms.

2.2.6 Bayesian estimation framework

From the computational perspective, two different approaches have been employed in SFA literature to isolate the inefficiency component from the idiosyncratic error. The first one is based on maximum likelihood methods, as proposed by Jondrow et al. (1982). In this case, given the parameters of the model $\Theta = (\beta, \sigma^{-2}, \eta, \varphi)$, the likelihood function takes the form:

$$L(y_{it}, \theta) = \prod p(y_{it}|x_{it}, \theta) \tag{6}$$

and so its maximization requires the derivation of marginal distributions $p(y_{it}|x_{it}, \theta)$ as an integral with respect to the measure induced by the assumed parametric distribution of u_{it}:

$$p(y_{it}|x_{it}, \theta) = \int_{\mathbb{R}_+} p(y_{it}|x_{it}, u_{it}, \theta) p(u_{it}|\theta) du_{it}.$$

The other approach is the Bayesian one, first applied to the context of SFA by van den Broeck et al. (1994). It relies on a posterior simulator, such as the currently popular Gibbs sampling, which is applied in order to determine the distribution of the inefficiency component u_{it} via draws from the posterior distribution $p(\theta|y_{it}, x_{it})$. Hence, as opposed to Jondrow et al.'s approach, no explicit analytical formula for the likelihood function is needed.

In our study, the stochastic frontier will be estimated with Bayesian techniques that naturally correspond to the latter approach. Thus, all structural parameters of the production function $(y_{it}|u_{it}, \Theta) \sim N(x'_{it}\beta - u_{it}, \sigma^2)$, contained in the vector β, as well the variance of disturbances v_{it} and u_{it}, the mean of the inefficiency term u_{it}, denoted by φ^{-1}, and the pace of technological progress η, will be estimated in a Bayesian procedure.

The first step of this procedure consists in making assumptions on the considered shapes of parameter distributions and endowing them with appropriate priors. The vector β is assumed to take the multivariate normal distribution (possibly truncated, to depict the regularity conditions), $\beta \sim N(\mu, \Sigma)$. The prior distribution of σ^{-2} is taken close to the "usual" flat prior, as in Koop, Osiewalski and Steel (1999). v_{it}'s are treated as independent normal variables with zero mean, unknown variance and with no autocorrelation over time (for all t, v_{it} is independent of $v_{i,t-1}$). The analysis starts with an assumption that u_{it}'s are independent exponentially distributed variables with mean φ^{-1} and no autocorrelation. In this case, $\varphi^{-1} \sim Exp(-\ln r^*)$, which implies that prior median efficiency is equal to r^*. According to the findings presented in the literature, r^* should take the values from the interval $[0.5, 0.9]$ (see Marzec and Osiewalski, 2008; Makieła, 2009). Having found that the final results are insensitive to any value choice out of the aforementioned interval, the prior efficiency median was set to 0.75. The alternative half-normal distribution of u_{it} has also been investigated, but it hardly affects the final outcome of the analysis.

2

The additional economic regularity conditions, imposed in a few considered cases, depend on the specific form of the production function under estimation. In case of the Cobb-Douglas specification, input elasticities, equal to the estimated β parameters, have been assumed to be non-negative. Similar assumptions have also been made in case of the translog specification; however, in this case input elasticities are linear combinations of input quantities and β parameters, so the restriction is applied to average elasticities only and not to their values for all units separately. All in all, for all production functions under consideration, the regularity conditions enter the estimation procedure through $p(\beta) \in [0, 1]$. Should average elasticities in the sample satisfy these assumptions, then $p(\beta) = 1$.

The complexity of stochastic frontier models makes numerical integration methods inevitable. In the current study, as in most recent Bayesian literature, this procedure is based upon Gibbs sampling, which involves taking sequential random draws from the full conditional posterior distribution (cf. e.g., Koop, Steel and Osiewalski, 1995). Under very mild assumptions (see Tierney, 1994), these draws converge to the distribution of draws from the joint posterior. In the current research, implemented in WinBUGS, the characteristics of joint posterior distribution have been calculated on the basis of 300 000 burn-in draws and 300 000 accepted (final) draws for different starting points.

To evaluate the convergence of the Markov Chain Monte Carlo (MCMC) estimation procedure, the following tests were done:

- assessment of the history plot (which plots the estimated parameter value against the iteration number),
- autocorrelation tests: high autocorrelation might imply slow exploration of the entire posterior distribution,
- evaluation of posterior kernel density plots.

Due to the obvious multi-collinearity present in our data, consisting of HP-filtered, constructed time series of annual frequency, this Bayesian procedure suffers from low estimation efficiency and may run into risk of leading to uninformative results. We have therefore limited our SF analysis to data of decadal frequency. As we shall see shortly, this is enough to show significant departures of the Cobb–Douglas and translog parametric results from the non-parametric DEA benchmark, and to characterize a number of intriguing properties of the estimated production function.

2.2.7 Advantages and limitations of the SFA approach

A large amount of work has been devoted in the literature to the development of Bayesian methods suitable for making inference in stochastic frontier models. Some of the important advantages of this approach include: (i) the possibility of exact inference on technical efficiency in the presence of idiosyncratic disturbances, (ii) the possibility of using prior knowledge on the shape of aggregate production functions, and (iii) relatively easy incorporation of ideas and restrictions such as regularity conditions, or the optimal treatment of parameter and model uncertainty.

Although applications of Bayesian approaches to SFA are widespread in the empirical literature, some competing methods, such as the aforementioned deterministic DEA, have also been strongly advocated. Undoubtedly, SFA makes it possible to account for the stochastic disturbances and measurement error to which the DEA method seems quite sensitive (cf. Koop and Steel, 2001). However, while choosing the SFA approach (based upon either classical or Bayesian econometrics), any researcher has to make far more assumptions than in the case of DEA. The utmost objective of comparing these two approaches is thus to make these assumptions testable.

Our work does not use, but is also closely related to a novel "compromise" approach, the semi-parametric StoNED (Stochastic Non-smooth Envelopment of Data, cf. Kuosmanen and Kortelainen, 2010) which shares a number of properties with both DEA and SFA. In a nutshell, StoNED combines nonparametric frontier estimation akin to DEA (however, in StoNED it is not necessary to approximate the aggregate production function with piecewise linear functions – they may be replaced by other increasing and concave but not necessarily differentiable functions) with a stochastic treatment of the composite error, $v_{it} - u_{it}$, under certain parametric assumptions. Its drawbacks, in relation to the purposes of the current paper, are that it does not yet allow one to deal with time-varying inefficiency in panel data, and that it does not provide an operationally useful method to estimate the frontier nonparametrically other than by applying DEA with variable returns to scale (which is the "lower bound" production function considered in StoNED, and which we compute here). Most importantly, it does not provide any value added for identifying the desired properties of the (parametric) frontier production function, and that is why we set this avenue aside.

3

3 Overview of the results

As mentioned above, there are multiple ways of characterizing the World Technology Frontier, i.e., estimating maximum potential output given inputs, and the results may vary depending on the approach. Under the fundamental assumption that the aggregate production function is a useful tool for approximating real-world production processes, and given the hypothesis that there exists a unique "true" production function, we proceed to catalog the discrepancies between the alternative estimates. When doing this, we shall keep in mind the specific assumptions underlying them, and carry out appropriate statistical tests aimed at verifying if these conditions are met.

DEA (and even more so, bias-corrected DEA), due to its nonparametric character, should in general allow for a better fit to the unknown production function than parametric SFA methods, potentially suffering from misspecification problems.[16] It is however relatively less useful for deriving secondary characteristics of the production function than the parametric SFA (e.g., there is no way to approximate its second derivative and hence, curvature measures such as the elasticity of substitution), and it does not extrapolate forward into regions with yet unobserved factor mixes.

3.1 The Cobb–Douglas function does not capture the curvature of the WTF: a graphical presentation

Given the preceding discussion, it seems to be a useful exercise to assess the goodness of fit of various parametric specifications of the aggregate production function to the DEA-based frontier. In the literature, Cobb–Douglas and translog production functions (as well as CES) have been frequently applied in this context. However, our findings provide strong evidence against the Cobb–Douglas specification, already at this stage, and regardless of the assumptions on returns to scale and inputs used for production. Our results regarding the translog specification paint a much more promising picture.

Figure 1 illustrates that even when human capital is not included in the production function, and the function itself is assumed to have constant returns to scale to capital and (unaugmented) labor: $Y/L = f(K/L)$, large deviations of the Cobb–Douglas specification from the nonparametric (or translog) aggregate frontier production function are already clearly visible. The fact is that the DEA frontier (whether augmented with the bootstrap or not) and the translog one have much more curvature than the Cobb–Douglas. The Cobb–Douglas function will thus systematically overestimate productivity for extremely low and high capital endowments, and underestimate it in the intermediate range.

[16] Obviously, this is only true unless the true data-generating process is based on one of the considered parametric forms. In such case, as shown by van Biesebroeck (2007) in a Monte Carlo study, best estimates are always obtained with methods which "know" the actual parametric form of the true production function, and not with DEA which ignores this information.

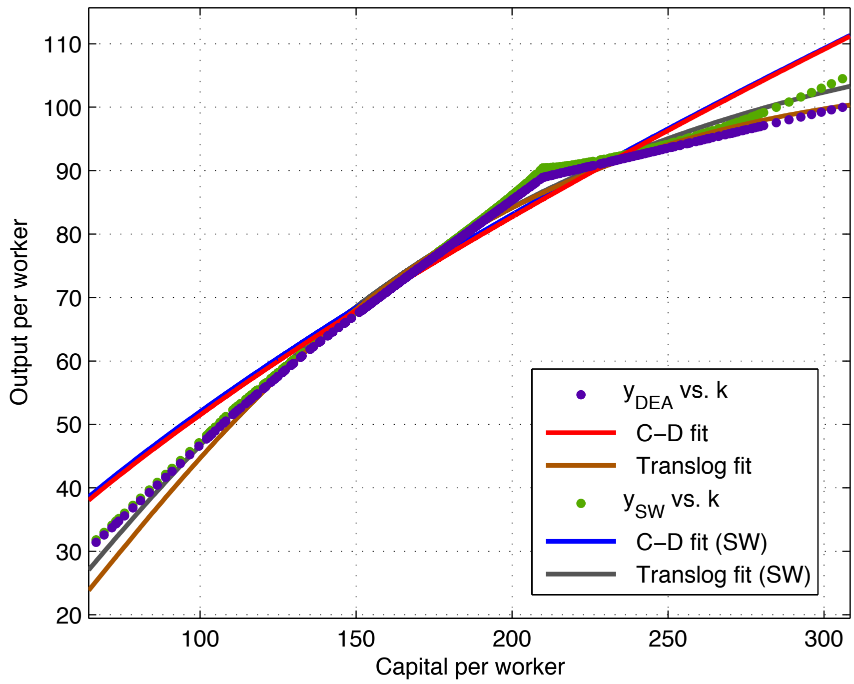

Figure 1: Potential output given inputs in 2004 – estimates of a two-factor production function with constant returns to scale. DEA, bootstrap-corrected DEA vs. the Cobb-Douglas and translog production functions.

The translog production function, on the other hand, can be fitted quite closely to the DEA-based frontier. This is due to its markedly higher flexibility thanks to the inclusion of second-order terms. This said, its generalization and extrapolation properties are still doubtful due to the fact that it constitutes a local log-*quadratic* approximation of the true production function, and the second-order terms render it convex or decreasing if factor endowments are sufficiently high. This will be commented upon when discussing the implied partial elasticities with respect to inputs as well as the implied Morishima and Allen–Uzawa elasticities of substitution.

3.2 Imperfect substitutability between unskilled and skilled labor

Another important building block of our study is the fact that our principal aggregate production function specifications assume that output is produced from physical capital and the stocks of unskilled and skilled labor, the latter two being mutually imperfectly substitutable inputs:

$$Y = F(K, H^U, H^S).$$

We do not make any prior assumptions on returns to scale.

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The reason for this extension of the traditional capital-and-labor-only approach is that neglecting human-capital augmentation of labor and assuming perfect substitutability between its unskilled and skilled part leads to serious misspecification problems (see also e.g., Caselli and Coleman, 2006; Growiec, 2010, 2012). A preliminary argument supporting this finding is presented in Figures 2–3. We see there that the estimates of technical efficiency (and thus, maximum attainable output) vary largely whether human capital augmentation of labor is included in the production function or not, and whether the human capital aggregate is further decomposed into unskilled and skilled labor. Since each of these figures illustrates the difference between two nested specifications, they provide an adequate measure of the extent of function misspecification due to improper aggregation.

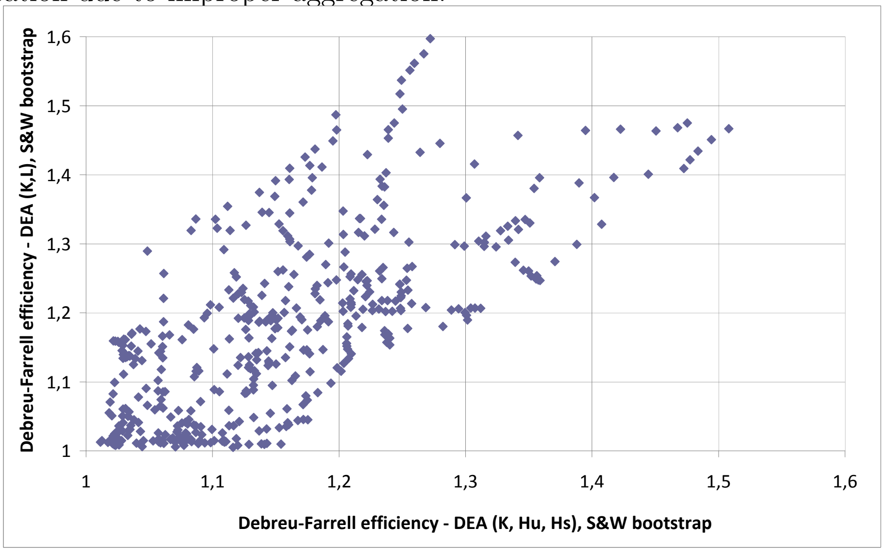

Figure 2: The importance of labor augmentation. Bias-corrected Debreu-Farrell efficiency measures computed for the cases $Y = F(K, L)$ and $Y = F(K, H^U, H^S)$.

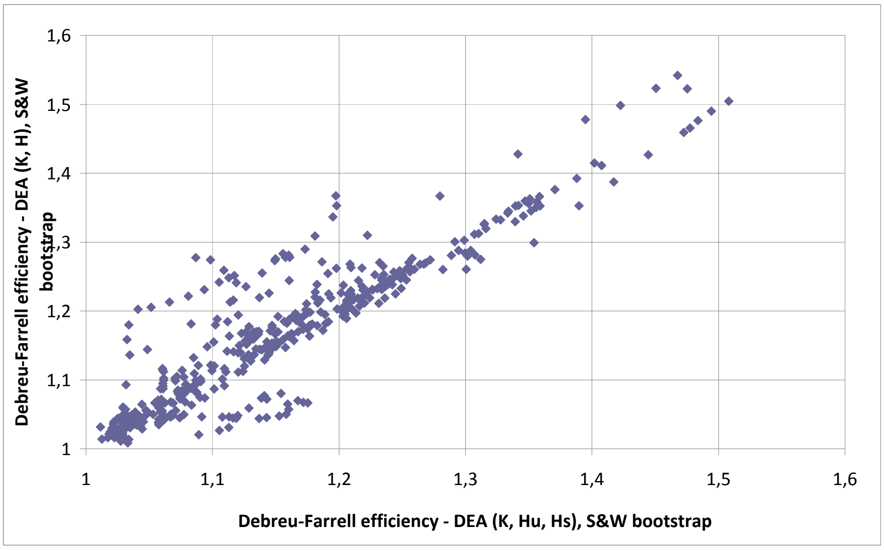

Figure 3: Bias-corrected Debreu-Farrell efficiency measures computed for the cases $Y = F(K, H)$ and $Y = F(K, H^U, H^S)$.

