NATIONAL BANK OF POLAND
WORKING PAPER
No. 144

Unemployment in the Estimated New Keynesian SoePL-2012 DSGE Model

Grzegorz Grabek, Bohdan Kłos

Warsaw 2013

Grzegorz Grabek – Economic Institute, National Bank of Poland.

Bohdan Kłos – Economic Institute, National Bank of Poland.

Design:

Oliwka s.c.

Layout and print:

NBP Printshop

Published by:

National Bank of Poland
Education and Publishing Department
00-919 Warszawa, 11/21 Świętokrzyska Street
phone: +48 22 653 23 35, fax +48 22 653 13 21

ISSN 2084–624X

http://www.nbp.pl

Contents

Abstract

The paper shows some new features implemented in SoePL-2012 DSGE model, namely explicitly modeled unobserved labour supply and observed unemployment rate. Our approach to labour market in the New Keynesian DSGE model follows papers of Galí et al. (2011); Galí (2011b), see also Christiano et al. (2010b). The Galí's idea has been implemented into medium-scale small open economy model estimated on Polish data. We analyze estimates of labour market shocks (the wage markup shock and the labour supply preference shock) and use the results to explain the evolution of unemployment in the period of 1999–2011.

JEL: D58, E24, E31, E37, E52

Keywords: estimated DSGE model, New Keynesian wage Phillips curve, unemployment, labour market shocks.

Introduction

In a series of papers J. Galí (Galí, 2011c,b,a; Galí et al., 2011) explained how to include unemployment into New Keynesian DSGE models. Many New Keynesian (representative agents) DSGE models attempt to explain employment and wages. Employment is a demand oriented variable driven by some well identified shocks (related to technology, consumption preferences and foreign trade). Unemployment is neglected. Sticky wages depend on labour market specific shocks, namely labour supply shock and wage markup shock. These shocks have very different economic meaning but cannot be identified (discriminated) simultaneously, see Erceg et al. (2000); Chari et al. (2009). Researchers applied *ad hoc* tricks to omit the issue, e.g. they imposed additional restriction on stochastic structure of the shocks (see for example Grabek et al. (2007, 2011), Justiniano et al. (2011)). The inclusion of unemployment rate into a model seems to have much more power to solve the issue.

Galí's idea may not be an exhaustive explanation of a developed market economy labour market and some drawbacks depicted by Christiano (2011) (see also Christiano et al., 2010b) are worthwhile to consider, nevertheless it offers somewhat better explanation of the contemporary economies than many older (estimated) New Keynesian DSGE models. Therefore, it is important to verify how the Galí's proposition works in practice. The paper of Galí et al. (2011) was the first attempt to check the proposition. The authors rebuilt the well known Smets-Wouters DSGE model (estimated on the US data) and got deeper insight into macroeconomic development of the US economy. Following that paper we included unemployment into our SoePL-2012 (a small open economy New Keynesian) DSGE model estimated on Polish data and checked how it explains recent events. In addition, we compared the performance of this version to the older one where unemployment has been omitted (SoePL-2011).

The implementation of the simplest version of Galí's proposition requires two additional equations: the labour supply equation and the unemployment rate equation. They are in some sense redundant, because they are post-recursive (the last items of the sequence without any feedback). Nevertheless they uncover the relation between deep parameters of the labour market. It pushed us to rethink and modify priors (or calibration) of several parameters. In addition, the inclusion of the unemployment rate to the set of observables used to estimate the SoePL model allows to identify and estimate labour market parameters better. Exercises which we have done suggest that the issue of shocks identification mentioned above is a result of improper calibration or invalid priors of the parameters. Having two version of the model, we analyzed the importance of the parameters related to labour market and investigated which parameters are responsible for the specific changes of the models' properties (e.g. impulse response function), in particular monetary transmission mechanism. The conclusions we have reached are specific to our model (the specification and the data set), however we believe that our experiences may be interesting and useful for researchers who develop larger estimated DSGE models.

The paper is organized as follows. In the first section, we review the main idea and the construction of Galí's New Keynesian wage Phillips curve. The derivation of the curve is the core of the concept of the labour market we apply. In the second section of this paper, we shortly describe economic background of SoePL model family, and specification changes we have done to include unemployment to the newest version of the model as well as results of the model estimation. The reader interested in more exhaustive presentation can find a longer description of agents decision problems as well as first order conditions in the Appendix A. Next, we analyze the relevance of labour market specific parameters (e.g. their impact on monetary transmission mechanism), and the decompositions of the observed unemployment rate, the level of employment and the growth rate of real wages into shocks. Finally, we present results of an additional exercise — the analysis of misspecification of SoePL-2012 based on DSGE-VAR approach. In the third section, we compare two versions of SoePL models: the older SoePL-2011 (where unemployment is excluded) to the newer SoePL-2012 (where unemployment is included). We check the identification of labour market shocks, discuss the forecasting precision of those models, and compare the impulse response functions focusing on the question of which parameters are responsible for the differences.

1 New Keynesian wage Phillips curve

Dynamic stochastic general equilibrium models, particularly the most conventional and influential models of Christiano et al. (2005) and Smets and Wouters (2007), originally neglected the phenomenon of unemployment. The need to address the problem of unemployment, however, has never been questioned and last couple of years have seen growing literature on incorporating unemployment into DSGE models, e.g. Christiano et al. (2010a), Christoffel et al. (2009), Gertler et al. (2008), Galí et al. (2011), Walsh (2005). In most of the studies reference to unemployment results from applying labour market search theory framework, based on the ideas of Mortensen and Pissarides (1994).

The search and match at the labour market framework is not the only way to tackle the phenomenon of unemployment. Particularly appealing approach, due to its simplicity and low cost of implementation, is that of Galí (2011b), applied to Smets and Wouters (2007) model in Galí et al. (2011). We are going to focus on this approach and start the paper with a brief review of the Galı's idea.

Galí's approach should be considered more as a reinterpretation of a standard New Keynesian model than an extension of the model. In this framework unemployment is an equilibrium phenomenon and results from market power i.e. wage markup, which itself is a result of imperfect substitution between different types of labour. Fluctuations of unemployment are due to nominal wage rigidities[1]. The framework allows to take unemployment into account *per se*, but also solves an identification problem between labour supply preference and wage markup shocks, pointed out by Chari et al. (2009). The distinction between the two is important from the point of view of policy implications. The labour supply shock is a manifestation of a preference shift and as such should be accommodated by the policy. The wage markup shock, on the other hand, is a result of inefficiency and policy makers may want to or should address this inefficiency to stabilize output fluctuations.

The framework assumes that within each representative household there is a continuum of members, specialized in particular type of labour. Members of each type of labour are represented by a "union", which task is to set the nominal wage. Process of wage setting is subject to a standard Calvo (1983) scheme. When a union is allowed to optimize, it chooses a wage which maximizes household's utility. The wage that follows is equal to marginal rate of substitution between consumption and employment augmented by a wage markup. When a household's member decides whether to participate in the labour market, she takes labour conditions (wages) as given and cares only about relation of consumption-equivalent of wage and disutility of work. Notice, however, that unemployment has no impact on consumption of the household's member because consumption is subject to risk sharing. Since prevailing wages are augmented by a wage markup, the supply of labour will hence exceed the demand for labour. In other words, exploitation of monopoly power condemns some individuals to unemployment. However, resulting unemployment should be seen as involuntary because it involves individuals willing to work (at the given wage) but unable to find employment. To be more specific we review the

[1] In the more general setup, when the wage markup (λ_t^w) is stochastic, one finds stochastic markup shocks the other source of unemployment fluctuation.

most important steps to be done to derive the wage equation, the labour supply equation, and the unemployment rate equation.

1

1.1 The wage inflation equation

Galí's model considers the economy containing a large number of identical (representative and „large") households, each composed of continuum of members. The members of a household are represented by the unit square and indexed by $(i, j) \in [0,1] \times [0,1]$, where i indicates specific labour service (e.g. profession) of the member and the second index j indicates disutility form work. The disutility is given by $\zeta_t^h j^{\sigma_L}$ for employed and zero otherwise. The stochastic term ζ_t^h, called labour supply shock, represents preference shift; σ_L determines the elasticity of marginal disutility of work, $\frac{1}{\sigma_L}$ is sometimes called Frisch elasticity of labour. The period utility of the household is defined as follows:

$$U\left(C_t, \{h_t(i)\}; \zeta_t^h\right) \equiv \ln C_t - \zeta_t^h \int_0^1 \int_0^{h_t(i)} j^{\sigma_L} \,\mathrm{d}\, j \,\mathrm{d}\, i = \ln C_t - \zeta_t^h \int_0^1 \frac{h_t(i)^{1+\sigma_L}}{1+\sigma_L} \,\mathrm{d}\, i \tag{1.1}$$

where C_t is the households' consumption basket[2] and $h_t(i) \in [0,1]$ is the fraction of the household's members employed in period t specialized in labour type i. Under such conditions a simplified decision problem of the representative household is fairly standard. One maximizes:

$$\mathop{\mathrm{E}}_0 \sum_{t=0}^{\infty} \beta^t U\left(C_t, \{h_t(i)\}, \zeta_t^h\right) \quad \text{subject to} \quad P_t C_t + Q_t B_t \leq B_{t-1} + \int_0^1 W_t(i) h_t(i) \,\mathrm{d}\, i + D_t \tag{1.2}$$

where P_t is the price index of the consumption basket, $W_t(i)$ is the nominal wage of labour type i, $Q_t B_t$ is the value of risk free bonds purchased by the household, D_t is a lump-sum component of household's income. Solving the problem (using Dixit-Stiglitz agregator) one may obtain demand function for C_t.

The labour is heterogeneous, hence workers (or unions representing each profession) have monopolistic power to set wages for the given profession $W_t(i)$. The aggregate demand for specific labour services, $h_t(i)$, is determined by firms. Thus, each household takes $W_t(i)$, $h_t(i)$ as given. In this setup, wages are assumed to be sticky of Calvo type. The unions reoptimize wages with probability $1-\xi_w$, but the fraction ξ_w of workers (unions) is unable to optimize so (in the simplest variant of the Calvo scheme) they keep wages unchanged. When nominal wages are set, the firms determinate their demand for labour and households send their members with the lowest work disutility.