The correlation coefficients between the three considered DEA-based efficiency measures presented in Figures 2–3 are substantial, but significantly different from unity (with $I = 475$ units). They can be viewed in Table 1:

Table 1: Correlation coefficients of bootstrap–corrected DEA Debreu–Farrell efficiency measures for three different input choices.

	$F(K, L)$	$F(K, H)$	$F(K, H^U, H^S)$
$F(K, L)$	1	0,803	0,705
$F(K, H)$		1	0,918
$F(K, H^U, H^S)$			1

The reasons for relaxing the constant returns to scale assumption will be discussed in greater detail at a later stage of the analysis.

3.3 WTF in 1980–2004, according to DEA

Let us now characterize the most general properties of the WTF in 1980–2004, viewed through the lens of the (bootstrap-augmented) DEA approach. It is a natural choice to begin with this specification since it has the least imposed structure and most flexibility, making it best suited to capturing the specific features of our data.

The results are the following. Efficiency and potential output measures for each country and year have been presented in Tables 2–4. Table 2 presents Debreu-Farrell efficiency measures, capturing the distance to the WTF (1 represents 100% efficiency, and the larger is the number, the more inefficient is the data unit). Table 3 presents bootstrap–corrected efficiency measures. As opposed to the original distances, these ones have been corrected for the inherent bias in DEA estimates. Table 4 presents "potential" (WTF-based, bias-corrected) output per worker in the considered 19 OECD countries in 1980–2004,[17] denominated in thousands of PPP converted US dollars under constant prices as of year 2008. By definition, it is the product of each country's actual GDP and the Debreu-Farrell efficiency measure, capturing their distance to the WTF.

There are interesting regularities visible in the observed trends, summarized in Tables 2–4. For example, as illustrated in Figure 4, growth in actual and potential productivity is often parallel, but sometimes we also observe sharp departures from the parallel pattern: while the USA maintained a relatively stable distance to the WTF across years, in Japan this gap has opened wide in the last years.

[17] We do not report the results for 1970-1979 because for these first few years of data, the DEA frontier is estimated quite roughly, due to a small sample size.

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Diverging stories can also be told about Greece and Ireland. In the former country, distance to the WTF in terms of technical efficiency was sizeable and increasing throughout the period; in the latter, it was much smaller, and distance to the WTF first increased but then decreased again.

Table 2: Distances to the World Technology Frontier. Debreu-Farrell efficiency measures.

	Australia	Austria	Belgium	Canada	Denmark	Finland	France	Greece	Ireland	Italy	Japan	Netherlands	Norway	Portugal	Spain	Sweden	Switzerland	UK	USA
1980	1,2022	1,1343	1,0686	1,0702	1,1995	1,2634	1,1237	1,2057	1,0085	1,0000	1,1874	1,0496	1,0000	1,0192	1,0000	1,1711	1,0000	1,0000	1,0000
1981	1,2034	1,1245	1,0765	1,0707	1,2032	1,2587	1,1241	1,1908	1,0125	1,0000	1,1882	1,0505	1,0000	1,0359	1,0000	1,1734	1,0000	1,0000	1,0000
1982	1,2195	1,1287	1,0800	1,1246	1,1879	1,2517	1,1192	1,2212	1,0163	1,0000	1,1851	1,0753	1,0000	1,0504	1,0000	1,1733	1,0000	1,0000	1,0000
1983	1,2484	1,1294	1,0850	1,1324	1,1733	1,2488	1,1188	1,2265	1,0212	1,0000	1,1708	1,1083	1,0000	1,0350	1,0000	1,1670	1,0000	1,0000	1,0000
1984	1,2315	1,1260	1,0810	1,1277	1,1489	1,2419	1,1207	1,2305	1,0224	1,0000	1,1636	1,1115	1,0000	1,0301	1,0000	1,1566	1,0000	1,0000	1,0000
1985	1,2158	1,1266	1,0749	1,1266	1,1244	1,2410	1,1242	1,2265	1,0000	1,0000	1,1522	1,1238	1,0000	1,0192	1,0021	1,1485	1,0000	1,0000	1,0000
1986	1,2064	1,1222	1,0727	1,1357	1,1075	1,2438	1,1249	1,2305	1,0000	1,0000	1,1410	1,1319	1,0000	1,0097	1,0058	1,1518	1,0000	1,0000	1,0000
1987	1,2168	1,1256	1,0707	1,1506	1,1068	1,2378	1,1216	1,2309	1,0000	1,0000	1,1373	1,1597	1,0000	1,0082	1,0000	1,1557	1,0000	1,0000	1,0000
1988	1,2162	1,1343	1,0674	1,1597	1,1150	1,2326	1,1206	1,2280	1,0000	1,0000	1,1260	1,1632	1,0000	1,0100	1,0000	1,1570	1,0000	1,0000	1,0000
1989	1,2089	1,1404	1,0679	1,1774	1,1216	1,2321	1,1125	1,2372	1,0000	1,0000	1,1128	1,1519	1,0000	1,0108	1,0000	1,1571	1,0000	1,0000	1,0000
1990	1,2161	1,1439	1,0739	1,2069	1,1382	1,2549	1,1110	1,2477	1,0000	1,0000	1,1010	1,1391	1,0000	1,0000	1,0000	1,1625	1,0000	1,0042	1,0000
1991	1,2421	1,1391	1,0757	1,2486	1,1380	1,2997	1,1125	1,2827	1,0000	1,0000	1,0909	1,1412	1,0000	1,0000	1,0000	1,1719	1,0000	1,0128	1,0000
1992	1,2472	1,1391	1,0832	1,2706	1,1432	1,3534	1,1199	1,2895	1,0000	1,0047	1,0904	1,1443	1,0000	1,0148	1,0211	1,1992	1,0000	1,0093	1,0000
1993	1,2421	1,1469	1,0960	1,2749	1,1533	1,3924	1,1358	1,3019	1,0000	1,0149	1,1008	1,1587	1,0000	1,0280	1,0512	1,2232	1,0334	1,0001	1,0000
1994	1,2277	1,1670	1,1127	1,2749	1,1526	1,3951	1,1492	1,3094	1,0000	1,0199	1,1240	1,1763	1,0000	1,0443	1,0680	1,2145	1,0941	1,0000	1,0000
1995	1,2044	1,1816	1,1168	1,2689	1,1632	1,3780	1,1514	1,3214	1,0000	1,0204	1,1450	1,1726	1,0000	1,0621	1,0767	1,1945	1,2231	1,0000	1,0000
1996	1,2034	1,2025	1,1278	1,2729	1,1684	1,3617	1,1617	1,3268	1,0000	1,0222	1,1681	1,1721	1,0000	1,0717	1,0881	1,1888	1,2887	1,0000	1,0000
1997	1,2115	1,2117	1,1401	1,2747	1,1741	1,3432	1,1795	1,3403	1,0000	1,0298	1,1988	1,1712	1,0000	1,0815	1,0983	1,1758	1,3383	1,0000	1,0000
1998	1,2145	1,2256	1,1507	1,2694	1,1901	1,3484	1,1812	1,3307	1,0000	1,0399	1,2498	1,1762	1,0000	1,1033	1,1111	1,1608	1,3693	1,0000	1,0000
1999	1,2216	1,2261	1,1608	1,2615	1,2047	1,3430	1,1820	1,3364	1,0000	1,0549	1,3074	1,1806	1,0000	1,1317	1,1063	1,1476	1,4011	1,0000	1,0000
2000	1,2244	1,2268	1,1629	1,1851	1,2158	1,3383	1,1811	1,3314	1,0000	1,0699	1,3567	1,1915	1,0000	1,1541	1,0987	1,1355	1,4318	1,0000	1,0000
2001	1,2317	1,2249	1,1623	1,0000	1,2206	1,3340	1,1798	1,3245	1,0000	1,0814	1,3856	1,1930	1,0000	1,1697	1,0964	1,1240	1,4393	1,0000	1,0000
2002	1,2309	1,2277	1,1722	1,0000	1,2265	1,3294	1,1815	1,3135	1,0000	1,0917	1,4128	1,2028	1,0000	1,1826	1,0990	1,1128	1,4516	1,0000	1,0000
2003	1,2296	1,2270	1,1812	1,0000	1,2320	1,3238	1,1850	1,3065	1,0000	1,0985	1,4302	1,2160	1,0000	1,1957	1,0881	1,1000	1,4686	1,0000	1,0000
2004	1,2288	1,2421	1,1881	1,0000	1,2376	1,3177	1,1875	1,3084	1,0000	1,1045	1,4384	1,2217	1,0000	1,2067	1,0683	1,0880	1,4848	1,0000	1,0000

Table 3: Distances to the World Technology Frontier. Bootstrap-based bias-corrected Debreu-Farrell efficiency measures.

	Australia	Austria	Belgium	Canada	Denmark	Finland	France	Greece	Ireland	Italy	Japan	Netherlands	Norway	Portugal	Spain	Sweden	Switzerland	UK	USA
1980	1,2087	1,1391	1,0730	1,0869	1,2161	1,2721	1,1362	1,2169	1,0389	1,0364	1,1976	1,0609	1,0340	1,0331	1,0293	1,1769	1,0543	1,0125	1,0589
1981	1,2095	1,1282	1,0812	1,0876	1,2202	1,2671	1,1348	1,2032	1,0358	1,0299	1,1980	1,0609	1,0379	1,0485	1,0289	1,1789	1,0590	1,0173	1,0576
1982	1,2265	1,1323	1,0852	1,1440	1,2047	1,2595	1,1293	1,2302	1,0359	1,0251	1,1952	1,0856	1,0353	1,0625	1,0293	1,1782	1,0593	1,0209	1,0530
1983	1,2573	1,1330	1,0908	1,1502	1,1894	1,2560	1,1285	1,2340	1,0425	1,0233	1,1809	1,1201	1,0348	1,0476	1,0288	1,1716	1,0570	1,0228	1,0602
1984	1,2384	1,1293	1,0868	1,1448	1,1643	1,2493	1,1299	1,2371	1,0473	1,0217	1,1732	1,1221	1,0430	1,0415	1,0247	1,1611	1,0569	1,0227	1,0745
1985	1,2208	1,1295	1,0803	1,1442	1,1411	1,2481	1,1332	1,2325	1,0310	1,0258	1,1611	1,1343	1,0571	1,0313	1,0210	1,1526	1,0596	1,0237	1,0741
1986	1,2116	1,1251	1,0785	1,1576	1,1241	1,2502	1,1347	1,2360	1,0575	1,0291	1,1498	1,1428	1,0627	1,0205	1,0233	1,1557	1,0608	1,0243	1,0776
1987	1,2220	1,1285	1,0758	1,1768	1,1203	1,2437	1,1326	1,2354	1,0891	1,0284	1,1493	1,1719	1,0686	1,0183	1,0223	1,1595	1,0605	1,0239	1,0809
1988	1,2215	1,1372	1,0723	1,1874	1,1253	1,2390	1,1324	1,2336	1,1055	1,0313	1,1392	1,1746	1,0614	1,0189	1,0276	1,1611	1,0604	1,0241	1,0830
1989	1,2148	1,1434	1,0727	1,2060	1,1302	1,2389	1,1266	1,2432	1,1128	1,0293	1,1263	1,1609	1,0444	1,0214	1,0330	1,1613	1,0612	1,0214	1,0796
1990	1,2222	1,1471	1,0787	1,2363	1,1476	1,2641	1,1256	1,2552	1,1130	1,0312	1,1139	1,1469	1,0399	1,0226	1,0338	1,1676	1,0612	1,0221	1,0757
1991	1,2482	1,1420	1,0796	1,2822	1,1461	1,3070	1,1268	1,2915	1,1188	1,0346	1,1037	1,1493	1,0453	1,0297	1,0260	1,1768	1,0614	1,0281	1,0690
1992	1,2545	1,1421	1,0868	1,3016	1,1515	1,3584	1,1323	1,2991	1,1163	1,0294	1,1022	1,1518	1,0575	1,0376	1,0415	1,2037	1,0484	1,0220	1,0784
1993	1,2491	1,1503	1,0996	1,3008	1,1632	1,4018	1,1472	1,3104	1,1200	1,0317	1,1119	1,1662	1,0740	1,0441	1,0670	1,2338	1,0831	1,0113	1,0902
1994	1,2339	1,1706	1,1165	1,2996	1,1623	1,4076	1,1627	1,3161	1,1288	1,0345	1,1370	1,1850	1,0916	1,0588	1,0829	1,2241	1,1446	1,0182	1,1014
1995	1,2097	1,1856	1,1211	1,2889	1,1752	1,3879	1,1656	1,3277	1,1386	1,0325	1,1605	1,1811	1,1078	1,0760	1,0905	1,2028	1,3005	1,0217	1,1093
1996	1,2093	1,2070	1,1326	1,2946	1,1828	1,3707	1,1771	1,3334	1,1410	1,0338	1,1865	1,1813	1,1369	1,0850	1,1014	1,2032	1,3542	1,0238	1,1078
1997	1,2189	1,2180	1,1457	1,3042	1,1916	1,3499	1,1984	1,3508	1,1430	1,0411	1,2225	1,1826	1,1427	1,0959	1,1114	1,1889	1,3898	1,0260	1,1105
1998	1,2263	1,2360	1,1588	1,3075	1,2134	1,3563	1,2038	1,3395	1,1541	1,0514	1,2797	1,1912	1,1527	1,1230	1,1284	1,1692	1,4172	1,0261	1,1093
1999	1,2368	1,2389	1,1716	1,3119	1,2315	1,3576	1,2076	1,3470	1,0939	1,0660	1,3414	1,1978	1,1583	1,1517	1,1288	1,1566	1,4446	1,0247	1,0985
2000	1,2421	1,2397	1,1750	1,2685	1,2447	1,3589	1,2097	1,3419	1,0835	1,0809	1,3949	1,2088	1,1603	1,1743	1,1203	1,1470	1,4726	1,0243	1,0887
2001	1,2489	1,2377	1,1738	1,0985	1,2491	1,3557	1,2068	1,3340	1,0708	1,0934	1,4226	1,2094	1,1595	1,1918	1,1165	1,1367	1,4776	1,0224	1,0774
2002	1,2486	1,2400	1,1833	1,1576	1,2533	1,3519	1,2062	1,3241	1,0670	1,1053	1,4506	1,2181	1,1675	1,2034	1,1188	1,1263	1,4838	1,0215	1,0707
2003	1,2487	1,2401	1,1936	1,1603	1,2538	1,3458	1,2062	1,3148	1,0668	1,1128	1,4677	1,2320	1,1753	1,2153	1,1090	1,1134	1,4943	1,0226	1,0703
2004	1,2479	1,2543	1,2016	1,1558	1,2579	1,3392	1,2071	1,3149	1,0774	1,1173	1,4754	1,2348	1,1719	1,2283	1,0869	1,1010	1,5080	1,0241	1,0718

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Table 4: Potential output based on bias-corrected Debreu-Farrell efficiency measures.

	Australia	Austria	Belgium	Canada	Denmark	Finland	France	Greece	Ireland	Italy	Japan	Netherlands	Norway	Portugal	Spain	Sweden	Switzerland	UK	USA
1980	62,26	57,83	63,57	61,09	56,70	49,47	62,35	57,10	42,13	55,47	50,58	62,50	61,24	30,47	50,57	52,34	62,27	45,18	65,99
1981	63,12	58,26	65,42	61,36	57,85	50,35	63,38	56,51	43,19	56,01	51,78	61,64	62,64	31,62	52,35	52,93	62,74	46,35	66,44
1982	64,88	59,50	66,95	65,08	58,10	51,15	64,20	57,77	44,43	56,54	52,81	62,53	63,70	32,74	54,13	53,55	62,89	47,67	66,89
1983	67,45	60,62	68,48	66,29	58,33	52,21	65,34	58,10	46,05	57,27	53,38	64,51	64,95	32,97	55,83	54,04	62,93	49,04	68,33
1984	67,35	61,49	69,34	67,01	58,07	53,28	66,68	58,66	47,72	58,16	54,40	65,10	66,80	33,46	57,26	54,44	63,15	50,32	70,42
1985	67,21	62,59	70,00	68,00	57,86	54,74	68,22	59,09	48,54	59,53	55,37	66,60	69,02	33,81	58,55	54,93	63,50	51,63	71,62
1986	67,40	63,46	70,98	69,68	57,94	56,49	69,76	60,07	51,52	60,99	56,51	68,01	70,71	34,13	59,99	55,94	63,66	52,82	73,03
1987	68,60	64,87	71,97	71,55	58,69	57,94	71,16	60,97	55,01	62,27	58,27	70,71	72,49	34,73	61,09	56,91	63,67	53,75	74,35
1988	69,20	66,68	73,00	72,76	59,93	59,45	72,73	61,91	57,96	63,69	59,53	71,83	73,55	35,42	62,53	57,70	63,69	54,47	75,53
1989	69,56	68,39	74,32	74,33	61,21	61,08	73,87	63,42	60,52	64,67	60,48	71,87	74,14	36,18	64,07	58,40	63,79	54,86	76,27
1990	70,97	69,86	76,04	76,61	63,24	63,89	75,19	64,95	62,67	65,74	61,15	71,78	75,84	36,90	65,47	59,52	63,85	55,47	76,94
1991	73,81	70,65	77,36	80,03	64,31	67,76	76,49	67,52	65,05	66,88	61,60	72,63	78,47	37,83	66,48	61,06	63,96	56,61	77,45
1992	75,79	71,66	79,10	82,09	65,83	72,50	77,99	68,32	66,90	67,62	62,25	73,46	81,78	38,80	69,11	63,90	63,42	57,41	79,23
1993	77,21	73,23	81,26	83,17	67,76	77,30	80,09	69,12	69,18	69,08	63,39	75,15	85,51	39,73	72,41	67,33	65,96	58,19	81,32
1994	78,03	75,78	83,79	84,39	69,01	80,26	82,26	69,66	71,95	70,72	65,43	77,23	89,32	40,99	74,84	68,85	70,33	60,08	83,49
1995	78,23	78,31	85,43	85,08	71,10	81,66	83,55	70,75	75,06	72,00	67,41	77,90	92,94	42,36	76,31	69,72	80,75	61,77	85,59
1996	79,97	81,57	87,62	86,93	72,88	83,01	85,51	71,86	77,89	73,30	69,53	78,89	97,54	43,43	77,61	71,77	85,05	63,31	87,16
1997	82,42	84,34	89,94	89,18	74,74	83,92	88,24	73,94	80,83	74,77	72,18	79,97	100,00	44,59	78,55	72,80	88,34	64,80	89,24
1998	84,68	87,68	92,24	91,12	77,43	86,33	89,79	74,77	84,49	76,21	76,11	81,57	102,68	46,43	79,75	73,28	91,14	66,14	91,12
1999	87,02	89,92	94,48	93,12	79,92	88,26	91,14	76,99	82,83	77,73	80,52	82,98	104,95	48,37	79,63	73,95	93,89	67,37	92,19
2000	88,85	91,83	95,89	91,50	82,11	90,04	92,26	78,79	84,81	79,06	84,75	84,61	106,95	50,10	78,79	74,65	96,58	68,65	93,22
2001	90,70	93,35	96,86	80,24	83,76	91,42	92,92	80,60	86,59	80,00	87,70	85,45	108,83	51,63	78,23	75,29	97,62	69,83	94,00
2002	91,99	95,06	98,73	85,36	85,42	92,76	93,81	82,29	89,09	80,80	90,92	86,95	111,74	52,93	78,04	76,12	98,67	71,07	95,09
2003	93,31	96,52	100,68	86,21	86,88	94,07	94,88	83,92	91,93	81,40	93,67	89,15	114,89	54,26	76,96	77,03	100,07	72,47	96,70
2004	94,59	99,03	102,41	86,51	88,64	95,42	96,17	85,95	95,83	82,08	95,92	90,91	117,18	55,65	75,00	78,12	101,77	73,87	98,43

Figure 4: Growth in output and potential output per capita, USA and Japan.

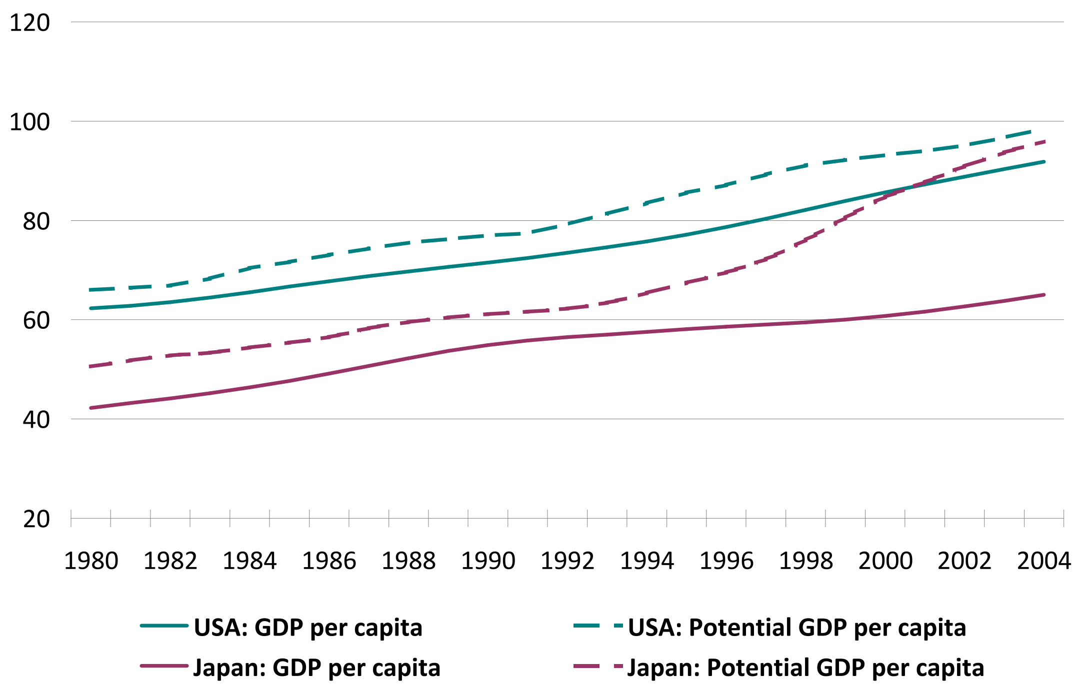
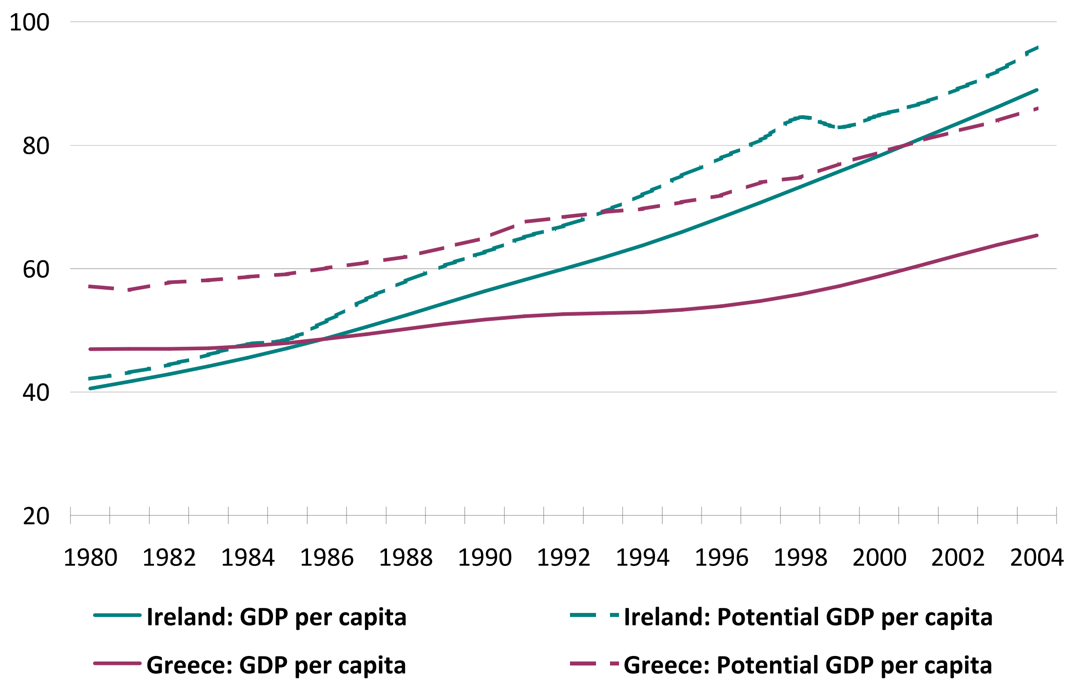


Figure 5: Growth in output and potential output per capita, Greece and Ireland.



Another finding is that DEA consistently diagnozes several countries as 100% efficient throughout the whole period 1980–2004: USA, Norway, UK (excluding 1990-93), and Ireland (excluding 1980-84). We may also single out another group of countries including Switzerland and Italy: they were efficient in the first half of the sample, but then they became increasingly inefficient in the second half of the sample (since 1993). Moreover, most countries recorded divergence with respect to the frontier in the considered period, especially after 1995, but there is a number of notable exceptions to this rule, including Sweden, Canada (efficient since 2001) and to a lesser extent – Greece and Spain, which recorded some convergence.[18]

Complementing DEA with the Simar–Wilson bootstrap eliminates the possibility of 100% efficiency, but otherwise does not change this picture much. The list of interesting differences includes Norway (which joins the "club" of countries gradually diverging from the frontier), and the USA and Ireland (which observed a decrease in efficiency around 1995-98 but then returned to their usual efficiency levels).

3.4 Parametric estimates of the aggregate production function

Before we put our parametric estimates and their nonparametric counterparts into "competition", let us also present the numerical results obtained for the parametric case.

To this end, three alternative parametric, SFA-based estimates of the aggregate production function have been contained in Table 5. Reported estimates of the parameter η refer to the technical change parameter in the Battese and Coelli (1995) intertemporal component of the inefficiency term z_t, $\lambda = \varphi^{-1}$ is the mean of the distribution of its time-invariant component u_i, whereas σ^2 refers to the estimated variance of the idiosyncratic error term v_{it}.

Under every Cobb–Douglas specification (the unrestricted case and the CRS case), the partial elasticity with respect to capital is estimated at $0.6 - 0.7$, which is a large number. The partial elasticity with respect to unskilled labor, on the other hand, is very low (between 0.05 and 0.1) and only marginally significantly different from zero. The estimated measure of scale elasticity is slightly less than unity, suggesting (mildly) decreasing returns at the aggregate level. Standard errors of estimation suggest that it might be indistinguishable from unity (representing constant returns), though.

Turning to the estimates of the translog production function reported in Table 5, we observe that the quadratic terms in the translog are generally statistically insignificant. Hence, because of this statistical imprecision, at this point we cannot infer if the departures from the Cobb–Douglas benchmark are economically important or not.

[18]Please recall that our dataset ends in 2004.

Furthermore, in the translog case, aggregate returns to scale, when not restricted to be constant, can be country-specific. The same applies to partial elasticities and elasticities of substitution – their magnitude will vary across countries and time. We shall document these meaningful variations in the following subsection.

At the general level, however, it should be noted that the average values of partial elasticities obtained under the translog specifications are somewhat closer to the ones postulated in related literature yet still far from consistent. The capital elasticity is estimated around 0.6 (i.e., way more than $\frac{1}{3}$ suggested by, e.g., Kydland and Prescott, 1982), the skilled labor elasticity – around 0.25, and the unskilled elasticity – around 0.1.

Table 5: Parameters of the estimated production functions (SFA).

	SFA-CD (K,Hu,Hs)	s.e.	SFA-CD (K,Hu,Hs) [CRS]	s.e.	SFA-Translog (K,Hu,Hs) [CRS]	s.e.	SFA-Translog (K,Hu,Hs)	s.e.
Constant	1,0050	0,6509	0,1892	0,2292	-0,8686	2,1510	12,9300	10,8300
LNK	0,7321	0,0520					2,1530	1,4820
LNHU	0,0565	0,0348					-2,0640	0,9748
LNHS	0,1715	0,0410					-0,9649	1,0230
LNK_INT			0,7628	0,0514	1,3330	0,9506		
LNHS_INT			0,1435	0,0398	-0,0802	0,6068		
LNK2							-0,2901	0,2172
LNHU2							0,0919	0,0559
LNHs2							-0,0070	0,0930
LNK2_INT					-0,1571	0,2110		
LNHS2_INT					0,0371	0,0940		
LNKHS_INT					0,0671	0,1349		
LNKHU							0,1238	0,1065
LNKHS							0,1520	0,1376
LNHUHS							-0,1151	0,0777
eK	0,7321		0,7628		~ 0,6115		~ 0,6048	
eHu	0,0565		0,1435		~ 0,1226		~ 0,1009	
eHs	0,1715				~ 0,2659		~ 0,2514	
eS	0,9601		1,0000		1,0000		0,9571	
η	-0,1385	0,0377	-0,1735	0,0339	-0,2136	0,0625	-0,2205	0,0720
σ^2	0,0039	0,0008	0,0042	0,0009	0,0047	0,0011	0,0042	0,0010
λ	4,1630	1,1710	4,9300	1,2830	6,7160	2,3660	6,3080	2,5690

The symbol ~ denotes the average value of the relevant (country-specific) elasticity.

It is worthwhile to comment on our estimates of η under the Cobb–Douglas and translog specifications. In both cases, the data suggest gradual divergence of actual productivity from the WTF, i.e., on average, OECD countries are found to systematically lag behind the frontier.

One may also draw a few conclusions on the preferred shape of the aggregate production function by comparing potential output excluding the idiosyncratic disturbance term, $Y_{it}^{**} = \theta_{it}F(K_{it}, H_{it}^U, H_{it}^S)$, to each country's actual output Y_{it}. Ratios of form Y^{**}/Y have been presented in Table 6, allowing us to see in which countries departures of output from the assumed functional form (controlling for inefficiency but not idiosyncratic disturbances) are most pronounced. What is especially notable there, there are some strong correlations between these departures and factor endowments. In particular, for all tested production functions, departures from the function are positively correlated with the stock of physical capital. Correlation with output is obvious; its magnitude varies across proposed specifications, though, being somewhat less pronounced for the cases of CRS Cobb–Douglas and unrestricted translog.

Table 6: Departures from the parametric production function in 2000, controlling for technical inefficiency.