Following Erceg et al. (2000), we consider optimal wage setting problem in Calvo framework. Workers (unions) choose wage $W_t^\star$ to maximize households utility taking into account budget constraints and aggregate demand for labour. We denote by $h_{t+k|t}$ the demand in period $t+k$ for labour whose wage was set in period t. The first order wage reoptimization condition of workers is given by:

[2] One may use Dixit-Stiglitz CES agregator to define the basket and the consumer price index. In the sake of simplicity, we neglect that dimension of the problem.

$$\sum_{k=0}^{\infty}(\beta\,\xi_w)^k \mathop{\mathrm{E}}_t\left[\frac{h_{t+k|t}}{C_{t+k}}\left(\frac{W_t^\star}{P_{t+k}}-\lambda^w mrs_{t+k|t}\right)\right]=0$$

where $W_t^\star$ is the optimal wage that maximizes household utility, $\lambda^w \equiv \frac{\epsilon^w}{\epsilon^w-1}$ is the wage markup and ϵ^w is the elasticity of substitution between different types of labour[3], $mrs_{t+k|t} \equiv \zeta_{t+k}^h\, C_{t+k}\, h_{t+k|t}^{\sigma_L}$ is the marginal rate of substitution between consumption and employment in period $t+k$ for the workers whose wage is reset in period t.

Log-linearizing the condition around deterministic (and zero inflation) *steady state* leads to the aggregate nominal wage equation; version derived by Galí is as follows (a tilde indicates variables in logs):

$$\widetilde{\pi}_t^w = \beta \mathop{\mathrm{E}}_t \widetilde{\pi}_{t+1}^w - \Lambda^w\left(\widetilde{\mu}_t^w - \widetilde{\lambda}^w\right) \tag{1.3}$$

where: $\widetilde{\pi}_t^w \equiv \widetilde{w}_t - \widetilde{w}_{t-1}$ is wage inflation; $\Lambda^w \equiv \Lambda^w\left(\beta, \lambda^w, \xi^w, \sigma_L\right) > 0$ is a function of parameters; $\widetilde{\mu}_t^w \equiv \left(\widetilde{w}_t - \widetilde{p}_t\right) - \widetilde{mrs}_t$ is average markup, and $\widetilde{mrs}_t$ is economy's average marginal rate of substitution[4].

Similar wage inflation equation may be found in many New Keynesinan DSGE models. Apart from different parametrization, the wage equation is usually composed of two blocks. The first block is related to wage expectations (possibly modified by more sophisticated indexation scheme applied by unions which cannot reoptimize wages). The second one expresses the role of imperfect competition and frictions on the labour market. When wage rigidity is high (Calvo probability ξ^w close to one), wage inflation is determined mainly by the first block. When wages are more flexible, impact of the second block dominates.

1.2 Labour supply and unemployment

It is worth emphasizing that much more sophisticated structure of the representative household designed by Galí does not change main features of the wage equation in New Keynesian DSGE models. To see why more complicated structure of households is useful, let's turn to labour supply. A household member of profession i has work disutility $\zeta_t^h\, j^{\sigma_L}$. Taking as given labour conditions and household welfare, she will work if real wage at least compensates her disutility expressed in term of consumption (household's marginal value of income), hence the marginal supplier, $h_t^s(i)$, is given by:

$$\frac{W_t(i)}{P_t} = \zeta_t^h\, C_t\, h_t^s(i)^{\sigma_L}. \tag{1.4}$$

The aggregate labour supply is $H_t^s = \int_0^1 h_t^s(i)\,\mathrm{d}\,i$. Logarithmic transformation and integration over professions leads Galí to the following approximation of aggregate labour supply:

$$\left(\widetilde{w}_t - \widetilde{p}_t\right) = \widetilde{C}_t + \sigma_L \widetilde{H}_t^s + \widetilde{\zeta}_t^h. \tag{1.5}$$

[3]It should be considered a measure of labour heterogeneity. In the papers of Galí λ^w is called *the desired or frictionless wage markup* to emphasize distortions generated by wage stickiness, when $\xi^w > 0$.

[4]Economy's average marginal rate of substitution is defined as follows:

$$mrs_t = \zeta^h\, C_t\, H_t^{\sigma_L}, \qquad \text{where} \qquad H_t = \int_0^1 h_t(i)\,\mathrm{d}\,i \qquad \text{and} \qquad \widetilde{mrs}_{t+k|t} = \widetilde{mrs}_{t+k} - \epsilon^w \sigma_L\left(\widetilde{w}_t^\star - \widetilde{w}_{t+k}\right)$$

The unemployment rate, defined as $u_t = \frac{L_t - E_t}{L_t}$, is approximated by $u_t = \widetilde{H}_t^s - \widetilde{H}_t$, which means that labour force (L_t) is reduced to labour supply (H_t^s) and employment (E_t) is approximated by labour demand (H_t). Neglecting the change of variables, the approximation is correct for small u_t.

1

1.3 Wage Phillips curve

Having the labour supply equation and the unemployment rate approximation one may reformulate the wage function and express the growth rate of nominal wages as a function of unemployment rate — this is basically the New Keynesian wage Phillips curve.

$$\widetilde{\pi}_t^w = \beta E_t \widetilde{\pi}_{t+1}^w - \Lambda_w \sigma_L \left(u_t - u^n\right) \tag{1.6}$$

where $u^n = \frac{\mu^w}{\sigma_L}$ is the "natural" rate of unemployment, the rate of unemployment that would prevail in the absence of nominal rigidities.

1.4 Comments and remarks

The framework presented above gives a very raw approximation of the labour market. The unemployment is a problem of households, but the unemployed household members are not forced to reduce their consumption. Due to perfect consumption insurance against labour market outcomes the consumption of employed and unemployed is the same. Due to preferences additively separable in consumption, the unemployed enjoys leisure keeping the level of consumption.

The firms always rent desired basket of labour services. Labour is heterogeneous, but firms never face a mismatch problem. They need not search properly skilled workers and they do not pay any cost of hiring etc. There is no cost of job searching. Labour market imperfections and frictions lead to inefficient allocation of labour. This is, however, a macroeconomic issue. The firms do not face microeconomic costs of any labour market failure. These drawbacks are apparent but they are a natural result of the approximation[5].

In the framework under consideration, labour is treated as indivisible, so a worker works a fixed number of hours. Hence variations of labour input result from variations of employment

[5] In the more general setup of the DSGE model with labour market friction, Galí (2011a) defines the household's period utility function of the form:

$$U(C_t, L_t) \equiv \ln C_t - \frac{\zeta^h}{1+\sigma_L} L_t^{1+\sigma_L}, \qquad L_t = H_t + \psi U_t \tag{1.7}$$

where L_t is an index of total effort, H_t is the fraction of employed household members, and U_t is the fraction of unemployed (job searching), $\psi \in [0,1]$ is the marginal disutility generated by unemployed relative to employed member of the household. The labor force is then given by the sum $H_t + U_t$. He assumes employment at firms to evolve according to:

$$H_t = (1-\delta^h) H_{t-1} + x_t U_t^0 \tag{1.8}$$

where δ^h is a constant separation rate, x_t is the job finding rate, and U_t^0 is the fraction of unemployed members at the beginning of the period t. In addition, to mention the most important extension, there is an exogenous cost of hiring payed by optimizing firms, and wages settings based on Nash bargaining protocol. In general, such specification solves some of issues indicated above. The model of Christiano and others (2010a) is another example. These models show how one may improve/adjust specification of the basic variant.

(the extensive margin), not working hours. Galí points out that the extensive margin dominates the observed variation of the total working hours so indivisible labour assumption seems to be good approximation. Christiano considers this point the core of re-interpretation of the standard New Keynesian model *in which variations in the number of hours worked by the representative household are interpreted as variations in number of people working* (see Christiano, 2011). It may, however, lead to nonstandard definition of unemployment[6].

In the Galı's perspective, unemployment and labour supply are — in fact — redundant variables, there is no impact of the unemployment on any agent's decision. Hence there is no change of equilibrium conditions that determine non-market variables. Unless the Taylor interest rate rule includes the unemployment rate, the feature does not influence monetary policy as well. On the other hand, these features (in fact drawbacks) could be also an advantage. The inclusion of new equations and the extension of the model should not change the properties of the original version. Reinterpretation of New Keynesian model is in some sense neutral. Therefore the framework under consideration seems to be particularly well suited for already large models, for which further extensions become problematic in terms of operationality and transparency.

[6] In general, according to Christiano et al. (2010a), to be unemployed one must (1) be available for work, (2) take steps to secure employment, and (3) the failure to find a job should make him worse off relative to one who got a job. Further, Christiano (2011), commenting on Galí, Smets and Wouters model, emphasizes three additional points:

1. The model implies that unemployed are better off than employed.
2. The conclusion regarding the rise of power of unions in US is at odds with the data of unions density.
3. The general conclusion that countries with the highest union power should face the highest unemployment does not agree with data of union density.

Taking into account official US definition of unemployment, Christiano questions existence of unemployment in the case at hand, because in the Galı's model the unemployment does not satisfy any one condition.

The critique of Christiano simplifies a concept of wage markup to power of labour unions. In the framework under consideration, labour heterogeneity is the primary source of wage markup, hence the wage markup is a measure of labour heterogeneity. This feature of labour is responsible for the structural mismatch and unemployment. The power of unions may be a shadow of the feature, but it may be also an independent source of labour market frictions. Moreover, unions are able to reduce the mismatch. Therefore we consider these points important but not conclusive.

2 The DSGE SoePL model

In this section we describe the basic structure of the SoePL model and the way the unemployment is incorporated into it. We limit the presentation in the main part of the paper to a brief and non-technical overview. The compact description of the model (in particular the agents' decision problems as well as equilibrium conditions' equations) has been moved to Appendix A.1, see also Grabek et al. (2011).

2.1 SoePL — A non-technical overview

SoePL is a model of small open economy estimated on Polish data. It features nominal prices and wages stickiness as in Calvo (1983), as well as real rigidities such as external habit persistence in consumption, capital adjustment costs, and Calvo-type rigidity for employment.

There are four types of agents in the model: households, firms, monetary and fiscal authorities. Households and firms are forward-looking and optimizing (maximizing utility/profits), whereas monetary and fiscal authorities follow *ad hoc* rules: interest rate rule and balanced budget rule.

There are five intermediate goods markets: domestic, imported consumption/investment/export goods, and export goods. Each of the market is characterized by monopolistic competition and stochastic markup. Price setting mechanisms are subject to Calvo schemes. Domestic intermediate goods producers use Cobb-Douglas production function with capital and labour services and common stochastic (stationary) productivity. At the final stage, domestic intermediate goods are combined with appropriate imported components to form final goods: private consumption, investment and exported goods (there is no import component in government spending).

Households derive utility from consumption, leisure and cash. Apart from consumption and cash, households allocate their financial resources into physical capital investments, which are subject to investment-specific technology shock, and domestic and foreign interest rate bearing assets, with foreign interest rates being subject to risk premium shock. Households' income consists of interest on financial assets holdings, wages, rent on physical capital services to domestic firms, firms' profits as dividends, and lump-sum government transfers. All of the income and consumption expenditures are appropriately taxed with payroll, labour-income, capital-income, and consumption tax. Heterogeneity of labour allows households to set wages, being subject to Calvo scheme, and exploit their monopoly power in the form of stochastic wage markup.

Monetary authority follows a Taylor rule with interest rate smoothing. Interest rate reacts to deviations of consumer inflation from stochastic inflation target and deviations of output from the *steady state*, as well as to growth rates (first differences) of inflation and output. Fiscal budget is assumed to be balanced in every period. Interactions of government spending and effective tax rates are described with a separate SVAR model.

Foreign economy is assumed to consist of the euro area and the US. Interactions of foreign interest rates, outputs, inflation rates and EUR/USD exchange rate are also provided by a SVAR model.

2.2 Extensions of DSGE SoePL model — SoePL-2012

In the older version of the DSGE SoePL model (e.g. SoePL-2011), there was no reference to unemployment, although all the prerequisites for the discussed type of unemployment — nominal wage rigidities and stochastic wage markup (as in Galí, Smets, Wouters model) — were present. Introducing unemployment, we do not follow Galí's framework precisely, although we believe that our proceedings are, in the end, equivalent to the solution presented in the section 1. In particular, we do not introduce explicitly the additional structure of households (the additional dimensions marked by i, j — see section 1), hence our utility function of household can be considered a packed (or a reduced form) version of the original function. In our opinion, the additional structure of households is important to motivate existence of unemployment at the level of a single person or members of the household (sub-micro level), however to solve the household's problem, all additional dimensions must be integrated out. Keeping in mind (hidden in formal notation that we apply) behaviour of households members, we focus on the decision maker — the representative household which is able to supply heterogeneous labour services[7].

2

The labour services, $h_{l,t}$, may be measured in hours worked or in persons (or full-time jobs). The standard DSGE New Keynesian models employ the former approach. Indivisibility of labour stressed by Galí could suggest the latter. The distinction may be regarded irrelevant if each member of household must work a constant number of hours (n). In this case variations in hours worked are variations at extensive margin. Therefore, the interpretation of the labour measurement does not seems to be crucial for the definition of unemployment[8].

2.2.1 Household's utility function

Taking into account interpretation presented above, the old version of the utility function of households does not require any adjustments to enhance the model and explain unemployment explicitly. Nevertheless, following Galí et al. (2011) we introduced an endogenous preference shifter $\tilde{n}_t$, hence the packed (reduced form) utility function is defined as follows:

$$\underset{t}{\mathrm{E}} \sum_{s=0}^{\infty} \beta^{t+s} \left[\zeta_t^c \ln\left(C_{l,t} - b\, C_{l,t-1}\right) - \tilde{n}_t\, A_L\, \zeta_t^h \frac{\left(h_{l,t}\right)^{1+\sigma_L}}{1+\sigma_L} + \zeta_t^q A_q \frac{\left(\frac{Q_{l,t}}{z_t^+ P_t}\right)^{1-\sigma_q}}{1-\sigma_q} \right] \tag{2.1}$$

where $C_{l,t}$ is the consumption, $h_{l,t}$ is the labour service provided by the household, $Q_{l,t}$ — cash holdings, ζ_t^c, ζ_t^h, ζ_t^q are preference shocks. We apply the following definition of the preference shifter:

$$\tilde{n}_t \equiv n_t U_{c_l,t} = n_t \left[\left(1+\tau_t^c\right)\gamma_t^{cd}\, \psi_{z^+,t}\right], \qquad n_t = n_{t-1}^{(1-\vartheta_n)} U_{c_l,t}^{-\vartheta_n} = n_{t-1}^{(1-\vartheta_n)} \left[\left(1+\tau_t^c\right)\gamma_t^{cd}\, \psi_{z^+,t}\right]^{-\vartheta_n}$$

[7] Notice that there is another, neglected (in this case, by many authors of DSGE models) dimension/part of the labour market — the transformation of heterogeneous labour supply into homogeneous labour services. Assuming neutrality of this transformation (zero costs, lack of any distortions or externalities) such simplification seems to be justified.

[8] It is worth noting that Galí et al. (2011) performed an empirical robustness check where they substituted between hours worked and employment as observable variable concluding that the main results are not affected.

2

and $U_{c,t}$ is the marginal utility of consumption, τ_t^c is the consumption tax rate, $\gamma_t^{cd} \equiv \frac{P_t^c}{P_t^d}$ is the ratio of consumer to producer prices, and $\psi_{z^+,t}$ is the marginal utility of income.

The purpose of the shifter is to counteract the wealth effect on labour supply. For example, in a standard model, due to wealth effect, labour supply rises after a positive interest rate shock — it moves counter-cyclically — whereas empirical estimates conducted by Christiano et al. (2010a) show procyclical response of labour force. By imposing negative correlation between consumption and disutility of labour in the form of endogenous preference shifter, we counteract the wealth effect and generate response of labour supply in accordance with empirical estimates. The specification of the shifter differs slightly from that of Gali's, which in turn was motivated by Jaimovich and Rebelo (2009). The above expression of marginal utility of consumption is derived from first order condition of household's utility maximisation subject to budget constraint (see Grabek et al., 2011).

The endogenous preference shifter has no impact on the first order conditions of the household decision problem. The usual set of equations derived from the household problem remains unaltered — see Appendix A.1 for details.

2.2.2 Real wages

Wages are set by unions, but they are subject to Calvo-type rigidity. Hence the ratio $1-\xi_w$ of households (unions) may set optimal wage; the others, ξ_w, cannot reoptimize, so they adjust (index) wages to inflation of consumer prices, π_t^c, the value of the expected stochastic inflation target $\overline{\pi}_{t+1}$ and the expected technology growth $\mu_{z^+,t+1}$:

$$W_{l,t+1} = \left(\pi_t^c\right)^{\kappa_w} \left(\overline{\pi}_{t+1}^c\right)^{1-\kappa_w} \mu_{z^+,t+1} W_{l,t}.$$

The optimal wage maximizes expected utility derived from the wage income after taking into account disutility of labour. Again, we apply the packed or reduced form utility function so the objective function is as follows:

$$\max_{W_t^{new}} \operatorname{E}_t \sum_{s=0}^{\infty} \left(\beta \xi_w\right)^s \Bigg[-\zeta_{t+s}^h \tilde{n}_{t+s} \frac{\left(h_{l,t+s}^d\right)^{1+\sigma_L}}{1+\sigma_L} + \upsilon_{t+s}\left(1-\tau_{t+s}^y\right)\left(1-\tau^w\right) \times \\ \times \left(\left(\pi_t^c \dots \pi_{t+s-1}^c\right)^{\kappa_w} \left(\overline{\pi}_{t+1}^c \dots \overline{\pi}_{t+s}^c\right)^{1-\kappa_w} \left(\mu_{z,t+1} \dots \mu_{z,t+s}\right) W_t^{new}\right) h_{l,t+s}^d \Bigg]$$

where υ_{t+s} is the marginal utility of income, τ^w is the rate of social security contribution payed by employees and τ_t^y is the effective personal income tax rate. When solving the maximization problem, household takes into account labour demand function (λ_t^w is the stochastic wage markup and H_t^d is aggregate labour demand):

$$h_{l,t}^d = \left[\frac{W_{l,t}}{W_t}\right]^{\frac{\lambda_t^w}{1-\lambda_t^w}} H_t^d. \tag{2.2}$$

By doing so, households (unions) make use of the imperfect substitution of labour — they exploit their monopoly power. After log-linearization, first order condition combined with a

Calvo-induced formula for the average real wage $\left(\bar{w}_t\right)$ produces the equation (a hat denotes a log-linearized variable):

$$\widehat{\bar{w}}_t = \frac{b_w\,\xi_w}{b_w\left(1+\beta\,\xi_w^2\right)-\lambda^w\,\sigma_L}\left[\left(\widehat{\bar{w}}_{t-1}+\beta\,\underset{t}{\mathrm{E}}\widehat{\bar{w}}_{t+1}\right)+\kappa_w\left(\widehat{\pi}^c_{t-1}-\beta\widehat{\pi}^c_t\right)\right.$$
$$\left.+\left(1-\kappa_w\right)\left(\widehat{\bar{\pi}}^c_t-\beta\,\underset{t}{\mathrm{E}}\,\widehat{\bar{\pi}}^c_{t+1}\right)-\left(\widehat{\pi}_t-\beta\,\underset{t}{\mathrm{E}}\,\widehat{\pi}_{t+1}\right)\right] \quad (2.3)$$
$$+\frac{\lambda^w-1}{b_w\left(1+\beta\,\xi_w^2\right)-\lambda^w\,\sigma_L}\left[\sigma_L\,\widehat{H}^d_t+\widehat{\zeta}^h_t+\widehat{\lambda}^w_t+\widehat{n}_t+\widehat{\gamma}^{cd}_t+\frac{\tau_c}{1+\tau^c}\,\widehat{\tau}^c_t+\frac{\tau_y}{1-\tau^y}\,\widehat{\tau}^y_t\right]$$

Apart from components related to the endogenous preference shifter the introduction of unemployment into SoePL has not changed the wage equation.