	SFA-CD (K,Hu,Hs)	SFA-CD (K,Hu,Hs) [CRS]	SFA-Translog (K,Hu,Hs) [CRS]	SFA-Translog (K,Hu,Hs)	mean
Australia	1,1129	1,2233	1,1092	1,0547	1,1250
Austria	1,1306	1,2146	1,0972	1,0864	1,1322
Belgium	1,1184	1,1945	1,0767	1,0362	1,1065
Canada	1,1231	1,1702	1,1365	1,1057	1,1339
Denmark	1,0669	1,1536	1,0396	1,0787	1,0847
Finland	1,1042	1,2452	1,0984	1,1186	1,1416
France	1,0900	1,2088	1,0982	1,0549	1,1130
Greece	1,1362	1,2724	1,1417	1,0936	1,1610
Ireland	0,9462	0,9461	0,8635	0,9356	0,9229
Italy	1,0934	1,2350	1,0980	1,1177	1,1360
Japan	1,2320	1,5028	1,2797	1,2822	1,3242
Netherlands	1,1239	1,2888	1,1143	1,0643	1,1478
Norway	1,0029	1,0963	0,8990	1,0026	1,0002
Portugal	1,1588	1,1473	1,0759	1,0413	1,1058
Spain	1,1893	1,2397	1,1312	1,1045	1,1662
Sweden	1,0284	1,1031	1,0338	1,0498	1,0538
Switzerland	1,2884	1,4762	1,2586	1,2976	1,3302
UK	0,9777	0,9951	0,9801	0,9411	0,9735
USA	0,9363	0,9488	0,9446	0,8993	0,9322
mean	1,0979	1,1927	1,0777	1,0718	1,1100
corr.with K/L	0,2529	0,4307	0,2452	0,3865	0,3498
corr.with Hu/L	0,2254	0,1186	0,0883	0,0400	0,1192
corr.with Hs/L	-0,0821	0,0811	0,1656	0,1065	0,0752
corr.with Y/L	-0,5055	-0,3724	-0,5042	-0,4040	-0,4511

After a brief presentation of our estimation results, let us pass to the presentation of their implications for the shape of the aggregate production function. In the following section, we shall dwell more on the discrepancies between the nonparametric DEA outcomes discussed above and their SFA counterparts obtained under the parametric assumption of a Cobb–Douglas or translog production function.

4 Implications for the shape of the aggregate production function

Our estimates provide testable implications on the following properties of the aggregate production function:

1. *Implied efficiency levels.* How far is a given country in a given year from the WTF if the latter takes the given functional form? Is there any congruence of these distance measures across different specifications?

2. *Partial elasticities.* Are partial elasticities constant (as in the Cobb–Douglas specification)? If not, are they *systematically* related to inputs? If so, what is the pattern of dependence? Do we observe meaningful shifts in partial elasticities across time (e.g., as in the case where technical change favors some factors at the expense of others)? Do the observed regularities agree or disagree with the hypothesis of skill-biased technical change?

3. *Returns to scale.* For each given country and year, can returns to scale be diagnozed as constant, decreasing or increasing? Viewed globally, are they constant or variable?

4. *Elasticities of substitution.* Are Morishima and Allen–Uzawa (two-factor) elasticities of substitution constant across countries and time (as they are in the Cobb–Douglas and CES specifications)? If not, can we observe indications of greater complementarity or substitutability of certain inputs in certain countries? Do the observed regularities agree or disagree with the hypothesis of capital-skill complementarity?

Our first broad finding is that the CRS Cobb–Douglas specification is the one which most frequently fails in our tests. Our data provide several arguments against its validity, corroborating the preliminary evidence illustrated in the previous section.

We are however not able to offer an alternative *parametric* form of the function that would be in good agreement with nonparametric (bias-corrected) DEA results. In particular, our SFA-based estimates of translog production functions indicate visible departures of this particular functional specification from the DEA results, too: the discrepancy pertains to implied efficiency levels, identified partial elasticities, and returns-to-scale properties. On the other hand, the same translog estimations provide a strong argument why the CRS Cobb–Douglas is too simple a specification to match the complex patterns present in the data: partial elasticities vary substantially across countries, they are heavily correlated with factor endowments, and a number of Morishima (and Allen–Uzawa) elasticities of substitution are far away from unity.

The available evidence on constant vs. variable returns to scale is ambiguous. In a series of DEA-based tests of *local* returns to scale (in a given country and year), the null of their constancy is relatively rarely rejected (although some countries do exhibit

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decreasing, rather than constant returns to scale, throughout the whole considered period). In a test of *global* constancy of returns to scale, however, the null of constant returns to scale can be rejected against the alternative of variable returns to scale with 99% confidence. Unlike the DEA, the translog specification diagnozes a sharp pattern of dependence of returns to scale on the size of the economy.

4.1 Implied efficiency levels

Table 7 presents a comparison of seven different characterizations of the World Technology Frontier in the year 2000, computed on the basis of data for 1970–2000. In consecutive columns, we document Debreu–Farrell efficiency measures θ_i (such that potential output of country i at WTF is $Y_i^* = \theta_i Y_i$) computed according to the following methodologies:

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1. Bias-corrected DEA with constant returns to scale, and aggregate capital and (raw) labor taken as inputs.

2. Bias-corrected DEA with variable returns to scale, and aggregate physical and human capital as inputs.

3. Bias-corrected DEA with variable returns to scale, and aggregate physical capital as well as unskilled and skilled labor as inputs (the difference between these estimates and the aforementioned ones capture the degree of imperfect substitutability between unskilled and skilled labor).

4. SFA under the assumption of a Cobb–Douglas production function with variable returns to scale and aggregate physical capital as well as unskilled and skilled labor as inputs.

5. SFA under the assumption of a Cobb–Douglas production function with constant returns to scale and aggregate physical capital as well as unskilled and skilled labor as inputs.

6. SFA under the assumption of a translog production function with constant returns to scale and aggregate physical capital as well as unskilled and skilled labor as inputs, estimated in an intensive form: $\frac{Y}{H^U} = F\left(\frac{K}{H^U}, \frac{H^S}{H^U}\right)$.

7. SFA under the assumption of a translog production function with variable returns to scale and aggregate physical capital as well as unskilled and skilled labor as inputs.

In Table 7, we report correlations between efficiency indexes computed on the basis of each specification. What is crucial here is that in the cross-sectional dimension, DEA-based and SFA-based predictions on technical efficiency are quite strongly positively correlated.[19] Broadly the same group of countries is found to be closest to the frontier in all considered cases: Ireland, UK, and USA, and broadly the same group of countries lags behind by most: Finland, Greece, Japan, and Switzerland.

Table 7: Technical efficiency – comparison of alternative measurements for the year 2000.

	DEA (K,L)	DEA (K,H)	DEA (K,Hu,Hs)	SFA-CD (K,Hu,Hs)	SFA-CD (K,Hu,Hs) [CRS]	SFA-Translog (K,Hu,Hs) [CRS]	SFA-Translog (K,Hu,Hs)	mean
Australia	1,2021	1,2381	1,2421	1,2029	1,0438	1,0979	1,0810	1,1583
Austria	1,1628	1,2407	1,2397	1,1959	1,0395	1,0869	1,0857	1,1502
Belgium	1,0734	1,1972	1,1750	1,1506	1,0328	1,0655	1,0580	1,1075
Canada	1,2075	1,2700	1,2685	1,1599	1,0320	1,0772	1,0590	1,1535
Denmark	1,2168	1,2248	1,2447	1,2376	1,0466	1,1070	1,1296	1,1724
Finland	1,2469	1,3526	1,3589	1,3015	1,0634	1,1446	1,1688	1,2338
France	1,1406	1,2041	1,2097	1,1552	1,0421	1,0884	1,0876	1,1326
Greece	1,3207	1,3532	1,3419	1,2452	1,0553	1,1263	1,1231	1,2237
Ireland	1,0133	1,0635	1,0835	1,1174	1,0166	1,0324	1,0922	1,0598
Italy	1,1824	1,2216	1,0809	1,1355	1,0424	1,0818	1,0870	1,1188
Japan	1,4641	1,4780	1,3949	1,2724	1,0728	1,1686	1,1991	1,2928
Netherlands	1,2522	1,2683	1,2088	1,2641	1,0637	1,1250	1,1016	1,1834
Norway	1,0377	1,0574	1,1603	1,2700	1,0563	1,1038	1,1409	1,1180
Portugal	1,2806	1,2104	1,1743	1,0205	1,0035	1,0118	1,0198	1,1030
Spain	1,1920	1,1944	1,1203	1,0346	1,0122	1,0243	1,0221	1,0857
Sweden	1,1964	1,1791	1,1470	1,2281	1,0476	1,1052	1,1073	1,1444
Switzerland	1,4088	1,4592	1,4726	1,3335	1,0688	1,1369	1,1547	1,2907
UK	1,0140	1,0263	1,0243	1,0396	1,0109	1,0235	1,0229	1,0231
USA	1,0104	1,1210	1,0887	1,0152	1,0051	1,0181	1,0355	1,0420
Corr. with DEA	0,8222	0,9110	1,0000	0,7615	0,7421	0,7908	0,7189	
RMSE Dev. / DEA	0,0748	0,0520	0,0000	0,0822	0,1993	0,1518	0,1467	
Corr. with SFA-TL	0,5566	0,6202	0,7189	0,9014	0,9027	0,9139	1,0000	
RMSE Dev. / SFA-TL	0,1428	0,1662	0,1467	0,1009	0,0625	0,0217	0,0000	

We do find some meaningful discrepancies, however. Firstly, in the case of CRS Cobb–Douglas (and only in that case), all countries are found to be close to the frontier (less than 10% inefficiency) and there is little variation across countries. This suggests potential difficulties in identifying the inefficiency distribution u_{it} under this functional specification, and thus indirectly provides evidence against it. Secondly, the correlation between the CRS translog and unrestricted translog results is very strong, suggesting that there are only minor departures from CRS in such case. Thirdly, correlations are generally stronger within DEA and SFA estimates than between these two groups, suggesting that functional specification of the aggregate production function is at least as important than the choice of its inputs.

Furthermore, treating the bootstrap-augmented DEA with $\theta_{DEA}(K, H^U, H^S)$ and the unrestricted translog SFA with $\theta_{TL}(K, H^U, H^S)$ as two "benchmarks", representing the most general, nesting specifications in each of the two approaches, we have also computed the RMSE distance measures, quantifying the differences in predicted

[19] We do not compare our DEA and SFA results across the time-series dimension here because, due to reasons discussed in preceding sections, our SFA results are based on data of decadal frequency only. Hence, only three observations are available across time (for 1980, 1990 and 2000) which is too little to draw reliable conclusions.

Debreu–Farrell technical inefficiency measures obtained from alternative methodologies. The results are in line with expectations: the distances are largest between the two general methodologies (DEA/SFA), and within these methodologies, the distances are the larger, the simpler is the measure in question (with the unrestricted Cobb–Douglas being an exception to this rule).

Table 8 complements Table 7 by presenting a comparison of seven alternative estimates of potential output per worker. Departures of parametric SFA results (both Cobb–Douglas and translog) from the DEA ones are visible but not dramatic. The discrepancy is the strongest with respect to the estimates of the CRS Cobb–Douglas function.

To make sure, stochastic estimates included in Table 8 are computed as

$$Y_{it}^{*} = \theta_{it} Y_{it} = \theta_{it} F(K_{it}, H_{it}^{U}, H_{it}^{S}) \exp\left(v_{it}\right),$$

where $\theta_{it} = \exp(-u_{it})$ is the Debreu-Farrell efficiency measure reported above. Hence by definition these estimates contain the idiosyncratic disturbance term v_{it} as well. Knowing that this term can dominate the result if the postulated functional form of the aggregate production function provides a bad fit to the data, and wanting to get rid of this feature of our results, we have also computed potential output according to:

$$Y_{it}^{**} = \theta_{it} F(K_{it}, H_{it}^{U}, H_{it}^{S}),$$

so that the idiosyncratic disturbance term is not included. The respective results are presented in Table 9 where it is observed that the differences between the DEA and SFA results are much smaller if idiosyncratic disturbances are not included. One has to keep in mind, though, that under production function misspecification, especially likely in the CRS Cobb–Douglas case, numbers reported in SFA columns of Table 9 will be biased. The reason is that they do not represent inefficiency-corrected measures of *actual* output, but of output *as if* the estimated production function provided a perfect fit to the data, which it likely does not.

Differences across different production function specifications, documented in Tables 7–9, suggest that the parametric functional forms used in our SFA analyses, especially the CRS Cobb–Douglas ones, are likely to be somewhat misspecified. They also constitute suggestive evidence that allowing for imperfect substitutability between unskilled and skilled labor helps obtain significantly different (and thus certainly better, since this step allows for more generality) results, supporting the related findings by Growiec (2010, 2012).

On the other hand, the discrepancy between our DEA and SFA results could also indicate that the former method provides a relatively rough approximation of the WTF due to, e.g., sharp underrepresentation of certain input–output mixes in our dataset (see Growiec, 2012).

In sum, despite several important differences listed above, the ranking of countries in terms of their technical efficiency is similar under all functional specifications of the WTF. Hence, according to this test, the translog production function and the nonparametric frontier seem to identify a similar *location* of the WTF. Let us now assess its *curvature* properties.

Table 8: Potential output – comparison of alternative estimates. Stochastic estimates include inefficiency and idiosyncratic errors.

	DEA (K,L)	DEA (K,H)	DEA (K,Hu,Hs)	SFA-CD (K,Hu,Hs)	SFA-CD (K,Hu,Hs) [CRS]	SFA-Translog (K,Hu,Hs) [CRS]	SFA-Translog (K,Hu,Hs)	mean
Australia	85,99	88,57	88,85	86,05	74,67	78,54	77,33	82,8567
Austria	86,14	91,91	91,83	88,59	77,00	80,51	80,42	85,1998
Belgium	87,60	97,69	95,89	93,89	84,28	86,95	86,34	90,3768
Canada	87,10	91,61	91,50	83,67	74,44	77,70	76,39	83,2018
Denmark	80,28	80,81	82,11	81,65	69,05	73,03	74,52	77,3491
Finland	82,61	89,62	90,04	86,23	70,45	75,84	77,44	81,7467
France	86,99	91,83	92,26	88,10	79,48	83,01	82,95	86,3726
Greece	77,54	79,45	78,79	73,11	61,96	66,13	65,94	71,8476
Ireland	79,31	83,25	84,81	87,46	79,57	80,82	85,49	82,9592
Italy	86,48	89,35	79,06	83,05	76,24	79,13	79,50	81,8308
Japan	88,96	89,80	84,75	77,31	65,18	71,00	72,85	78,5501
Netherlands	87,65	88,77	84,61	88,48	74,45	78,74	77,11	82,8320
Norway	95,65	97,47	106,95	117,07	97,37	101,74	105,17	103,0599
Portugal	54,63	51,63	50,10	43,54	42,81	43,16	43,51	47,0539
Spain	83,84	84,00	78,79	72,77	71,18	72,04	71,88	76,3560
Sweden	77,87	76,74	74,65	79,93	68,18	71,93	72,07	74,4804
Switzerland	92,40	95,70	96,58	87,45	70,09	74,56	75,73	84,6452
UK	67,96	68,79	68,65	69,68	67,76	68,60	68,56	68,5712
USA	86,52	95,99	93,22	86,93	86,07	87,18	88,67	89,2248
Corr. with DEA	0,9184	0,9494	1,0000	0,9254	0,8521	0,8973	0,8864	
RMSE Dev. / DEA	5,5053	3,9030	0,0000	5,4995	13,3124	10,0614	9,7482	
Corr. with SFA-TL	0,8072	0,8347	0,8864	0,9608	0,9833	0,9916	1,0000	
RMSE Dev. / SFA-TL	9,0778	11,0974	9,7482	7,1465	4,3597	1,6533	0,0000	

Table 9: Potential output – comparison of alternative estimates. Stochastic estimates include inefficiency but not the idiosyncratic errors.