2.2.3 Labour supply

The existence of nominal wage rigidities justifies treating wage as exogenous in the process of setting optimal labour supply. Hence, household l, taking its wage as given, sets labour supply $h^s_{l,t}$ by maximizing household's utility:

$$\max_{h^s_{l,t}}\ \underset{t}{\mathrm{E}}\sum_{s=0}^{\infty}\beta^s\left[-\zeta^h_{t+s}A_L\tilde{n}_{t+s}\frac{\left(h^s_{l,t+s}\right)^{1+\sigma_L}}{1+\sigma_L}+\psi_{z^+,t+s}\left(1-\tau^y_{t+s}\right)\left(1-\tau^w\right)\overline{w}_{t+s}\,h^s_{l,t+s}\right]. \quad (2.4)$$

The first order condition describes labour supply of household. The log-linearization and some additional manipulation give the following equation:

$$\widehat{H}^s_t=\frac{1}{\sigma_L}\left[\left(\widehat{\bar{w}}_t-\widehat{\gamma}^{cd}_t-\frac{\tau^y}{(1-\tau^y)}\widehat{\tau}^y_t-\frac{\tau^c}{1+\tau^c}\widehat{\tau}^c_t\right)-\left(\widehat{\zeta}^h_t+\widehat{n}_t\right)\right]. \quad (2.5)$$

2.2.4 Unemployment

We define two unemployment rates:

$$un^g_t=\frac{H^s_t/n-H^d_t/n}{H^s_t/n}=\frac{E^s_t-E^d_t}{E^s_t}\quad\text{and}\quad un_t=\frac{\tilde{E}^s_t-\tilde{E}_t}{\tilde{E}^s_t}\approx\frac{H^s_t-\tilde{E}_t}{H^s_t} \quad (2.6)$$

where n is the number of hours worked per employee and E^s_t, E^d_t are the full-time employment (supply and demand, respectively). The first rate is casted to conditions of the model (the model's unemployment rate), the second refers to observed variables: $\tilde{E}_t$ is observed employment and $\tilde{E}^s_t$ is the labour force (or number of persons looking for a job given the wage), however we assume $H^s_t=\tilde{E}^s_t$ and $n=1$ (see equation (A.14) and comments below this formula), to use the approximation. The *steady state* of these rates is the same:

$$un^g=un=1-\left(\frac{1}{\lambda^w}\right)^{\frac{1}{\sigma_L}} \quad (2.7)$$

and

$$un_t = un_t^g - \frac{\tilde{E}_t - H_t^d}{H_t^s} = un_t^g + dun_t.$$

Two different unemployment rates are necessary because aggregate demand for labour services H_t^d set by firms is not equal to employment $\tilde{E}_t$ (times number of hours worked per employee). The firms determine demand for labour services (hours) but, due to a kind of labour market imperfection, cannot adjust employment level immediately. There exists a Calvo type rigidity and $1-\xi^e$ ratio of firms can set optimal employment, others have to keep employment unchanged. The solution to the Calvo problem[9] gives the aggregate employment $\tilde{E}_t$. If $\xi^e \to 0$, then $un_t \to un_t^g$. The labour market Calvo-type friction influences the real variable but it has a nominal nature — the *steady state* unemployment as well as the *steady state* employment (etc.) are not altered. We included this feature to solve an empirical (measurement) problem: to match the variance of observed employment (a very smooth variable) to variance of modeled employment (a very erratic variable). The impact of this rigidity is very limited, because we neglect any costs of the discrepancy between the labour demand and the employment. Perhaps, it should be a subject of further research to remove the constraint.

2.2.5 Wage Phillips curve

Taking into account the wage equation, the labour supply equation as well as the definitions of the unemployment rate, after some manipulations, one can derive the following log-linearized relation between (nominal) wage inflation and the unemployment rate – the New Keynesian wage Phillips curve:

$$\begin{aligned}\widehat{\pi}_t^w = {} & \beta \operatorname*{E}_t \widehat{\pi}_{t+1}^w - \left[\left(1-\kappa_w\right)\left(\beta \operatorname*{E}_t \widehat{\bar{\pi}}_{t+1}^c - \widehat{\bar{\pi}}_t^c\right) + \kappa_w\left(\beta \widehat{\pi}_t^c - \widehat{\pi}_{t-1}^c\right)\right] \\ & - \left(\beta \operatorname*{E}_t \widehat{\mu}_{z^+,t+1} - \widehat{\mu}_{z^+,t}\right) \\ & + \frac{\left(1-\beta\,\xi_w\right)\left(1-\xi_w\right)}{\xi_w}\left[\frac{\left(1-\frac{1}{\lambda^w}\right)}{1+\frac{1}{\sigma_L}\left(1-\frac{1}{\lambda^w}\right)}\right]\left[\frac{1}{\sigma_L}\widehat{\lambda}_t^w - \frac{1-\left(\frac{1}{\lambda^w}\right)^{\frac{1}{\sigma_L}}}{\left(\frac{1}{\lambda^w}\right)^{\frac{1}{\sigma_L}}}\widehat{un}_t^g\right].\end{aligned} \tag{2.8}$$

The wage Phillips curve derived using SoePL-2012 is presented here to illustrate (dis-) similarity of Galı's model and DSGE SoePL — see equation (1.6). The equation is not a part of SoePL-2012 model.

2.3 Results of estimation

2.3.1 Data and estimation procedure

The log-linearized structural form of the model is composed of the first order equilibrium conditions of agents and market clearing conditions. The model is solved using Anderson and Moore algorithm to obtain the reduced form. Next, we transform the reduced form into the so-called transition block of the state space form. Adding equations that link vector of state

[9] The decision problem solved by firms to find optimal employment is shown by equation (A.14) in Appendix A.1.

variables and observed variables — the measurement block — we get the following state space form of the model:

$$\begin{cases} \widehat{\xi}_{t+1} = F_{\xi}(\wp)\,\widehat{\xi}_{t-1} + e_{t+1}, & e_{t+1} \sim N(0, \text{diag}(\sigma_e^2)); \quad \text{/transition block/} \\ \underline{y}_t = A'_x(\wp, \hbar) + H'(\wp)\,\widehat{\xi}_t + u_t, & u_t \sim N(0, \text{diag}(\sigma_u^2)), \quad \text{/measurement block/} \end{cases} \tag{2.9}$$

where $\widehat{\xi}_t$ is a vector of state variables, $\underline{y}_t$ is a vector of observed variables, and matrices $F_{\xi}(\wp)$, $H(\wp)$, $A_x(\wp, \hbar)$ depend on the vector of deep parameters $\wp$ to be estimated (or calibrated). The $\hbar$ is a vector of parameters set to adjust measurement (expert's adjustment of constants). We use $\hbar$ to correct bias which occurs for some variables due to simplification of the theoretical model. The vectors σ_u, σ_e contain standard deviations of the structural shocks e and measurement errors u. The standard deviations of the shocks are estimated (or calibrated) parameters of the model, standard deviations of u are calibrated.

2

In the general case, the deep parameters $\wp$ and variances σ_u^2 (hence matrices $F_{\xi}(\wp)$ and $H(\wp)$ as well) may not be constant over the whole sample — this is the case of (deterministic) regime shift. We allowed for the change of the deep parameters in 1999, when the central bank introduced direct inflation targeting strategy and the fully floating exchange rate regime was applied in Poland. In addition, the government implemented important social reforms (in particular pension's system and health service system) which affected firms' as well as households' behaviour. The regime shift existed in all versions of DSGE SoePL up to the version of 2011. The newest version SoePL-2012 has been estimated over the shorter sample and the period before 1999:1 is not used to compute the likelihood[10], therefore we removed this structural change from the model.

To estimate the vectors $\wp$ and σ_e we apply the standard Kalman Filter based Bayesian estimator. The estimation procedure is composed of two steps: optimization (estimation of posterior mode and the approximation of the covariance matrix at the mode) and Markov Chain Monte Carlo sampling (namely random walk Metropolis-Hastings sampler) to get sample form the posterior distribution of parameters. In what follows we report results of the first step — the optimization.

We use 21 (in most cases seasonally adjusted) quarterly time series (quarterly growth rates of GDP, consumption, investments, exports, imports, real wages, real exchange rate (USD/PLN), employment, GDP deflator, investment deflator, CPI, US GDP, euro area GDP, US GDP deflator, euro area GDP deflator, crude oil prices, real EUR/USD exchange rate and 3-months interbank interest rate (Wibor-3m), unemployment rate, Libor-3m (dollar), Euribor-3m) and the sample covers the period 1996:1–2011:3, with 1996:1–1998:4 as the learning period for Kalman filter.

2.3.2 Transition from DSGE SoePL-2011 to DSGE SoePL-2012

The newer version of our DSGE model, SoePL-2012, includes two additional state variables (and two equations: the labour supply and the endogenous preference shifter), one additional parameter, ϑ_n, related to the endogenous preference shifter[11] as well as one additional observed

[10]The pre 1999:1 observations are still in learning subsample of the Kalman filter, however, so there may be an impact of the older regime on the estimates of parameters. Our sample is still very short and very heterogeneous, hence there is no a first best choice.

[11]The parameter ϑ_n may be used to switch off the preference shifter.

variable (and an equation in measurement block) — the unemployment rate. We also changed one observed variable, namely HP-detrended level of employment used in SoePL-2011 has been replaced by the growth rate of employment[12]. Nevertheless, the specification of the older version SoePL-2011 is nested in the newer version SoePL-2012. Neglecting the preference shifter, the labour supply equation is a post-recursive one and it influences observed unemployment rate (in the measurement block of state space form) exclusively. Since the observed variables cannot alter state variables in the state space form models, there is no additional impact of unemployment on any state variable. Therefore the specification's changes, in particular the inclusion of (state) labour supply and observed unemployment rate, do not impose any additional restrictions on agents' behaviour. There is no impact on macroeconomic equilibrium as well. The new state equation (labour supply) is — from this point of view — a redundant one[13]. Anyway, we use the labour supply state variable (or components of the equation) to define, explain, and forecast observed unemployment rate. On the other hand, the specification's changes (and inclusion of the additional observed variable that alters our information set) are very important because they reveal relations between parameters of the model. The most important one shows how deep parameters σ_L and λ^w determine the *steady state* of the unemployment rate (see equation (2.7))[14]. Therefore there is a reason to rethink Bayesian priors and, perhaps, alter the calibration of other parameters. These changes allow us to identify and estimate the parameters more precisely.

The transition from DSGE SoePL-2011 to DSGE SoePL-2012 involves the change of equations' specification, the change of data set, the change of parameters' calibration and the changes of parameters' estimates. However these changes cannot be assigned to the introduction of unemployment into DSGE model exclusively. We applied a sequential search procedure that relies on recalibration of fixed (data independent) parameters, adjustment of priors and re-estimation of all others parameters. The final variant of the SoePL-2012 has been chosen using several selection criteria, in particular we took into account Laplace approximation of the marginal likelihood and the root mean squared error of forecasts (forecast accuracy measure), because forecasting is one of the most important application of the model. Since our search procedure has been rather informal (heuristic), we cannot guarantee that all changes of priors and fixed parameters are due to inclusion of unemployment.

Table 4 (see Appendix A.2) presents results of optimization (the mode of posterior) for a set of the most important parameters. Table 5 in the Appendix shows *steady state* of the model, which is basically, consistent with sample averages.

[12]In addition, we corrected investment equation in the measurement block of the state space form. The older measurement method applied for SoePL-2011 and earlier versions did not protect cohesion of national account in some specific cases.

[13]Formally, the inclusion of the labour supply equation increases the size of the transition matrix (F_ξ) in the transition block of the state space form (see equation (2.9)) but the rank of the matrix does not change. One may define a kind of output gap based on unemployment (labour supply) and insert it into interest rate rule, see for example (Galí et al., 2011). In this case labour supply becomes an important part of a feedback (e.g. wages — unemployment — interest rate). Such a feedback is an additional behavioural restriction and it alters (for example) agents' reactions to shocks. We tested similar interest rate rule but it did not work well in SoePL-2012.

[14]Perhaps one should estimate the pair $\{un, \sigma_L\}$ or $\{un, \lambda^w\}$ rather than $\{\sigma_L, \lambda^w\}$, because it is easier to set priors for *un*.

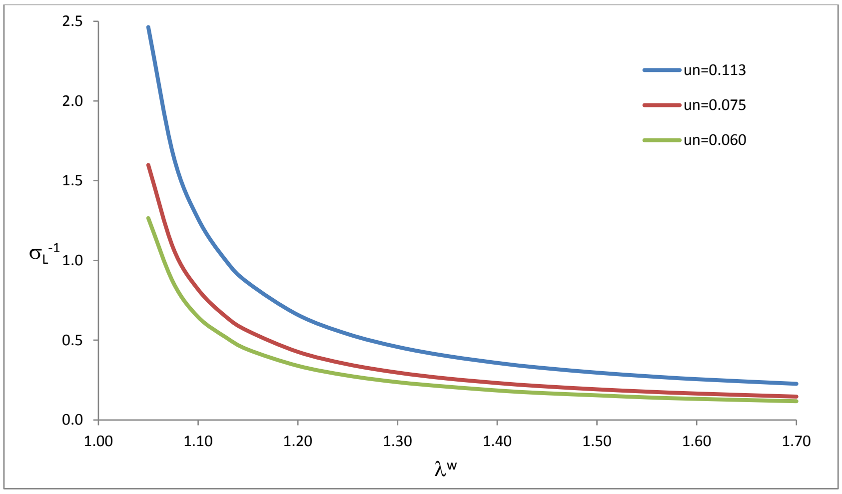
2.4 Parameters of labour market

The *steady state* condition given by the equation (2.7) reveals a relation between the labour market parameters. Figure 1 illustrates the relation between labour supply elasticity σ_L^{-1} and wage markup λ^w for a given level of *steady state* unemployment rate *un*.

The economic meaning of σ_L^{-1} parameter and the estimates of the labour supply elasticity have been discussed by several authors. Keane and Rogerson (2011) provide, for example, an extensive survey of literature and empirical studies. In the context of DSGE models some remarks of Christiano and others (2010b) are especially interesting. We cite here some of their conclusions.

The elasticity of labour supply with respect to the real wage keeping consumption constant[15] (the change of the real wage is of very short duration hence the wealth of households is unaffected) is equal to σ_L^{-1}. If labour supply is measured in hours, which a typical person wants to work (and the economy is populated by identical households), then σ_L^{-1} is called the *Frisch labour supply elasticity*. When labour is measured in number of people, who want to work (employed and unemployed) σ_L^{-1} measures the elasticity with which marginal people substitute in and out of employment in response to change in the real wage (Christiano et al., 2010b, p. 16). The

Figure 1. Labour supply elasticity σ_L^{-1} and wage markup λ^w for a given *steady state* unemployment rate



literature cited by Christiano and others indicates very low values of Frisch elasticity estimated using household data, however it is at odds with business cycle data, since over the cycle (when variations in employment reflect fluctuations in the quantity of people working) the employment fluctuates more than real wages. In terms of DSGE models' macro data we can see people respond elastically to a change in real wages. Therefore one should distinguish between micro- and macro- elasticity. Frisch labour supply elasticity, usually a small number, indicates microeconomic feature; it is the personal elasticity. Aggregate employment fluctuate in response to even small change in the real wage, so the aggregate elasticity can be large (Keane

[15]This type of elasticity is derived from the utility function (2.1).

and Rogerson, 2011). Christiano and others (2010b) consider the concept of differentiated micro and macro elasticities consistent with the household structure, in which a household is composed of large number of members (for technical reasons mapped into points in the interval $(0,1)$), but each member can work full time or not at all. Large macro elasticity just indicates many unemployed near the margin between taking a job and staying at home. The household structure of Christiano is very similar to the structure of Galí described in the paragraph 1.

To be in line with the discussion mentioned above, many authors of macro models constrain the value of σ_L^{-1} to interval of $[1,2]$. Higher value of the elasticity is consistent with aggregated data, the value close to lower bound perhaps does not contradict micro evidences[16]. The calibration of σ_L in the extended model of Galí (2011a) is consistent with micro data ($\sigma_L = 5.0$), the estimates of the parameters in the model of Galí (2011b) is consistent with micro data as well (mode posterior is 3.99). For the SoePL-2012 we obtained value 2.86, which is also consistent with micro data, and perhaps underestimates macroeconomic elasticity of labour supply to real wages.

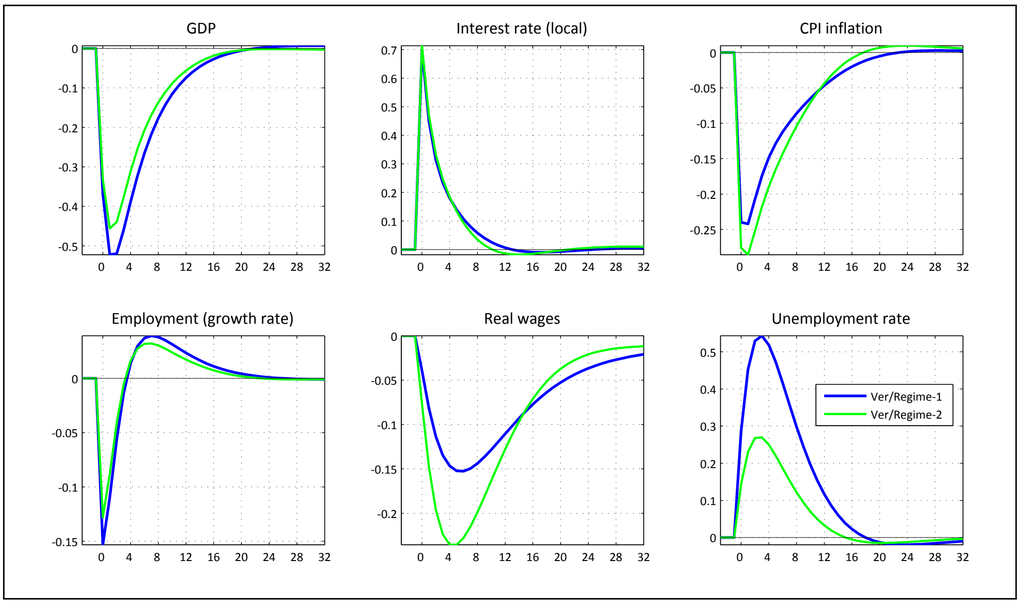
Table 1. Estimates of labour market parameters for the given *steady state* unemployment rate

Log Marginal Like-lihood	Implied st.state unemp. rate (un)	Calibrated value		Estimated value					
		Wage markup (λ^w)	Inv. labour supply el. (σ_L)	Wages Calvo params.		Wage markup shock		Labour supply shock	
				(ξ^w)	(κ^w)	(ρ_{λ^w})	(σ_{λ^w})	(ρ_{ζ^h})	(σ_{ζ^h})
-2234.94	0.113	1.410	2.859	0.660	0.286	0.522	0.471	0.799	0.673
-2236.48	0.113	1.350	2.497	0.658	0.285	0.493	0.474	0.801	0.693
-2238.35	0.113	1.300	2.184	0.654	0.282	0.467	0.476	0.807	0.717
-2241.03	0.113	1.250	1.857	0.648	0.277	0.437	0.478	0.815	0.751
-2244.88	0.113	1.200	1.518	0.637	0.270	0.404	0.479	0.817	0.796
-2250.20	0.113	1.150	1.163	0.619	0.259	0.364	0.479	0.778	0.827
-2253.75	0.113	1.128	1.000	0.601	0.256	0.351	0.478	0.746	0.861
-2260.03	0.113	1.100	0.793	0.571	0.253	0.337	0.477	0.720	0.933

Table 1 shows results of an exercise where we estimated several variants of the model keeping *steady state* unemployment rate constant (σ_L and λ^w were calibrated). The computation has been done using the final version of SoePL-2012. The results suggest that by calibrating a low value of σ_L one forces low Calvo probability for wages ξ^w and lower persistence of labour market shocks ρ_{λ^w}, ρ_{ζ_h}, hence the labour market is, in that case, much more flexible. The Laplace approximation of logarithm of marginal likelihood rejects the flexible labour market variant of the model, however. According to the log marginal likelihood model selection criteria, the variant with higher wage markup λ^w (1.41) and higher σ_L (2.86) fits data better.

[16]However, the rule of a uniform calibration is criticized by Keane and Rogerson (2011). They point out a bad practice of estimation the elasticity in one context and movement the value into other context. There are models where micro and macro elasticities are the same, but there some exceptions. Some life-cycle models differentiate labour supply elasticities, e.g. the elasticity is a function of (for example) age and sex.

Figure 2. Impulse response function of interest rate shock — low and high labour supply elasticity



To check the impact of these parameters on monetary transmission mechanism we performed some additional comparative exercises. We computed the impulse response functions (IRFs) for several shocks using two sets of parameters: the baseline of the SoePL-2012 model (see Table 4 in Appendix) where labour supply is inelastic (denoted Regime-2) and the other (denoted Regime-1) where the labour supply is elastic ($\sigma_L = 0.79$, $\lambda^w = 1.10$, all other parameters are unchanged). Figure 2 compares IRFs of interest rate shock (we moved other results to Appendix A.3.). Taking into account CPI inflation and GDP exclusively, the possible bias due to imprecise estimate of σ_L for a given *steady state* unemployment rate seems to be moderate, hence the monetary policy focused on these variables (and approximated by the Taylor rule) is relatively robust. However, the bias may be important for other variables, especially unemployment and real wages.