	DEA (K,L)	DEA (K,H)	DEA (K,Hu,Hs)	SFA-CD (K,Hu,Hs)	SFA-CD (K,Hu,Hs) [CRS]	SFA-Translog (K,Hu,Hs) [CRS]	SFA-Translog (K,Hu,Hs)	mean
Australia	85,99	88,57	88,85	95,76	91,34	87,11	81,56	88,4554
Austria	86,14	91,91	91,83	100,15	93,52	88,34	87,37	91,3228
Belgium	87,60	97,69	95,89	105,01	100,68	93,62	89,46	95,7065
Canada	87,10	91,61	91,50	93,97	87,11	88,31	84,46	89,1510
Denmark	80,28	80,81	82,11	87,11	79,65	75,92	80,38	80,8942
Finland	82,61	89,62	90,04	95,22	87,73	83,30	86,63	87,8765
France	86,99	91,83	92,26	96,03	96,07	91,16	87,50	91,6912
Greece	77,54	79,45	78,79	83,07	78,84	75,50	72,12	77,9021
Ireland	79,31	83,25	84,81	82,76	75,29	69,79	79,98	79,3121
Italy	86,48	89,35	79,06	90,80	94,16	86,88	88,86	87,9423
Japan	88,96	89,80	84,75	95,24	97,96	90,86	93,42	91,5687
Netherlands	87,65	88,77	84,61	99,44	95,96	87,74	82,07	89,4644
Norway	95,65	97,47	106,95	117,41	106,75	91,47	105,44	103,0207
Portugal	54,63	51,63	50,10	50,45	49,11	46,44	45,30	49,6670
Spain	83,84	84,00	78,79	86,54	88,25	81,49	79,39	83,1846
Sweden	77,87	76,74	74,65	82,20	75,21	74,36	75,66	76,6692
Switzerland	92,40	95,70	96,58	112,68	103,47	93,85	98,26	98,9913
UK	67,96	68,79	68,65	68,12	67,42	67,24	64,52	67,5301
USA	86,52	95,99	93,22	81,39	81,66	82,35	79,74	85,8386
Corr. with DEA	0,9184	0,9494	1,0000	0,9093	0,8613	0,8704	0,9057	
RMSE Dev. / DEA	5,5053	3,9030	0,0000	8,5495	7,1559	6,6828	5,9600	
Corr. with SFA-TL	0,9552	0,9137	0,9057	0,9502	0,9449	0,9110	1,0000	
RMSE Dev. / SFA-TL	4,7802	6,2994	5,9600	9,7674	6,4254	5,1412	0,0000	

4.2 Partial elasticities

Another test, aiming at defining the desirable properties of the aggregate production function, is to check if its partial elasticities tend to vary across countries and time if they are not restricted against such behavior.[20] To this end, we have computed the partial elasticities of the aggregate production function both with DEA and SFA (under the translog specification).

In the DEA approach, partial elasticities have been computed on the basis of the solution to each unit's optimal program. Knowing its maximum attainable output given inputs as well as the neighboring efficient units, we have identified each of its partial elasticities on the basis of the local slope of the (piecewise linear) production function, projected along the axis associated with the respective factor of production.

We have also endowed these partial elasticities with confidence intervals and corrected them for the DEA bias using the Simar–Wilson bootstrap. Unfortunately, these augmented results are somewhat less convincing than original ones. The reason is that whereas DEA guarantees the production function to be increasing and concave, the bootstrap-based production function need not satisfy these properties. It turns out that when the bootstrap predicts large DEA biases, it also suggests unacceptably high partial elasticities there, in line with local convexity of the function. For this reason, we have decided to concentrate only on the original DEA results in the current discussion.

Another related issue is that under the DEA approach, efficient units are located in vertices of the technology set. For such units, left-sided and right-sided partial elasticities do not coincide. We have decided to report only the right-sided partial elasticities here (i.e., percentage changes in output given a 1% *increase* in the respective input, holding everything else constant).[21]

Tables 10–11, based on the DEA approach, document a negative correlation between the estimated partial elasticities and the scale of the economy. Hence, they confirm that the nonparametric WTF has more curvature than the Cobb–Douglas production function for which this correlation is zero. Another finding is that while the average partial elasticities are generally in line with the ones present in the established literature, they tend to vary largely across countries and have visible trends across time. The DEA production function specification implies a consistent falling trend in the partial elasticity of unskilled labor,[22] a moderately increasing trend in the physical capital elasticity, and an essentially flat trend in skilled labor elasticity.

[20] In related studies, Bernanke and Gürkaynak (2001) as well as Gollin (2002) have documented substantial variability of capital and labor income shares across countries. Our current exercise, documenting the variability of implied partial elasticities, is complementary to theirs: partial elasticities and factor shares coincide under the Cobb–Douglas specification but the former depend on factor endowments otherwise.

[21] Left-sided partial elasticities as well as partial elasticities based on the Simar–Wilson bootstrap are available from the authors upon request.

[22] The huge drop in this partial elasticity in 1996 remains a caveat, though. We cannot offer an explanation of this apparent "discontinuity" in our results, apart from the fact that it coincides in time with Switzerland and Japan's significant departures from full efficiency.

Table 10: Partial elasticities estimated from the DEA (piecewise linear) production function. Cross-country averages.

	E_K	E_{H^U}	E_{H^S}	Scale
Australia	0,35	0,16	0,48	0,99
Austria	0,39	0,21	0,41	1,01
Belgium	0,32	0,31	0,37	1,00
Canada	0,32	0,21	0,65	0,90
Denmark	0,45	0,20	0,43	1,03
Finland	0,46	0,24	0,32	1,02
France	0,21	0,25	0,51	0,98
Greece	0,47	0,30	0,24	1,01
Ireland	0,26	0,25	0,38	0,37
Italy	0,57	0,00	0,23	0,53
Japan	0,00	0,00	0,75	0,75
Netherlands	0,13	0,35	0,55	0,94
Norway	0,00	0,20	0,32	0,40
Portugal	0,54	0,25	0,15	0,94
Spain	0,34	0,25	0,23	0,65
Sweden	0,55	0,18	0,43	1,03
Switzerland	0,30	0,73	0,57	1,05
UK	0,65	0,06	0,07	0,70
USA	0,00	0,00	0,00	0,00
mean	0,42	0,27	0,40	0,85
corr.with Y/L	-0,19	-0,22	-0,15	-0,39

Note: means have been computed excluding zeros.

The finding that some of the reported right-sided partial elasticities are very low or even zero, is an artifact of the construction of the DEA frontier as a convex hull of observed input-output pairs, with zero slope imposed on the function to the right of the highest efficient unit. Also by construction, left-sided partial elasticities must be greater or equal to the right-sided ones here. Hence, on the basis of right-sided partial elasticities reported here, one cannot make any inference regarding returns to scale. This will be done separately in the following subsection.

Under the translog specification, partial elasticities are computed as follows:

$$EL_K = \frac{\partial \ln Y_{it}}{\partial \ln K_{it}} = \alpha_K + \alpha_{KK} \ln K_{it} + \alpha_{KHu} \ln H_{it}^U + \alpha_{KHs} \ln H_{it}^S, \tag{7}$$

$$EL_{H^U} = \frac{\partial \ln Y_{it}}{\partial \ln H_{it}^U} = \alpha_{Hu} + \alpha_{KHu} \ln K_{it} + \alpha_{HuHu} \ln H_{it}^U + \alpha_{HuHs} \ln H_{it}^S, \tag{8}$$

$$EL_{H^S} = \frac{\partial \ln Y_{it}}{\partial \ln H_{it}^S} = \alpha_{Hs} + \alpha_{KHs} \ln K_{it} + \alpha_{HuHs} \ln H_{it}^U + \alpha_{HsHs} \ln H_{it}^S. \tag{9}$$

They are then, by definition, dependent on factor endowments. It is however not automatically certain that this would generate substantial variability of partial elasticities across countries and time. This is only the case if second-order terms are important in the above specification.

Tables 12–13, based on the translog specification, show partial elasticities do vary strongly across countries and time. Moreover, similar patterns are observed both in the CRS case (estimated according to an intensive form of the translog production function), and in the VRS case.

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In sum, partial elasticities presented above share a few common properties. First, they vary largely across countries and time. Second, they are generally negatively correlated with output per worker (apart from the skilled labor elasticity under the translog specification), indicating that the frontier production function has more curvature than suggested by the Cobb–Douglas production function. Both these findings provide evidence against the latter functional specification.

Third, we also find that the unskilled labor elasticity is robustly falling over time, in line with the concept of *skill-biased technical change*: the larger and more technologically advanced is the economy, the less it relies on unskilled labor for production.

Table 11: Partial elasticities estimated from the DEA (piecewise linear) production function. Annual averages.

	E_K	E_{HU}	E_{HS}	Scale
1980	0,30	0,43	0,39	0,90
1981	0,31	0,41	0,40	0,90
1982	0,30	0,39	0,40	0,88
1983	0,28	0,42	0,39	0,89
1984	0,35	0,38	0,41	0,91
1985	0,33	0,43	0,41	0,93
1986	0,42	0,33	0,41	0,85
1987	0,36	0,33	0,42	0,83
1988	0,35	0,33	0,40	0,82
1989	0,33	0,32	0,42	0,82
1990	0,31	0,29	0,43	0,80
1991	0,31	0,29	0,41	0,83
1992	0,39	0,30	0,43	0,91
1993	0,45	0,30	0,41	0,93
1994	0,47	0,31	0,39	0,94
1995	0,46	0,32	0,40	0,92
1996	0,46	0,15	0,43	0,82
1997	0,50	0,14	0,42	0,80
1998	0,55	0,12	0,38	0,79
1999	0,53	0,12	0,40	0,81
2000	0,50	0,16	0,38	0,81
2001	0,49	0,17	0,37	0,81
2002	0,51	0,14	0,38	0,79
2003	0,51	0,12	0,36	0,77
2004	0,57	0,11	0,34	0,76
mean	0,42	0,27	0,40	0,85
corr.with Y/L	-0,19	-0,22	-0,15	-0,39

Note: means have been computed excluding zeros.

Fourth, this fall is counteracted by respective increases in the skilled labor elasticity (in the translog specification), and also partially by increases in the physical capital elasticity (in the DEA case). Both these trends are in line with the skill-biased technical change hypothesis, too, although the latter is conditional on some degree of capital–skill complementarity. As we shall see shortly, the analysis of Allen–Uzawa and Morishima elasticities of substitution provides evidence of such complementarity. Fifth, we find a marked difference between partial elasticities estimated on the basis of DEA and SFA:

in the former case, partial elasticities are much closer to the benchmark values found in other (not WTF-based) literature (e.g., Kydland and Prescott,1982) than in the latter case. The average capital elasticity is around 0.4 in DEA as compared to 0.6 in SFA translog, and the skilled labor elasticity is around 0.4 in DEA as compared to 0.25 in SFA translog. This could be suggestive of some production function misspecification issues inherent in the parametric estimations.

Table 14 documents the correlations between partial elasticities computed for each of the 19 countries in each of the 25 years in question according to different specifications. Some of coefficients are driven by the relatively poor quality of bootstrap-based elasticity measures (by construction, Simar–Wilson bootstraps are not meant for capturing the curvature of the WTF but for improving the estimates of its location). We also conjecture, however, based on our other results, that (i) the DEA provides biased predictions on the elasticities in the cases of "atypical" units due to its piecewise linear character, (ii) the translog function provides a relatively poor fit to the data in the cases of "typical" units.

Table 12: Partial elasticities estimated from the translog production function. Cross-country averages.

	E_K	E_{HU}	E_{HS}	Scale
Australia	0,61	0,06	0,28	0,95
Austria	0,65	0,02	0,22	0,89
Belgium	0,66	0,09	0,18	0,93
Canada	0,51	0,02	0,40	0,93
Denmark	0,66	-0,04	0,21	0,83
Finland	0,66	0,00	0,18	0,84
France	0,60	0,21	0,29	1,10
Greece	0,67	0,09	0,16	0,92
Ireland	0,72	-0,06	0,12	0,78
Italy	0,57	0,33	0,23	1,13
Japan	0,53	0,28	0,37	1,17
Netherlands	0,56	0,14	0,26	0,96
Norway	0,50	0,01	0,28	0,80
Portugal	0,68	0,15	0,09	0,91
Spain	0,63	0,26	0,18	1,07
Sweden	0,64	-0,01	0,24	0,87
Switzerland	0,49	0,00	0,34	0,83
UK	0,72	0,16	0,23	1,11
USA	0,45	0,20	0,50	1,15
mean	0,60	0,10	0,25	0,96
corr.with Y/L	-0,55	-0,06	0,58	0,05

Note: translog parameters, computed for the 1980–2000 dataset, have been assumed constant over time.

Table 13: Partial elasticities estimated from the translog production function. Annual averages.

	E_K	E_{HU}	E_{HS}	Scale
1980	0,63	0,13	0,20	0,96
1981	0,63	0,13	0,20	0,96
1982	0,63	0,12	0,20	0,96
1983	0,63	0,12	0,21	0,96
1984	0,63	0,12	0,21	0,96
1985	0,63	0,11	0,21	0,96
1986	0,63	0,11	0,22	0,96
1987	0,63	0,11	0,22	0,96
1988	0,62	0,11	0,23	0,96
1989	0,62	0,11	0,24	0,96
1990	0,61	0,11	0,24	0,96
1991	0,61	0,11	0,24	0,96
1992	0,61	0,10	0,25	0,96
1993	0,62	0,10	0,25	0,96
1994	0,62	0,09	0,25	0,96
1995	0,61	0,09	0,26	0,96
1996	0,61	0,09	0,26	0,96
1997	0,60	0,09	0,27	0,96
1998	0,59	0,09	0,28	0,95
1999	0,58	0,08	0,29	0,95
2000	0,57	0,08	0,30	0,95
2001	0,56	0,08	0,30	0,95
2002	0,56	0,08	0,31	0,95
2003	0,55	0,08	0,32	0,94
2004	0,54	0,08	0,32	0,94
mean	0,60	0,10	0,25	0,96
corr.with Y/L	-0,55	-0,06	0,58	0,05

Note: translog parameters, computed for the 1980–2000 dataset, have been assumed constant over time.

Table 14: Correlations between partial elasticities estimated from the DEA, DEA+SW bootstrap, and translog production function.

	corr(DEA,TL)	corr(SW,TL)	corr(DEA,SW)
E_K	0,5234	0,0377	-0,0631
E_{HU}	-0,2717	-0,3654	0,0986
E_{HS}	0,0954	0,0191	-0,1364
Scale	-0,2337	-0,3458	-0,3997

Table 15: Statistical tests of local returns to scale (Löthgren and Tambour, 1999). Results based on bootstrap–based DEA estimates.