2.5 Shock structure of labour market variables

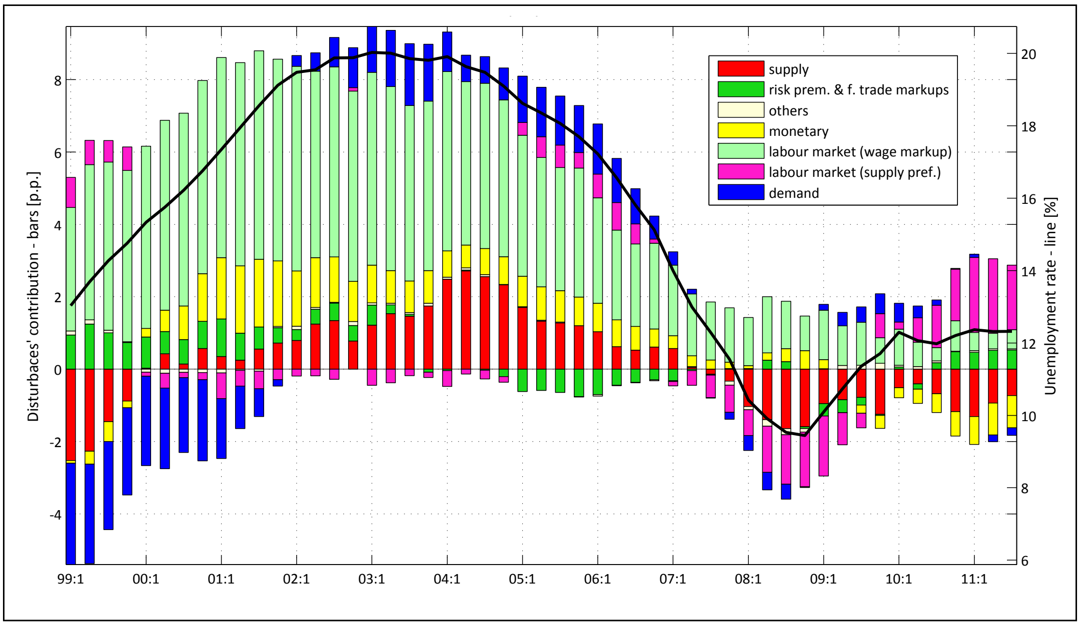
Figures 3-5 show contribution of disturbances to labour market observed variables: unemployment rate, employment level and real wage growth rate[17]. The disturbances have been aggregated into relatively homogeneous blocks[18], and the net impact is presented.

[17]The shock structure for the level of employment is much more readable, hence we computed the structure for level rather than growth rate. It is possible because employment is considered a stationary variable.

[18]The supply block is composed of domestic markup shock, technological (TFP) shock, effective rate of indirect taxes shock, world prices of raw materials shock, US/euro exchange rate shock; the demand block contains: investment technological shock, consumption preference shock, non-stationary technological shocks, public consumption shocks, euro area and US output shocks; the risk premium and foreign trade markup blok contains: risk premium shock, export markup shock, import markups shocks; the monetary block is composed of interest rate shock and inflation target shock.

2

Figure 3. Shocks structure of quarterly unemployment rate.



The unemployment rate exceeded *steady state* till the end of 2007. The excessive unemployment rate (the excess over *steady state*) was generated up to 2006 mainly by a series of wage markup shocks . Taking into account the role assigned to the stochastic wage markup by the Galí's idea, one may consider it an evidence of stronger labour heterogeneity and structural mismatch of labour[19] in the period under consideration. It is likely, since after the Russian crisis of 1998 (after pre-accession trade agreements with the EU), facing much more volatile (uncertain) demand for exported goods (see the net impact of risk premium and foreign trade markups shocks — Figure 3) firms adjusted their production profiles and adjusted (not simply reduced) employment[20]. The adjustment is perhaps one of the most important real processes represented by the series of wage markup shocks in the model. In this period, a new generation of young people (the echo of the baby boom of the end of seventies) entered the labour marked, so the high level of the wage markup just represents a part of the phenomenon[21]. Therefore one may suspect a rise of structural mismatch of labour supply and demand since firms were modifying their production profiles at the same time. In addition, institutional regulation of the labour market (in most cases implemented before 1989 when Polish economy started the transition to market economy) slowed down the reconstruction of labour market and perhaps also kept the level of unemployment rate above the *steady state* — the (institutional) labour market rigidity due to excessive regulations is captured by wage markup.

[19]Formally, it is a mismatch of labour supply. Wage markup (elasticity of substitution) represents degree of labour heterogeneity, hence any mismatch is due to inadequacy of labour supply structure. However, taking into account very special nature of this production factor and simplicity of our description of the labour market, it seems to be reasonable to emphasize inadequacy of labour demand as well. Therefore the term *structural mismatch of labour* means structural problems of supply as well as demand of labour.

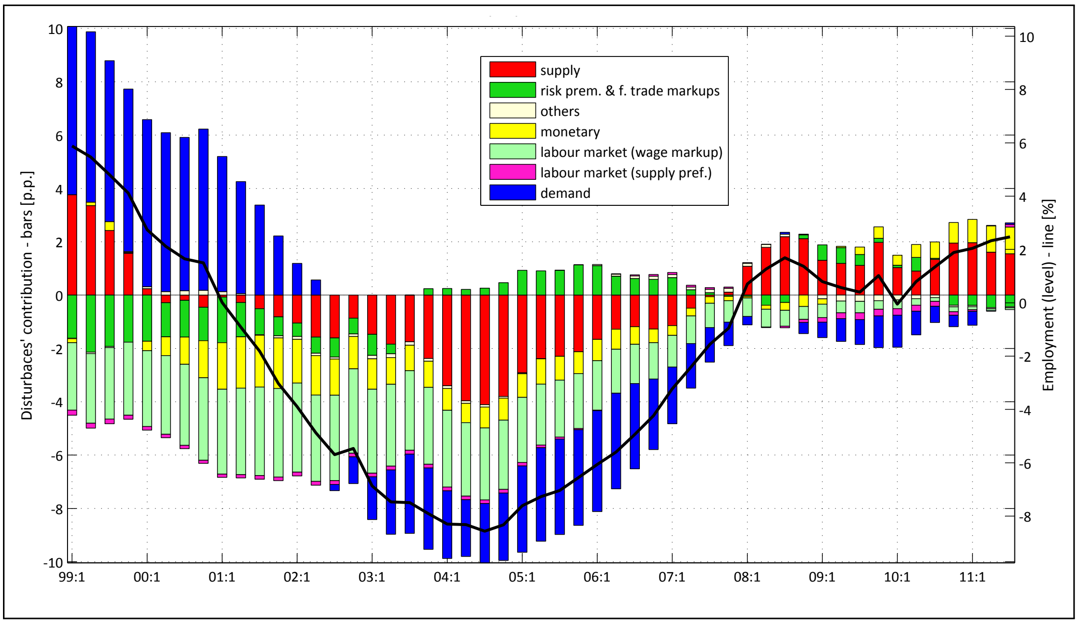
[20]Despite a slowdown of economic growth at the beginning of the century, demand shocks still increased employment and reduced unemployment till the middle of 2002, hence the demand shocks were not a reason why the unemployment rate exceeded the *steady state* level.

[21]The (exogenous) changes of labour supply, the changes of labour force due to migrations and retirement are not explicitly captured by the labour market of the DSGE model. These unrepresented demographic effects likely biased estimates of the wage markup and labour supply preferences shocks — see also Figure 8.

The SoePL model takes into account social contributions payed by employer and employee, however the changes of social contributions collection system are not described. The current form of the system, implemented in 1999, revealed total labour costs paid by employers and costs of pension system paid by employers and employees. Being much more transparent, the newer form of social contributions limited avoidance of the contributions. It increased total labour costs. This phenomenon is also represented by the series of positive wage markup shocks.

On the other hand, the negative impact of the wage markup shocks on employment was relatively stable till 2004 (the date of accession to the EU), when the outflow of labour force (migration) began (see Figure 4). However the unemployment rate started to decline earlier, in 2003, mainly — in terms of DSGE model — due to decline of the wage markup shocks. In statistical terms, unemployment rate started to decline around 2003 because of an increase in retired (the reduction of the labour force). The retirement is another feature not explicitly captured by the model, hence these effects spilled over into wage markups[22]. Employment started to rise ca. 6 quarters later, in the middle of 2004, when negative supply shocks started to disappear as well as the negative impact of wage markup shock declined. The further decline in wage markup impact on employment after the accession to the EU (when migration started

Figure 4. Shocks structure of quarterly employment level.



and labour force declined) looks puzzling. One may expect a rise in the markup[23]. The fact that we observe no such thing — there is very little impact on labour market mismatch and

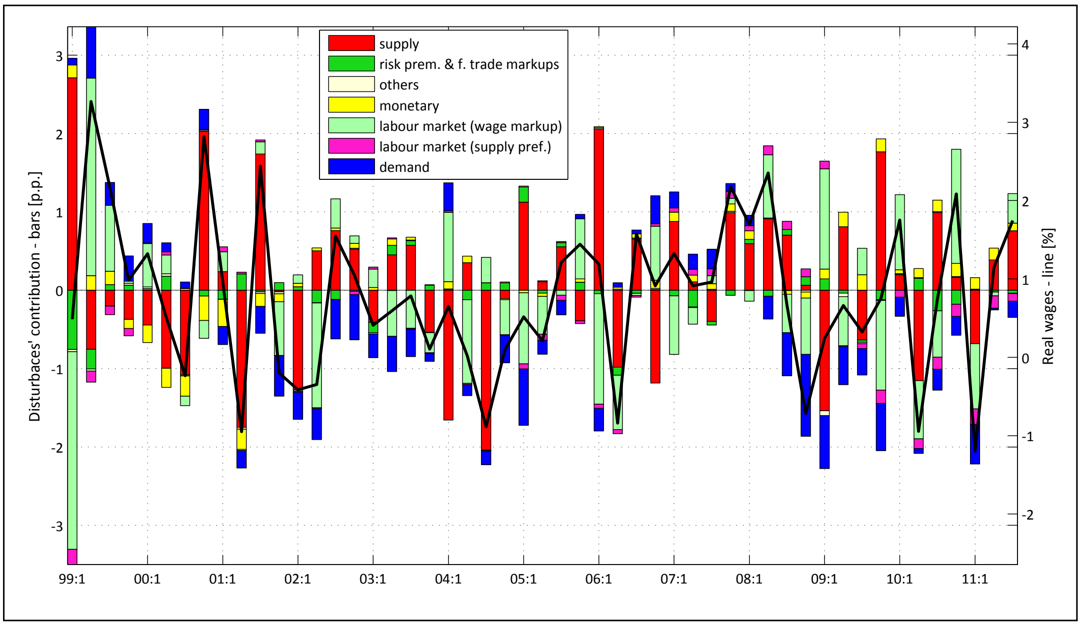
[22]For example, the above mentioned labour market regulations protect employment of the workers which are closed to the retirement age. In many cases, retired wanted to keep their job (perhaps on part-time basis) accepting more flexible (less expensive form the employer point of view, e.g. free from a part of social contributions) contracts. Therefore an increase in working retired reduces the markup but full-time employment declines.