	Australia	Austria	Belgium	Canada	Denmark	Finland	France	Greece	Ireland	Italy	Japan	Netherlands	Norway	Portugal	Spain	Sweden	Switzerland	UK	USA
1980	CRS	CRS	CRS	CRS	CRS	CRS	DRS	CRS	CRS	CRS	DRS	CRS	CRS	CRS	CRS	IRS	CRS	CRS	CRS
1981	CRS	CRS	CRS	CRS	CRS	IRS	DRS	CRS	CRS	CRS	DRS	DRS	CRS	CRS	CRS	CRS	CRS	CRS	CRS
1982	CRS	CRS	CRS	CRS	CRS	IRS	DRS	CRS	CRS	DRS	DRS	DRS	CRS	CRS	CRS	CRS	CRS	CRS	CRS
1983	CRS	CRS	CRS	CRS	CRS	IRS	DRS	CRS	CRS	DRS	DRS	DRS	CRS	CRS	CRS	CRS	CRS	CRS	CRS
1984	CRS	CRS	CRS	CRS	CRS	IRS	DRS	CRS	CRS	DRS	DRS	DRS	CRS	CRS	CRS	CRS	CRS	CRS	CRS
1985	CRS	CRS	CRS	CRS	CRS	IRS	DRS	CRS	CRS	DRS	DRS	DRS	CRS	CRS	CRS	CRS	CRS	CRS	CRS
1986	CRS	CRS	CRS	CRS	CRS	IRS	DRS	CRS	CRS	DRS	DRS	DRS	CRS	CRS	CRS	IRS	CRS	CRS	CRS
1987	CRS	CRS	CRS	CRS	CRS	IRS	DRS	IRS	CRS	DRS	DRS	DRS	CRS	CRS	CRS	IRS	CRS	CRS	CRS
1988	CRS	CRS	CRS	CRS	CRS	IRS	DRS	CRS	CRS	DRS	DRS	DRS	CRS	CRS	CRS	CRS	CRS	CRS	CRS
1989	CRS	CRS	CRS	CRS	CRS	IRS	DRS	CRS	CRS	DRS	DRS	DRS	CRS	CRS	CRS	CRS	CRS	CRS	CRS
1990	CRS	CRS	CRS	CRS	CRS	CRS	DRS	CRS	CRS	DRS	DRS	DRS	CRS	CRS	CRS	CRS	CRS	CRS	CRS
1991	CRS	CRS	CRS	CRS	CRS	IRS	DRS	CRS	CRS	DRS	DRS	DRS	CRS	CRS	CRS	CRS	CRS	CRS	CRS
1992	CRS	CRS	CRS	CRS	CRS	CRS	DRS	CRS	CRS	DRS	DRS	DRS	CRS	CRS	CRS	CRS	CRS	CRS	CRS
1993	CRS	IRS	CRS	CRS	CRS	IRS	DRS	CRS	CRS	DRS	DRS	DRS	CRS	CRS	CRS	CRS	CRS	CRS	CRS
1994	CRS	CRS	CRS	CRS	CRS	CRS	DRS	CRS	CRS	DRS	DRS	DRS	CRS	CRS	DRS	CRS	CRS	CRS	CRS
1995	CRS	CRS	IRS	CRS	CRS	CRS	DRS	CRS	CRS	DRS	DRS	DRS	CRS	CRS	DRS	CRS	CRS	CRS	CRS
1996	CRS	IRS	DRS	CRS	CRS	CRS	DRS	DRS	CRS	DRS	DRS	DRS	CRS	CRS	DRS	CRS	CRS	CRS	CRS
1997	DRS	CRS	DRS	CRS	CRS	CRS	DRS	DRS	CRS	DRS	DRS	DRS	CRS	CRS	DRS	CRS	CRS	CRS	CRS
1998	DRS	CRS	DRS	CRS	IRS	CRS	DRS	DRS	CRS	DRS	DRS	DRS	CRS	CRS	DRS	CRS	CRS	CRS	CRS
1999	DRS	CRS	DRS	CRS	IRS	CRS	DRS	DRS	CRS	DRS	DRS	DRS	CRS	CRS	DRS	CRS	CRS	CRS	CRS
2000	DRS	CRS	DRS	CRS	CRS	CRS	DRS	DRS	CRS	DRS	DRS	DRS	CRS	CRS	DRS	CRS	CRS	CRS	CRS
2001	DRS	DRS	DRS	CRS	CRS	CRS	DRS	DRS	CRS	DRS	DRS	DRS	CRS	CRS	DRS	CRS	CRS	CRS	CRS
2002	DRS	DRS	DRS	CRS	CRS	CRS	DRS	DRS	CRS	DRS	DRS	DRS	CRS	CRS	DRS	CRS	CRS	CRS	CRS
2003	DRS	DRS	DRS	CRS	CRS	CRS	DRS	DRS	CRS	DRS	DRS	DRS	CRS	CRS	DRS	CRS	CRS	CRS	CRS
2004	DRS	DRS	DRS	CRS	IRS	CRS	DRS	DRS	CRS	DRS	DRS	DRS	CRS	CRS	DRS	CRS	CRS	CRS	CRS

4

4.3 Returns to scale

Apart from the issues discussed above, our WTF estimates also provide interesting conclusions on local and global returns to scale. One advantage of methods used in the current analysis is that they do not require the researcher to impose *a priori* restrictions on whether returns to scale are decreasing, increasing, or constant. Instead, this property is obtained as a result and can be statistically tested against the null of constant returns. We have conducted such tests for our DEA-based estimates of the aggregate production function, according to Löthgren and Tambour (1999) and Simar and Wilson (2002) procedures.

Results of tests carried out for all units in the sample separately are summarized in Table 15. Comparing the bias-corrected DEA-based efficiency estimates under variable, non-increasing, and constant returns to scale leads, in most cases, to the conclusion that local returns to scale are constant. It is not always the case, though. In particular, decreasing returns have been found in 29.1% of all cases, and in some countries such as Japan, France, Italy and the Netherlands, they have been found for all or almost all considered years. We also observe a tendency of decreasing returns becoming more widespread in the recent years. On the other hand, increasing returns are found rarely (in 4.4% of all considered cases), and Finland in 1981-89 is the only case when increasing returns were found in more than two consecutive years.

Going beyond local returns to scale, measured for individual observations, we have also carried out the Simar–Wilson (2002) statistical test of global returns to scale. The results are illustrated in Figure 6, showing how each of the quantiles of the Shephard distance ratios shifts across time. The most important feature of this Figure is that all these lines are located below $\hat{S}_{obs}^{CRS}$, for all years under consideration. Therefore, under all conventional significance levels (including $\alpha = 1\%$), the null of constant returns to scale has to be rejected for the alternative of variable returns to scale, in all years.[23]

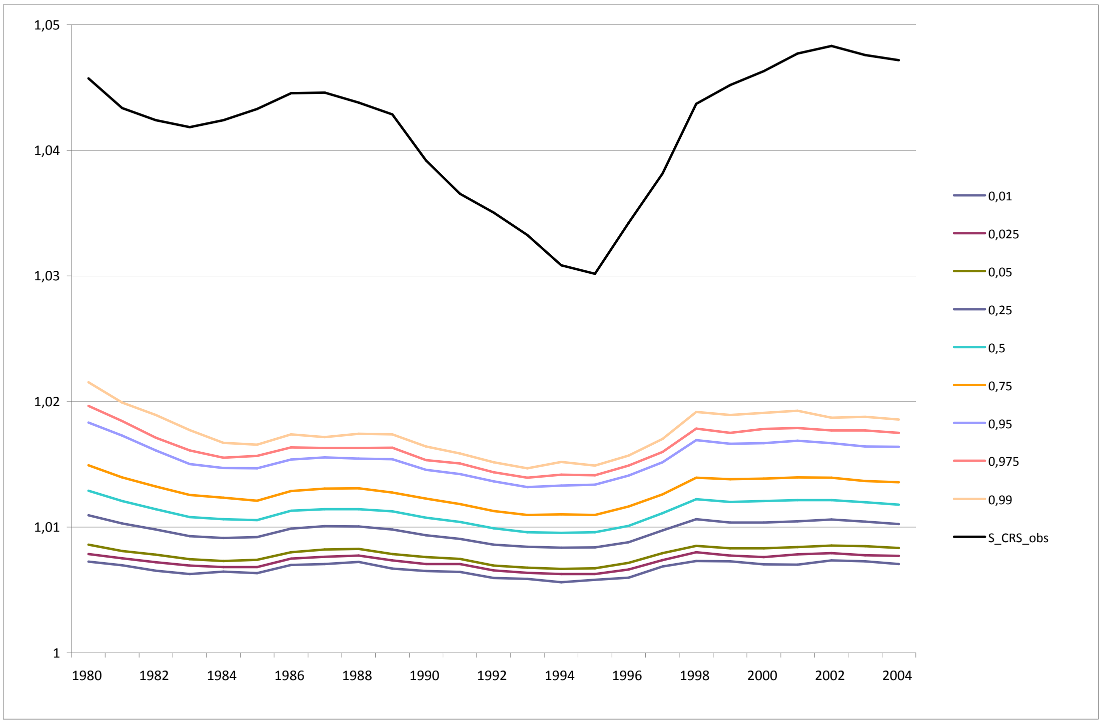
In sum, DEA-based returns-to-scale tests provide mixed evidence on this property. On the one hand, the aggregate production function is often locally indistinguishable from CRS; on the other hand, it is also robustly identified as globally VRS.

Some inference on returns to scale can also be done using our SFA results. As is visible in Table 16, results of estimations of the Cobb–Douglas and the translog production function without the CRS restriction lead to a conclusion that returns to scale are generally country-specific, yet globally close to constant. When computed for the entire sample of countries, the scale elasticity is slightly below unity but not distinguishable from unity in the statistical sense. Country-specific translog production function estimates indicate, however, that returns to scale depend on the size of the economy. Unlike in the DEA case, they are decidedly increasing in the US and decreasing in smaller economies such as Norway and Ireland. This result might also

[23] A change in pattern of development of all our DEA-based returns-to-scale statistics is observed in 1995–96, coinciding with a sudden drop in the average unskilled labor elasticity and marked departures of Switzerland and Japan from full efficiency.

reflect the misspecification of the estimated translog production function, though, so it should be treated with care. The relationship between the estimated scale elasticities and per capita variables is generally very weak.

Figure 6: Simar and Wilson's (2002) test of global returns to scale. The black line corresponds to the test statistic $\hat{S}^{CRS}_{obs}$, other lines are respective quantiles of the underlying distribution.



In sum, the parametric and nonparametric approaches both tend to invalidate the assumption of constancy of global returns to scale (although *on average*, returns to scale might be approximately constant), and the constancy of local returns to scale, in numerous cases. At the level of individual observations, there is however little congruence between results obtained with either method.

Table 16: Returns to scale – evidence from stochastic frontier estimates.

	mean (IRS >1, DRS <1)	1970	2000
SFA-CD(K,Hu,Hs)	0,960		
Australia	0,947	0,939	0,953
Austria	0,889	0,896	0,881
Belgium	0,929	0,924	0,933
Canada	0,939	0,962	0,897
Denmark	0,835	0,840	0,827
Finland	0,840	0,827	0,842
France	1,104	1,111	1,087
Greece	0,911	0,890	0,929
Ireland	0,780	0,771	0,782

Italy	1,119	1,096	1,139
Japan	1,177	1,180	1,160
Netherlands	0,956	0,939	0,957
Norway	0,804	0,821	0,763
Portugal	0,898	0,868	0,926
Spain	1,057	1,023	1,089
Sweden	0,879	0,893	0,854
Switzerland	0,830	0,834	0,822
UK	1,114	1,125	1,091
USA	1,164	1,199	1,124
Translog(K,Hu,Hs) mean	0,956	0,955	0,950
Corr. with K/L	0,041	-0,065	-0,028
Corr. with Hu/L	-0,017	-0,227	0,136
Corr. with Hs/L	0,212	0,214	0,078
Corr. with Y/L	0,028	0,203	-0,059
Corr. with L	0,709	0,759	0,638
Corr. with Y	0,654	0,708	0,582

4.4 Morishima and Allen–Uzawa elasticities of substitution

Another important characteristic of the shape of a production function is its elasticity of substitution. In the two-input world, this characteristic is uniquely defined and interpreted as local curvature of the isoquant (contour line of the production function), i.e., percentage change in the marginal rate of substitution between inputs given a 1% change in their relative price. The elasticity of substitution is an important measure of flexibility of production processes or the ease with which the inputs can be substituted. However, since our results described above (as well as the respective ones due to e.g., Caselli and Coleman, 2006; Growiec, 2012) provide evidence against homogeneity of human capital, we are considering three-input production functions here, for which the elasticity of substitution is no longer a unique concept.

The two most frequently mentioned concepts of elasticity of substitution for n-input functions are the Allen–Uzawa and the Morishima elasticity (cf. Blackorby and Russell, 1989). The first one is defined as (cf. Hoff, 2004):

$$\sigma_{ij}^{A} = \frac{\sum_{k=1}^{n} X_k F_{X_k}}{X_i X_j} \frac{H_{ij}}{|H|}, \qquad i \neq j, \tag{10}$$

for any two inputs $X_i, X_j \in \{K, H^U, H^S\}$, with $|H|$ being the determinant of the bordered Hessian matrix:

$$H = \begin{bmatrix} 0 & F_K & F_{H^U} & F_{H^S} \\ F_K & F_{KK} & F_{KH^U} & F^{KH^S} \\ F_{H^U} & F_{KH^U} & F_{H^U H^U} & F_{H^U H^S} \\ F_{H^S} & F_{KH^S} & F_{H^U H^S} & F_{H^S H^S} \end{bmatrix}, \tag{11}$$

and H_{ij} being the cofactor of (i, j)-th element in the H matrix. The Allen–Uzawa elasticity of substitution is symmetric and simplifies to the unique elasticity of substitution in the two-input case. Unfortunately, as forcefully argued by Blackorby and Russell (1989), it does not measure the curvature of the underlying production function or the ease of input substitution appropriately, nor does it provide information about the comparative statics of income shares.

These two important critisms do not apply to the Morishima elasticity of substitution, which it thus a more theoretically sound concept of elasticity of substitution. The Morishima elasticity of substitution is defined as

$$\sigma_{ij}^{M} = \frac{F_{X_j}}{X_i}\frac{H_{ij}}{|H|} - \frac{F_{X_j}}{X_j}\frac{H_{ij}}{|H|}, \qquad i \neq j, \tag{12}$$

and thus $\sigma_{ij}^{M} \neq \sigma_{ji}^{M}$, signifying that the current measure is not symmetric.

It is not possible to compute meaningful estimates of the elasticity of substitution for the DEA-based WTF, because – by construction – the production function is then piecewise linear, and for any linear function, the elasticity of substitution must be infinite. In turn, we have computed these estimates only under the translog specification. The results are presented in Tables 17–18.

The translog-based estimates of Morishima and Allen–Uzawa elasticities of substitution imply the following regularities:

- According to Allen–Uzawa elasticities of substitution, capital and unskilled labor, as well as capital and skilled labor, are gross substitutes on average. Skilled and unskilled labor are generally complementary. There is substantial variation in these elasticities of substitution across countries.
- According to Allen–Uzawa elasticities of substitution, for all pairs of factors, substitutability does not exhibit any clear time trend. In some countries, the trend is increasing, whereas in others it is decreasing.
- According to Morishima elasticities of substitution, when capital price increases, capital can be relatively easily substituted with unskilled labor, but not with skilled labor.
- According to Morishima elasticities of substitution, when unskilled labor wage increases, some of it can be substituted with capital, somewhat more easily than with skilled labor.
- According to Morishima elasticities of substitution, when skilled labor wage increases, it can be relatively easily substituted with capital, easier than with unskilled labor.
- Neither definition of the elasticity of substitution supports its constancy across countries and time (required in the CES case). None of the computed values of elasticity is close to unity on average (as required in the Cobb–Douglas specification).

Table 17: Morishima and Allen-Uzawa elasticities of substitution, inferred from the translog production function. Cross-country averages.

	Morishima EoS						Allen–Uzawa EoS		
	$E(K,H^U)$	$E(K,H^S)$	$E(H^U,K)$	$E(H^U,H^S)$	$E(H^S,K)$	$E(H^S,H^U)$	$E(K,H^U)$	$E(K,H^S)$	$E(H^U,H^S)$
Australia	-16,37	-7,19	0,93	-0,93	0,29	0,61	1,46	0,46	-4,92
Austria	-2,37	-8,57	0,73	-0,39	0,41	0,06	1,02	0,57	-3,45
Belgium	-8,06	-6,61	0,69	-0,11	0,33	0,07	0,99	0,47	-8,21
Canada	2,46	-12,51	0,89	-0,89	0,30	0,32	1,65	0,57	-2,69
Denmark	6,13	-8,21	0,74	-0,31	0,45	-0,14	0,94	0,58	-2,22
Finland	0,13	-8,12	0,67	-0,15	0,41	-0,03	0,87	0,54	-3,31
France	16,80	-32,20	-0,26	1,29	1,02	-2,10	-0,51	1,92	8,88
Greece	-5,30	-7,61	0,58	1,05	0,38	-0,52	0,81	0,53	-12,18
Ireland	3,62	-4,33	0,57	-0,03	0,27	-0,01	0,65	0,31	-2,89
Italy	-9,71	-30,44	0,22	-0,45	0,60	0,33	0,45	1,20	3,41
Japan	12,73	-36,51	-0,07	0,85	0,76	-2,05	-0,17	1,73	4,13
Netherlands	-63,48	23,46	2,09	-2,51	-0,70	1,93	3,56	-1,18	-24,26
Norway	2,32	-20,30	0,75	-0,29	0,34	-0,03	1,20	0,56	-2,69
Portugal	0,46	-16,35	-0,11	3,87	1,17	-2,27	-0,14	1,57	-22,14
Spain	-7,57	-16,32	0,33	-0,83	0,51	0,60	0,56	0,89	5,84
Sweden	3,58	-9,17	0,76	-0,41	0,44	-0,10	1,05	0,61	-2,84
Switzerland	2,16	-19,60	0,79	-0,57	0,36	-0,02	1,35	0,62	-1,95
UK	-190,52	66,19	5,84	-12,44	-3,00	20,51	9,90	-5,31	-50,22
USA	431,60	-65,92	-1,33	4,15	1,22	-21,34	-3,62	3,26	10,12
mean	9,40	-11,59	0,78	-0,48	0,29	-0,22	1,16	0,52	-5,87

Note: translog parameters, computed for the 1980–2000 dataset, have been assumed constant over time.

Table 18: Morishima and Allen-Uzawa elasticities of substitution, inferred from the translog production function. Annual averages.

	Morishima EoS						Allen–Uzawa EoS		
	$E(K,H^U)$	$E(K,H^S)$	$E(H^U,K)$	$E(H^U,H^S)$	$E(H^S,K)$	$E(H^S,H^U)$	$E(K,H^U)$	$E(K,H^S)$	$E(H^U,H^S)$
1980	-2,36	-15,43	0,42	0,29	0,52	-0,53	0,60	0,86	-2,36
1981	-1,87	-15,26	0,42	0,34	0,52	-0,60	0,61	0,85	-2,11
1982	-0,61	-15,37	0,41	0,25	0,54	-0,63	0,58	0,88	-0,88
1983	0,64	-15,57	0,39	0,26	0,55	-0,71	0,55	0,91	-0,03
1984	1,78	-15,64	0,39	0,19	0,56	-0,72	0,53	0,92	0,42
1985	3,08	-16,04	0,37	0,17	0,57	-0,77	0,51	0,95	0,84
1986	4,59	-16,90	0,35	0,18	0,59	-0,84	0,47	0,98	1,34
1987	7,22	-19,03	0,29	0,23	0,65	-0,92	0,36	1,08	2,34
1988	13,74	-25,47	0,09	0,39	0,80	-1,08	0,04	1,35	5,38
1989	-41,73	33,85	2,03	-1,13	-0,71	0,01	3,31	-1,21	-29,35
1990	-0,40	-9,36	0,65	0,09	0,38	-0,93	0,97	0,65	-5,37
1991	6,14	-13,95	0,49	0,36	0,51	-1,25	0,70	0,87	-3,09
1992	11,15	-15,40	0,42	0,46	0,56	-1,51	0,57	0,96	-1,44
1993	16,97	-16,33	0,35	0,57	0,61	-1,86	0,44	1,04	0,52
1994	23,80	-17,46	0,29	0,69	0,66	-2,23	0,32	1,12	1,78
1995	33,94	-18,96	0,22	0,87	0,70	-2,77	0,18	1,22	2,90
1996	45,90	-21,15	0,10	1,19	0,79	-3,50	-0,04	1,38	4,79
1997	55,27	-23,40	-0,02	1,49	0,87	-4,05	-0,24	1,52	6,15
1998	74,04	-31,55	-0,63	3,06	1,31	-6,49	-1,24	2,26	15,58
1999	-99,19	46,92	5,99	-13,50	-3,41	19,36	9,59	-5,57	-85,27
2000	23,23	-10,96	1,20	-1,54	0,00	0,87	1,81	0,10	-12,21
2001	29,28	-14,43	0,96	-0,91	0,17	-0,05	1,42	0,40	-8,37
2002	30,78	-15,34	0,91	-0,78	0,20	-0,19	1,37	0,45	-7,42
2003	23,73	-12,85	1,10	-1,24	0,08	0,61	1,72	0,23	-9,14
2004	-24,10	5,18	2,28	-3,96	-0,70	5,31	3,82	-1,16	-21,78
mean	9,40	-11,59	0,78	-0,48	0,29	-0,22	1,16	0,52	-5,87

Note: translog parameters, computed for the 1980–2000 dataset, have been assumed constant over time.

Thus, our analysis provides some evidence for the disputed concept of capital-skill complementarity. Using the translog specification instead of the CES, and basing our discussion on Morishima elasticities of substitution, we can say more on this issue that is usually said in the related literature. In particular, we observe a one-sided relationship here: capital-skill complementarity is observed on average only when their relative price changes due to changes in capital price, not the skilled wage.

5 Conclusion

Summing up, the objective of the current paper has been to investigate the shape of the aggregate (country-level) production function based on the estimates of the World Technology Frontier (WTF). Using annual data on inputs and output in 19 highly developed OECD countries in 1970–2004, we have estimated the WTF both non-parametrically and parametrically (using the bias-corrected DEA and SFA approach, respectively) and then used these estimates to assess several properties of the implied production function.

We have obtained the following principal results:

- the CRS Cobb–Douglas production function fails to reproduce the important properties of our data (inferred inefficiency levels, estimated partial elasticities, elasticities of substitution),
- the (non-parametric) bootstrap-augmented DEA frontier is not only markedly different from the CRS Cobb–Douglas production function specification, but also from the unrestricted Cobb–Douglas and the translog, even though the latter offers much more flexibility and can be fitted to the data relatively well,
- regardless of the approach taken, the ranking of countries with respect to their technical efficiency is relatively stable (although individual distances to the frontier may vary),
- partial elasticities of the aggregate production function are correlated with inputs both in the DEA and in the translog case, and they vary substiantially across countries and time, providing strong evidence against the Cobb–Douglas specification, and also providing support for the skill-biased technical change hypothesis,
- tests of returns to scale based on the DEA, Cobb–Douglas and translog representations of the frontier provide mixed evidence on this property, although DRS seems more prevalent in smaller economies, and IRS – in larger economies,
- unskilled and skilled labor are not perfectly substitutable,
- elasticities of substitution vary largely across countries and time, but there are some indications of capital–skill complementarity, postulated in the related literature.

5

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A Appendix

Apart from the positive motivation, followed throughout this article, knowing the shape of the aggregate production function also has important corollaries for development and growth accounting exercises. As is clear from numerous earlier contributions (e.g. Koop, Osiewalski and Steel, 1999, 2000; Kumar and Russell, 2002; Henderson and Russell, 2005; Jerzmanowski, 2007; Bos et al., 2010), the fractions of cross-country productivity differentials attributed to differences in efficiency, technology, and inputs are largely dependent on the methodology and dataset used in each study. The same caveat applies to decompositions of total GDP growth.

Detailed development and growth accounting exercises can (and should) also be conducted on the basis of the alternative WTF production functions identified in the current study. Let us now discuss several of such results in the form of the current appendix.

A.1 Development accounting: DEA vs. the Cobb–Douglas production function

The DEA-based non-parametric production frontier approach is very useful for the purposes of development accounting (cf. Kumar and Russell, 2002; Henderson and Russell, 2005; Jerzmanowski, 2007; Growiec, 2012). Within the DEA paradigm, the ratio of GDP per worker between two countries (here, between each particular OECD country and the US) can be easily decomposed into a product of (i) the efficiency ratio, and (ii) fractions of the potential output ratio attributed to differences in the endowment of each separate factor of production.

The latter group of factors cannot be determined uniquely because when we assess the impact on output of differences in one factor holding other factors constant, *we can hold them constant at different levels*: either at US levels, or country's levels, or a mixture of the two. For three factors of production (physical capital K, unskilled labor H^U and skilled labor H^S; see also Growiec, 2012), the "Fisher-ideal" decomposition (cf. Henderson and Russell, 2005) has to satisfy the following:

$$\begin{aligned} \frac{y_C(K_C, H_C}{y_U(K_U, H_U)} &= \frac{E_C}{E_U} \cdot \frac{y^*(K_C, H_C)}{y^*(K_U, H_U)} \\ &= \frac{E_C}{E_U} \cdot\ K \text{ diff } \cdot\ H^U \text{ diff } \cdot\ H^S \text{ diff} \end{aligned} \tag{13}$$

where

$$\begin{aligned} K \text{ diff} &= \sqrt[6]{\left(\frac{y^*(K_C, H_C^U, H_C^S)}{y^*(K_U, H_C^U, H_C^S)}\right)^2 \frac{y^*(K_C, H_C^U, H_U^S)}{y^*(K_U, H_C^U, H_U^S)} \frac{y^*(K_C, H_U^U, H_C^S)}{y^*(K_U, H_U^U, H_C^S)} \left(\frac{y^*(K_C, H_U^U, H_U^S)}{y^*(K_U, H_U^U, H_U^S)}\right)^2}, \\ H^U \text{ diff} &= \sqrt[6]{\left(\frac{y^*(K_C, H_C^U, H_C^S)}{y^*(K_C, H_U^U, H_C^S)}\right)^2 \frac{y^*(K_C, H_C^U, H_U^S)}{y^*(K_C, H_U^U, H_U^S)} \frac{y^*(K_U, H_C^U, H_C^S)}{y^*(K_U, H_U^U, H_C^S)} \left(\frac{y^*(K_U, H_C^U, H_U^S)}{y^*(K_U, H_U^U, H_U^S)}\right)^2}, \end{aligned}$$

$$H^S \text{ diff } = \sqrt[6]{\left(\frac{y^*(K_C, H_C^U, H_C^S)}{y^*(K_C, H_C^U, H_U^S)}\right)^2 \frac{y^*(K_C, H_U^U, H_C^S)}{y^*(K_C, H_U^U, H_U^S)} \frac{y^*(K_U, H_C^U, H_C^S)}{y^*(K_U, H_C^U, H_U^S)} \left(\frac{y^*(K_U, H_U^U, H_C^S)}{y^*(K_U, H_U^U, H_U^S)}\right.}$$

Please note that in each of the fractions indicated above, the numerator and denominator differ by a single variable only, being the variable whose contribution to the total GDP ratio we are about to measure.

The results according to the above decomposition, for 1980 and 2004, are presented in Table 19. Results for other years are available from the authors upon request.

Shifting to the parametric approach, and taking the well-established assumption of a Cobb–Douglas production function, coupled with the usual assumption of perfect substitutability between skilled and unskilled labor (made here to attain comparability to the established literature), the development accounting relationship can be written down as:

$$\frac{y_C(K_C, H_C^U, H_C^S)}{y_U(K_U, H_U^U, H_U^S)} = \underbrace{\frac{TFP_C}{TFP_U}}_{\text{appropriate tech.}} \cdot \underbrace{\frac{K_C^\alpha}{K_U^\alpha}}_{K \text{ diff}} \cdot \underbrace{\frac{(H_C^U + H_C^S)^{1-\alpha}}{(H_U^U + H_U^S)^{1-\alpha}}}_{H \text{ diff}}, \tag{14}$$

where α is the capital share in output. We take its (country-specific) values from Gollin (2002).[24] The results are viewed in Table 20.

Development accounting exercises may also be conducted on the basis of SFA, under the assumption of Cobb–Douglas or translog frontiers (cf. Koop, Osiewalski and Steel, 1999, 2000). We leave this for further research.

A.2 Growth accounting: DEA vs. the Cobb–Douglas production function

Analogously to the development accounting exercise described above, we have also conducted a growth accounting exercise where we decomposed the total 1980–2004 increase in GDP per worker into the impacts of (i) change in efficiency relative to the WTF, (ii) technological progress at the WTF, (iii) factor accumulation.

As compared to development accounting, there is one additional factor which ought to be disentangled here: technological progress at the frontier which pushes the WTF forward so that potential productivity is increased. Formally, with three factors of production, K, H^U, H^S, the "Fisher-ideal" (cf. Henderson and Russell, 2005; Growiec, 2012) decomposition of the 2004/1980 (or 2004/1990)[25] productivity ratio is the following (denoting $s = 1980, 1990, n = 2004$):

[24]Specifically, we apply Gollin's adjustment no. 2, where capital and labor shares are adjusted for self-employment in the economy (self-employed income is attributed to capital and labor in the same proportion as it is split in the rest of the economy.)

[25]Results for other years are available from the authors upon request.

$$\frac{y_n(K_n,H_n^U,H_n^S)}{y_s(K_s,H_s^U,H_s^S)} = \frac{E_n}{E_s}\cdot\frac{y_n^*(K_n,H_n^U,H_n^S)}{y_s^*(K_s,H_s^U,H_s^S)} = \tag{15}$$

$$= \underbrace{\frac{E_n}{E_s}}_{\text{efficiency}}\cdot\underbrace{\sqrt{\frac{y_n^*(K_n,H_n^U,H_n^S)}{y_s^*(K_n,H_n^U,H_n^S)}\frac{y_n^*(K_s,H_s^U,H_s^S)}{y_s^*(K_s,H_s^U,H_s^S)}}}_{\text{techn. progress}}\cdot\underbrace{\sqrt{\frac{y_n^*(K_n,H_n^U,H_n^S)}{y_n^*(K_s,H_s^U,H_s^S)}\frac{y_s^*(K_n,H_n^U,H_n^S)}{y_s^*(K_s,H_s^U,H_s^S)}}}_{\text{factor accumulation}}.$$

The decomposition of GDP growth defined by Eq. (15) singles out dynamic changes in technical efficiency, shifts in the technology frontier given factor endowments, and factor accumulation holding the technological frontier fixed.

The product of the "efficiency change" and "technological progress" factors is also known as the (output-oriented) Malmquist productivity index in the DEA literature (cf. Fried, Knox Lovell, and Schmidt, 1993). It measures, for each country and time

Table 19: Development accounting results – DEA method.