[23]As we noticed, the intensive migration between 2004 and 2008 is not explicitly captured by the model, but the most straightforward outcomes of this process, e.g. decline of labour force and decline of unemployment rate, the rise of wage markup and the rise of wages (etc.), should be observed. It is not the case, hence this phenomenon is likely much more complex.

wage pressure — suggests that it was unemployed or relatively easily replaceable workers who emigrated[24]. After the accession to the UE one may also observe a negative impact of supply shocks on employment. There likely occurred a deeper supply side adjustment (e.g. an upgrade of production technology). We suppose that this supply side adjustment reduced the structural mismatch, however it temporarily increased unemployment (see Figure 3).

The role of labour supply preferences shock was marginal up to 2007 as long as the wage markup shocks dominated unemployment rate. The shrinkage of the labour force — or in model's terms the shift in households preferences towards leisure — which started in the second half of 2006, led to a decline in unemployment and was a significant factor in reaching unprecedentedly low level of unemployment rate in 2008. Had it been not for still positive impact of the wage markup, the level of unemployment rate would have been even lower. After the financial crisis, which led to a significant slowdown in economic growth in Poland, the labour supply preferences have reversed yet again, keeping the unemployment rate above the *steady state* in recent quarters. The effects of labour supply shocks for real wage growth and employment are very limited — in the case of the latter almost negligible as it is mainly demand and supply shocks which drive employment, with non-negligible role of monetary policy shocks, wage markup shock, and exchange rate shocks as determinants of export demand.

Figure 5. Shocks structure of quarterly real wage growth rate.



After 2009, the supply shocks (in particular the negative TFP shocks) stimulated employment, which means that firms decided to use labour less effectively. Perhaps they kept higher employment level even when facing a decline in aggregated demand — they were hoarding labour[25]. It

[24] Analyses of the Polish Labour Force Survey (see e.g. Grabowska-Lusińska and Okólski, 2008; Mioduszewska, 2008) indicate, for example, that amidst emigrants there was an overrepresentation of young men with vocational education from rural areas (there was a general overrepresentation of young individuals with secondary and higher education). Mioduszewska (2008) suggests that it may indeed indicate that the labour market mismatch and the lack of demand for those types of labour led many to leave the country.

[25] Labour hoarding is inadequate adjustment of employment during economic slowdown motivated by hight costs of job rotation. It is considered an additional labour market rigidity, see for example Strzelecki et al. (2009).

agrees with findings of Strzelecki et al. (2009) who studied the phenomenon of labour hoarding using very different analytical methods. They identified some other periods of more intensive labour hoarding: 1998–1999 and 2000–2001, but we do not note the positive impact of supply shocks on employment level in these years.

The shock structure of real wage growth rate does not exhibit any clear patterns. One may observe a more frequent negative rather than positive impact of demand shocks on the growth rate especially after 2001. The supply shocks more frequently increased the growth rate of real wages. The impact of the wage markup shocks was strong but rather erratic. Since 2002 the demand shocks were negative (e.g. they led to a decline in employment), hence these shocks hit nominal wages stronger than inflation. Provided the wage markup shock captures at least a part of migration effects, one may expect an impact of the shock on wages. It is hard to observe such feature.

The analysis of the labour market through the lenses of the DSGE model indicates that the role of the main driver of unemployment in the period under consideration — the wage markup shock — has greatly diminished over the last 10 years. Interpreted as a measure of the labour mismatch this finding suggests much better match of labour supply (i.a. in terms of education and skills) with the demand for labour and hence better overall condition and low inefficiency in the labour market. If this interpretation is correct it might in turn indicate that there is less room and need for active policy to address unemployment fluctuations.

2.6 Misspecification of SoePL-2012 — DSGE-VAR analysis

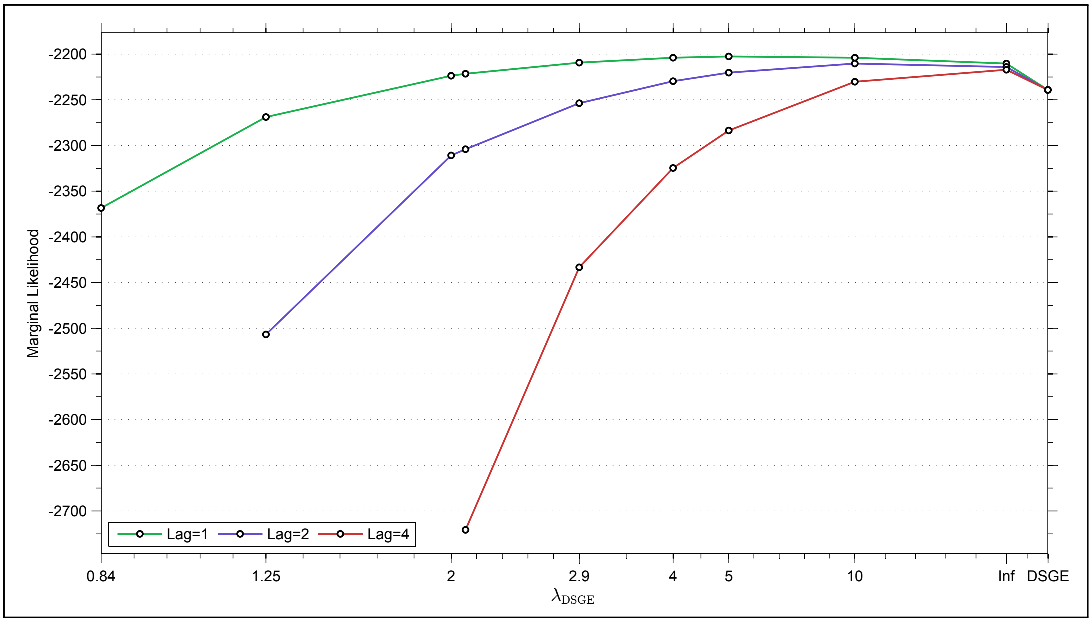
Microeconomic models of rational and forward-looking firms and households used to create theoretical background of DSGE macromodels are usually very simple. The simplicity of micro-foundation as well as a raw approximation of macro policy reaction functions (to mention just the two reasons) may lead to misspecification of the DSGE model. Transition from SoePL-2011 (which neglects unemployment) to SoePL-2012 which explicitly includes unemployment is an additional source of doubts. Description of the labour market we applied in DSGE SoePL-2012 is not very sophisticated, it is rather a crude approximation. One may therefore doubt whether it improves the model or it is a source of an additional bias. For that reason, we focused on potential misspecification of the model.

The issue of the model misspecification is, to some degree, masked by Bayesian approach. The approach, in contrast to classical statistics and inference, neglects the misspecification issue or — at least — it is not of primary interest. Hence the researchers may omit questions regarding specification errors and potential bias. One may call the model *false*, next estimate it using real data, and formulate conclusions on real economic phenomena. It is, however, justified, since the Bayesian model selection criteria (e.g. posterior odd ratio) asymptotically indicate less distorted model in the sense of Kullback-Leibler measure (see e.g. Fernández-Villaverde and Rubio-Ramírez, 2004). In this framework, the Bayesian estimation is the selection procedure of parameters space best suitable to express the data properties rather than an attempt to find „true" parameters.

2

Nevertheless the scale of misspecification is important because the bias caused by the misspecification distorts (e.g.) policy motivated by the model's forecasts. In this context, the idea of Del Negro and Schorfheide (2004) is particularly interesting (see also Del Negro et al. (2007) and Warne (2012); Del Negro and Schorfheide (2006) offer less technical presentation). They borrowed from the indirect inference literature (see for example Gourieroux et al. (1993)) an idea to use VAR as an alternative data generating process and suggest to use the VAR model to approximate the DSGE model[26]. Next, one may construct a mapping from the DSGE model to the VAR parameters. It defines a set of cross-coefficient restrictions for the VAR. One considers the deviations from these restrictions the evidence for DSGE model misspecification. In the Bayesian framework we formulate *priors* centered on the cross-coefficient restrictions. The tightness of the *priors* is scaled by a hyperparameter $\lambda_{\text{DSGE}} \in (0, \infty)$, so when λ_{DSGE} is zero, one gets an unrestricted VAR (DSGE restrictions are not used), when λ_{DSGE} goes to infinity (all restrictions have been strictly imposed) one gets an alternative representation of the DSGE model. Therefore the misspecification of the model (the derivations from DSGE restrictions) is measured by the hyperparameter λ_{DSGE}. Next, the (B)VAR is estimated using *priors* based on cross-coefficient restrictions implied by the DSGE model and the resulting (log) marginal likelihood forms basis for Bayesian model comparison (selection). The log marginal likelihood is a function of the hyperparameter, hence selecting the best fitting model (in terms of log marginal likelihood) one finds the measure of misspecification.

Figure 6. DSGE-VAR — The log marginal likelihood function of λ_{DSGE}



The computation procedure is quite well documented so the reader interested in technical aspects of estimation is referred to consult the papers of (e.g.) Del Negro. We followed Warne (2012) and used selected YADA package functions (adapted to our model's suits of Matlab scripts) to estimate several DSGE-VAR models for given values of the hyperparameters and VAR

[26]Under certain conditions (see Fernández-Villaverde et al., 2005) a DSGE model may be transformed into infinite order VAR(n), $n \to \infty$. For sufficiently large n (maximum lag) one may obtain a reasonable precise approximation.

orders (lags)[27]. The Figure 6 shows the summary of the results — the relations between log marginal likelihood of the estimated models and the hyperparamter λ_{DSGE}.