		GDP ratio	Effic.	K diff	H^U diff	H^S diff
Australia	1980	0,826	0,814	1,006	1,175	0,860
Austria	1980	0,815	0,834	0,989	1,233	0,801
Belgium	1980	0,951	0,903	1,017	1,393	0,743
Canada	1980	0,902	0,933	1,008	1,072	0,894
Denmark	1980	0,748	0,767	0,990	1,170	0,841
Finland	1980	0,624	0,731	0,987	1,324	0,653
France	1980	0,881	0,879	1,022	1,292	0,758
Greece	1980	0,753	0,805	1,015	1,492	0,618
Ireland	1980	0,651	0,881	0,777	1,321	0,720
Italy	1980	0,859	0,984	1,029	1,621	0,523
Japan	1980	0,678	0,744	1,008	1,191	0,759
Netherlands	1980	0,945	0,945	1,065	1,504	0,625
Norway	1980	0,950	0,991	1,056	1,377	0,659
Portugal	1980	0,473	0,979	0,635	1,503	0,507
Spain	1980	0,788	1	0,878	1,645	0,546
Sweden	1980	0,714	0,830	0,998	1,203	0,716
Switzerland	1980	0,948	0,944	1,037	1,151	0,841
UK	1980	0,716	0,986	0,754	1,141	0,844
USA	1980	1	1	1	1	1
Australia	2004	0,825	0,760	0,964	1,125	1,002
Austria	2004	0,860	0,774	0,928	1,168	1,025
Belgium	2004	0,928	0,772	0,941	1,273	1,004
Canada	2004	0,815	1	1,036	0,832	0,946
Denmark	2004	0,767	0,806	0,843	1,118	1,011
Finland	2004	0,776	0,759	0,854	1,169	1,025
France	2004	0,868	0,774	0,964	1,131	1,028
Greece	2004	0,712	0,747	0,790	1,174	1,027
Ireland	2004	0,969	1	0,874	1,228	0,903
Italy	2004	0,800	0,728	0,885	1,298	0,957
Japan	2004	0,708	0,635	0,990	1,112	1,012
Netherlands	2004	0,802	0,737	0,933	1,168	0,998
Norway	2004	1,089	1	1,014	1,199	0,895
Portugal	2004	0,493	0,829	0,684	1,254	0,694
Spain	2004	0,751	0,772	0,897	1,292	0,839
Sweden	2004	0,773	0,910	0,785	1,065	1,017
Switzerland	2004	0,735	0,664	1,004	1,128	0,977
UK	2004	0,786	1,000	0,707	1,087	1,022
USA	2004	1	1	1	1	1

period, the total change in productivity which resulted from anything but factor accumulation. In other words, the Malmquist productivity index captures the total productivity improvement under technologies *actually used* in the given country, whereas our "technological progress" index measures the total productivity improvement under *frontier* technology, given the country's factor endowments.

The results are presented in the form of annualized growth rates in Table 21.

The parametric approach, based on the Cobb-Douglas production function assumption, the 2004 / 1980 (or 2004 / 1990) provides an alternative decomposition of the productivity ratio into contributions attributable to technological progress shifting Total Factor Productivity, and factor accumulation: Formally, the "Fisher-ideal" decomposition, taking full account of technological change, is obtained from the following

Table 20: Development accounting results – the Cobb–Douglas production function.

		GDP ratio	TFP ratio	K diff	H diff
Australia	1980	0,826	0,810	1,011	1,009
Austria	1980	0,815	0,832	0,989	0,990
Belgium	1980	0,951	0,888	1,031	1,038
Canada	1980	0,902	0,910	1,032	0,960
Denmark	1980	0,748	0,766	0,992	0,984
Finland	1980	0,624	0,696	0,989	0,906
France	1980	0,881	0,845	1,046	0,997
Greece	1980	0,753	0,742	1,043	0,973
Ireland	1980	0,651	0,794	0,882	0,929
Italy	1980	0,859	0,846	1,084	0,936
Japan	1980	0,678	0,729	1,011	0,920
Netherlands	1980	0,945	0,796	1,208	0,983
Norway	1980	0,950	0,608	1,564	0,999
Portugal	1980	0,473	1,090	0,443	0,981
Spain	1980	0,788	0,997	0,879	0,900
Sweden	1980	0,714	0,792	0,998	0,903
Switzerland	1980	0,948	0,893	1,116	0,951
UK	1980	0,716	0,848	0,884	0,956
USA	1980	1	1	1	1
Australia	2004	0,825	0,814	0,997	1,017
Austria	2004	0,860	0,826	0,981	1,061
Belgium	2004	0,928	0,837	1,008	1,100
Canada	2004	0,815	0,787	1,001	1,034
Denmark	2004	0,767	0,837	0,929	0,988
Finland	2004	0,776	0,787	0,943	1,044
France	2004	0,868	0,813	0,997	1,070
Greece	2004	0,712	0,781	0,872	1,045
Ireland	2004	0,969	1,173	0,946	0,872
Italy	2004	0,800	0,797	0,968	1,036
Japan	2004	0,708	0,670	1,009	1,047
Netherlands	2004	0,802	0,815	0,985	0,998
Norway	2004	1,089	0,897	1,216	0,998
Portugal	2004	0,493	0,948	0,529	0,984
Spain	2004	0,751	0,806	0,966	0,965
Sweden	2004	0,773	0,855	0,896	1,010
Switzerland	2004	0,735	0,726	1,027	0,986
UK	2004	0,786	0,887	0,889	0,996
USA	2004	1	1	1	1

formula:

$$\frac{y_n}{y_s} = \frac{TFP_n}{TFP_s} \cdot \underbrace{\frac{K_n^\alpha}{K_s^\alpha}}_{K \text{ diff}} \cdot \underbrace{\frac{(H_n^U + H_n^S)^{1-\alpha}}{(H_s^U + H_s^S)^{1-\alpha}}}_{H \text{ diff}}. \tag{16}$$

The results obtained taking the (country-specific) values of the capital share α from Gollin (2002) are viewed in Table 22.

Growth accounting exercises may also be conducted on the basis of SFA, under the assumption of Cobb–Douglas or translog frontiers (cf. Koop, Osiewalski and Steel, 1999, 2000). This is beyond the scope of the current paper.

Table 21: Growth accounting results – DEA method.

		GDP growth	Effic.	Techn.	Factors
Australia	1980-2004	1,56%	-0,27%	0,89%	0,93%
Austria	1980-2004	1,78%	-0,30%	0,75%	1,32%
Belgium	1980-2004	1,47%	-0,63%	0,85%	1,25%
Canada	1980-2004	1,15%	0,28%	3,21%	-2,27%
Denmark	1980-2004	1,67%	0,20%	0,71%	0,75%
Finland	1980-2004	2,45%	0,15%	0,88%	1,41%
France	1980-2004	1,50%	-0,51%	0,94%	1,07%
Greece	1980-2004	1,33%	-0,30%	0,58%	1,05%
Ireland	1980-2004	3,19%	0,51%	0,89%	1,76%
Italy	1980-2004	1,28%	-1,20%	0,72%	1,78%
Japan	1980-2004	1,74%	-0,63%	1,12%	1,25%
Netherlands	1980-2004	0,90%	-0,99%	1,02%	0,88%
Norway	1980-2004	2,12%	0,03%	1,63%	0,44%
Portugal	1980-2004	1,73%	-0,66%	0,33%	2,07%
Spain	1980-2004	1,37%	-1,03%	0,60%	1,81%
Sweden	1980-2004	1,89%	0,37%	0,67%	0,83%
Switzerland	1980-2004	0,53%	-1,40%	1,64%	0,32%
UK	1980-2004	1,94%	0,05%	0,21%	1,67%
USA	1980-2004	1,56%	0,00%	2,61%	-1,02%
Australia	1990-2004	1,79%	-0,46%	1,17%	1,08%
Austria	1990-2004	1,74%	-0,56%	0,99%	1,32%
Belgium	1990-2004	1,27%	-1,10%	1,05%	1,33%
Canada	1990-2004	1,27%	1,28%	3,15%	-3,07%
Denmark	1990-2004	1,65%	-0,28%	0,83%	1,09%
Finland	1990-2004	2,32%	-0,03%	1,19%	1,14%
France	1990-2004	1,18%	-0,93%	1,14%	0,99%
Greece	1990-2004	1,57%	-0,25%	0,60%	1,22%
Ireland	1990-2004	3,10%	0,38%	1,13%	1,55%
Italy	1990-2004	0,95%	-2,04%	1,12%	1,91%
Japan	1990-2004	1,13%	-1,63%	1,61%	1,19%
Netherlands	1990-2004	1,09%	-0,97%	1,77%	0,30%
Norway	1990-2004	2,13%	0,00%	2,43%	-0,29%
Portugal	1990-2004	1,53%	-1,14%	0,34%	2,36%
Spain	1990-2004	0,57%	-1,71%	0,90%	1,40%
Sweden	1990-2004	2,23%	0,57%	0,60%	1,04%
Switzerland	1990-2004	0,77%	-2,17%	2,27%	0,72%
UK	1990-2004	1,92%	0,03%	0,18%	1,70%
USA	1990-2004	1,68%	0,00%	2,36%	-0,67%

In sum, the principal findings of our development accounting and growth accounting studies are the following:

- according to DEA, differences in GDP per worker between the USA and most Western European countries in 1980 have been mostly due to differences in efficiency and skilled labor endowments, whereas in 2004 they have been mostly due to differences in efficiency and physical capital endowments. Average efficiency differences have grown visibly between 1980 and 2004;

- according to the Cobb–Douglas production function specification, the differences in GDP per worker between the USA and other countries in the sample have been predominantly Total Factor Productivity (TFP)-driven, with a few exceptions where physical capital differences played an equally important role;

Table 22: Growth accounting results – the Cobb–Douglas production function.

		GDP growth	TFP ratio	K diff	H diff
Australia	1980-2004	1,56%	0,68%	0,62%	0,26%
Austria	1980-2004	1,78%	0,50%	0,80%	0,47%
Belgium	1980-2004	1,47%	0,35%	0,67%	0,44%
Canada	1980-2004	1,15%	0,12%	0,50%	0,54%
Denmark	1980-2004	1,67%	0,96%	0,47%	0,23%
Finland	1980-2004	2,45%	1,09%	0,56%	0,78%
France	1980-2004	1,50%	0,45%	0,55%	0,50%
Greece	1980-2004	1,33%	0,53%	0,36%	0,44%
Ireland	1980-2004	3,19%	2,31%	0,87%	-0,01%
Italy	1980-2004	1,28%	0,18%	0,51%	0,58%
Japan	1980-2004	1,74%	0,51%	0,43%	0,79%
Netherlands	1980-2004	0,90%	0,65%	-0,02%	0,27%
Norway	1980-2004	2,12%	1,22%	0,89%	0,00%
Portugal	1980-2004	1,73%	-0,84%	2,56%	0,03%
Spain	1980-2004	1,37%	-0,37%	1,28%	0,47%
Sweden	1980-2004	1,89%	0,92%	0,29%	0,67%
Switzerland	1980-2004	0,53%	-0,01%	0,14%	0,41%
UK	1980-2004	1,94%	0,93%	0,58%	0,41%
USA	1980-2004	1,56%	0,84%	0,45%	0,27%
Australia	1990-2004	1,79%	0,82%	0,68%	0,28%
Austria	1990-2004	1,74%	0,50%	0,82%	0,42%
Belgium	1990-2004	1,27%	0,21%	0,78%	0,29%
Canada	1990-2004	1,27%	0,53%	0,45%	0,28%
Denmark	1990-2004	1,65%	0,76%	0,65%	0,23%
Finland	1990-2004	2,32%	1,14%	0,40%	0,76%
France	1990-2004	1,18%	0,21%	0,57%	0,39%
Greece	1990-2004	1,57%	0,41%	0,67%	0,48%
Ireland	1990-2004	3,10%	2,99%	0,95%	-0,84%
Italy	1990-2004	0,95%	-0,23%	0,56%	0,62%
Japan	1990-2004	1,13%	-0,24%	0,37%	1,00%
Netherlands	1990-2004	1,09%	0,78%	0,13%	0,18%
Norway	1990-2004	2,13%	1,30%	0,82%	0,00%
Portugal	1990-2004	1,53%	-1,27%	2,80%	0,04%
Spain	1990-2004	0,57%	-0,80%	1,22%	0,16%
Sweden	1990-2004	2,23%	0,95%	0,39%	0,88%
Switzerland	1990-2004	0,77%	-0,14%	0,12%	0,79%
UK	1990-2004	1,92%	0,76%	0,64%	0,50%
USA	1990-2004	1,68%	0,93%	0,51%	0,23%

- according to DEA, factor accumulation and technological progress have provided significant positive contributions to GDP growth in 1980–2004, with technological progress being particularly powerful in 1990–2004. Average efficiency levels have been declining, on the other hand, providing negative contributions to GDP growth;
- according to the Cobb–Douglas production function specification, TFP growth, physical capital accumulation, and human capital accumulation have all provided positive contributions to GDP growth throughout 1980–2004. The variance of their relative strength across countries and time was large.

A.3 DEA results based on decadal data – a robustness check

There is an indication that when the DEA-based frontier is computed on the basis of decadal data (instead of annual ones), and is not corrected for DEA bias with the Simar and Wilson bootstrap, then the cross-country correlation between Debreu–Farrell technical efficiencies computed on the basis of DEA and SFA is significantly reduced. Some indicative results are summarized in Table 23.

It is not clear how to interpret this shift in correlation, since at least three effects could be at work here. On the one hand, it may be true that by reducing the dataset in the current robustness check, we have substantially increased the randomness in our results, which should naturally drive all correlations towards zero. On the other hand, the impact of bootstrapping out the bias in DEA efficiency estimates should not be neglected either: with a much smaller dataset, the percentage of frontier observations goes up so the expected value of this bias goes up as well. Thirdly and most interestingly, it may also be the case that due to having an expanded time-series dimension of our original dataset in the DEA case, we have been able to capture more of its time-series variation then than in the SFA case, estimated using primarily the cross-sectional variation of data. Consequently, when we apply both methods to exactly the same dataset, we may get some new information on the distance of each given parametric form of the aggregate production function to its nonparametric benchmark.

Our tentative result is somewhat mixed here, though: the Cobb–Douglas function exhibits slightly larger average deviations of technical efficiency estimates from their nonparametric counterparts, but the estimates are slightly more correlated.

Table 23: Technical efficiency – comparison of alternative measurements for the year 2000. All results based on decadal data.

	DEA (K,Hu,Hs) [CRS]	DEA (K,Hu,Hs) [VRS]	SFA-CD (K,Hu,Hs)	SFA-CD (K,Hu,Hs) [CRS]	SFA-Translog (K,Hu,Hs) [CRS]	SFA-Translog (K,Hu,Hs)	mean
Australia	1,157	1,057	1,2029	1,0438	1,0979	1,0810	1,1067
Austria	1,000	1,000	1,1959	1,0395	1,0869	1,0857	1,0680
Belgium	1,000	1,000	1,1506	1,0328	1,0655	1,0580	1,0512
Canada	1,242	1,185	1,1599	1,0320	1,0772	1,0590	1,1259
Denmark	1,059	1,000	1,2376	1,0466	1,1070	1,1296	1,0967
Finland	1,005	1,000	1,3015	1,0634	1,1446	1,1688	1,1139
France	1,017	1,000	1,1552	1,0421	1,0884	1,0876	1,0651
Greece	1,581	1,579	1,2452	1,0553	1,1263	1,1231	1,2850
Ireland	1,000	1,000	1,1174	1,0166	1,0324	1,0922	1,0431
Italy	1,002	1,000	1,1355	1,0424	1,0818	1,0870	1,0581
Japan	1,025	1,019	1,2724	1,0728	1,1686	1,1991	1,1262
Netherlands	1,444	1,437	1,2641	1,0637	1,1250	1,1016	1,2392
Norway	1,129	1,000	1,2700	1,0563	1,1038	1,1409	1,1167
Portugal	1,153	1,153	1,0205	1,0035	1,0118	1,0198	1,0603
Spain	1,000	1,000	1,0346	1,0122	1,0243	1,0221	1,0155
Sweden	1,285	1,279	1,2281	1,0476	1,1052	1,1073	1,1753
Switzerland	1,035	1,000	1,3335	1,0688	1,1369	1,1547	1,1215
UK	1,000	1,000	1,0396	1,0109	1,0235	1,0229	1,0162
USA	1,000	1,000	1,0152	1,0051	1,0181	1,0355	1,0123
Corr. with DEA	0,9748	1,0000	0,2231	0,2356	0,2553	0,0534	0,8581
RMSE Dev. / DEA	0,0430	0,0000	0,1918	0,1681	0,1588	0,1689	0,1101
Corr. with SFA-TL	0,1077	0,0534	0,9014	0,9027	0,9139	1,0000	0,5251
RMSE Dev. / SFA-TL	0,1663	0,1689	0,1009	0,0625	0,0217	0,0000	0,0617