The results indicate rather moderate misspecification of the DSGE SoePL-2012 model. The hyperparameter $\widehat{\lambda}_{\mathrm{DSGE}}$ is not lower than 5 for the shortest lag, VAR(1). In the case of the longest lag, VAR(4), the hyperparameter is higher than 10. The short sample has not allowed to estimate the function for longer lags. Moreover one may even criticize these results, since the ratio of sample size to number of observables has not allowed to form proper priors for $\lambda_{\mathrm{DSGE}} < 0.8$ for the VAR(1). All the estimates are therefore very imprecise[28].

[27]The Metropolis-Hastings sampler was applied to obtain the Markov chain composed of 1250000 elements. We took last 250000 elements to estimate the log marginal likelihood using Geweke harmonic means procedure.

[28]Under an alternative interpretation, the DSGE restrictions represented by *priors* are imposed by inclusion of an artificial data generated by the DSGE model. When $\lambda_{\mathrm{DSGE}} = 1$ the sample used to estimate DSGE-VAR model is composed of 50% of real data and 50% of artificial data. Therefore the interval $\lambda_{\mathrm{DSGE}} \in (0, 1)$ of is of special interest.

3

3 DSGE SoePL-2011 versus DSGE SoePL-2012

3.1 Framework of models' comparison

Let $\mathscr{E}$ be the specification of the (DSGE) model (the set/vector of equation expressing the economic behaviour of agents and the whole economy), $\mathscr{D}_E$ — the data set/matrix specific for the economy under consideration, $\mathscr{P}_E$ — the set/vector of model's $\mathscr{E}$ parameters. Hence the DSGE model $\mathscr{M}$ is the triple:

$$\mathscr{M} = \left\{\mathscr{E}, \mathscr{P}_E, \mathscr{D}_E\right\}. \tag{3.1}$$

In general, the set of parameters includes parameters specific to the economy, $\mathscr{P}^e(.)$, (e.g. the scale and persistence of shocks) which should be estimated as well as calibrated parameters (we call them *data independent parameters* to emphasize the general nature assigned them by researches), $\mathscr{P}^c(.)$:

$$\mathscr{P}_E\left(\mathscr{D}_E, \mathscr{I}_E\right) = \left[\mathscr{P}^e(\mathscr{E}, \mathscr{D}_E, \mathscr{I}_E), \mathscr{P}^c(\mathscr{E}, \mathscr{I}_E)\right], \tag{3.2}$$

where $\mathscr{I}_E$ is the information set (e.g. priors) relevant to the model specification $\mathscr{E}$. In addition, the model specification $\mathscr{E}$ may assume existence of unobserved (state) variables S. One may need estimates of S to understand and/or analyze and/or forecast changes in the economy. Estimates of the unobserved state variables S_M are functions of model specification, data matrix, and (values of) parameters:

$$S_M = S\left(\mathscr{E}, \widehat{\mathscr{P}^e}(\mathscr{E}, \mathscr{D}_E, \mathscr{I}_E), \widehat{\mathscr{P}^c}(\mathscr{E}, \mathscr{I}_E), \widehat{\mathscr{D}_E}\right). \tag{3.3}$$

The formula shows that even when all parameters are data independent (calibrated), estimates of states still depend on data. In contrast, the impulse response functions IRF_M depend exclusively on the model's specification $\mathscr{E}$ and parameters $\mathscr{P}_E$. There is not a direct impact of data on IRF:

$$IRF_M = IRF\left(\mathscr{E}, \widehat{\mathscr{P}^e}(\mathscr{E}, \mathscr{D}_E, \mathscr{I}_E), \widehat{\mathscr{P}^c}(\mathscr{E}, \mathscr{I}_E)\right). \tag{3.4}$$

We will use concepts defined in equations (3.1)–(3.4) to structure our analyses. We are going to compare the older version of our DSGE model (SoePL-2011) to the newer version (SoePL-2012), hence using defined concepts we have:

$$\begin{gathered} \text{SoePL-2011: } \mathscr{M}^{11} = \left\{\mathscr{E}^{11}, \mathscr{P}_{11}, \mathscr{D}_{11}\right\}, \qquad \text{SoePL-2012: } \mathscr{M}^{12} = \left\{\mathscr{E}^{12}, \mathscr{P}_{12}, \mathscr{D}_{12}\right\}, \\ \text{and } \mathscr{P}_{11} \subset \mathscr{P}_{12}, \quad \mathscr{E}^{11} \subset \mathscr{E}^{12}|\mathscr{P}_{11}, \quad \mathscr{I}_{11} \subset \mathscr{I}_{12}, \quad \mathscr{D}_{11} \neq \mathscr{D}_{12}, \quad \mathscr{P}^c_{11} \neq \mathscr{P}^c_{12}. \end{gathered} \tag{3.5}$$

Now we can define some special variants of the model, for example:

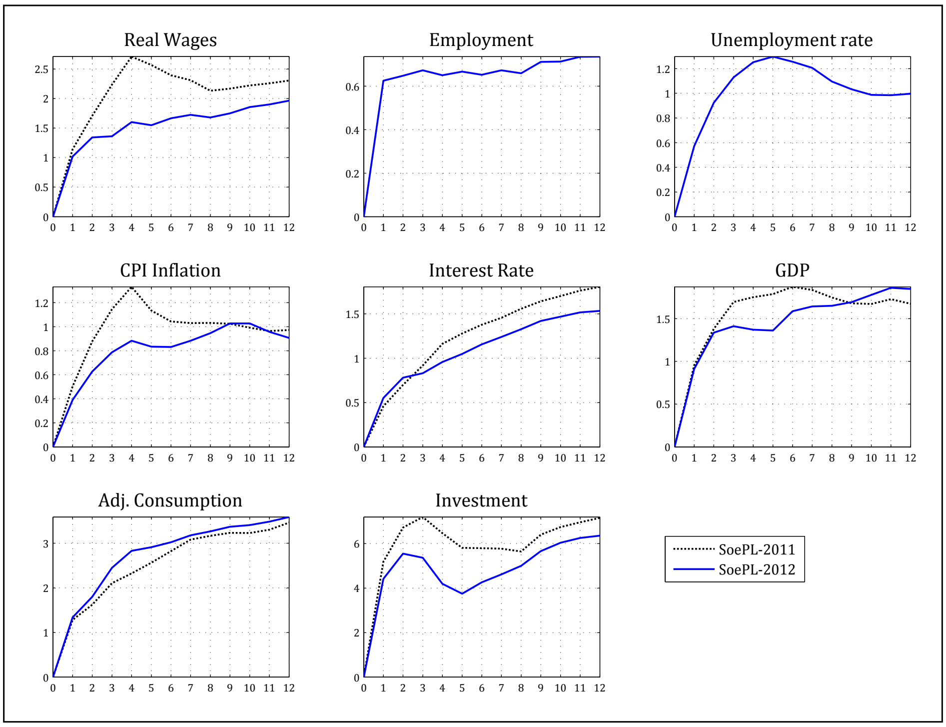
$$\begin{gathered} \mathscr{M}^{12A} = \left\{\mathscr{E}^{12}, \mathscr{P}_{11}, \mathscr{D}_{12}\right\}; \quad \mathscr{M}^{12B} = \left\{\mathscr{E}^{12}, \mathscr{P}_{12}, \mathscr{D}_{12A}\right\}; \quad \mathscr{M}^{12C} = \left\{\mathscr{E}^{12}, \mathscr{P}_{12}, \mathscr{D}_{12B}\right\}; \\ \mathscr{M}^{12D} = \left\{\mathscr{E}^{12}, \mathscr{P}_{12}, \mathscr{D}_{00}\right\}; \quad \mathscr{M}^{12E} = \left\{\mathscr{E}^{12}, \mathscr{P}_{11}, \mathscr{D}_{12C}\right\}; \quad \mathscr{M}^{12F} = \left\{\mathscr{E}^{12}, \mathscr{P}_{11}, \mathscr{D}_{11}\right\}; \\ \mathscr{M}^{11A} = \left\{\mathscr{E}^{11}, \mathscr{P}_{12}, \mathscr{D}_{11}\right\}; \quad \mathscr{M}^{11B} = \left\{\mathscr{E}^{11}, \mathscr{P}_{11}, \mathscr{D}_{12A}\right\}; \\ \text{where: } \mathscr{D}_{12A} \subset \mathscr{D}_{12}, \quad \mathscr{D}_{12B} \subset \mathscr{D}_{12}, \quad \mathscr{D}_{00} = \mathscr{D}_{11} \cap \mathscr{D}_{12}. \end{gathered} \tag{3.6}$$

These variants help us to trace the impact of model components on *IRF* and estimates of states S. In particular, we are interested in identification (estimation) of labour market shocks (wage markup shock and labour supply shock) in the discussed versions. Notice, that since the information sets of these models are different, in general case, the priors and calibration of the parameters are different[29].

3.2 Forecasting properties of the DSGE SoePL model

To assess forecasting precision of the models we use rolling *ex post* forecasts (a set of 12 quarters ahead forecasts, starting from 2005:1 up to 2011:3) of eight main observed variables generated by DSGE SoePL-2011 ($\mathscr{M}^{11}$) and DSGE SoePL-2012 ($\mathscr{M}^{12}$) (see Figure 13–14 in the Appendix A.4). Figure 7 compares mean root squared forecast errors (MRSFE) of SoePL-2012 and SoePL-2011 computed for the rolling forecasts.

Figure 7. Root mean squared forecast errors of rolling forecasts in SoePL-2012 and SoePL-2011.



The newest version of the model can forecast the unemployment rate, the older cannot, so this is the main advantage of the implemented changes. In addition, DSGE SoePL-2012 is able to predict annual CPI inflation and annual GDP growth rate better than the older version, at least for the first 8 quarters of forecast horizon. Except for very short term forecasts (up to two

[29]Taking into account equation (3.4) one can notice that, given the set of parameters (calibrated and estimated for the older version SoePL-2011 with endogenous preference shifter switched off), inclusion of labour supply and unemployment rate cannot change model's impulse response functions (IRF of $\mathscr{M}^{11}$ equals IRF of $\mathscr{M}^{12F}$, apart from unemployment rate), estimates of shocks (S of $\mathscr{M}^{11}$ equals S of $\mathscr{M}^{12F}$, but the labour supply) as well as forecasting accuracy of observed variables (the unemployment rate is an exception) comparing to SoePL-2011.

quarters), the interest rate is predicted much more accurately by the newest version. The newer version dominates the older one in the case of annual wages growth rate forecast and annual growth rates of investments. There are however few observables with higher MRSFE, private consumption computed with changes in inventories (adjusted consumption) is an example.

Usually, the MRSFE is an increasing function of forecast's horizon. In the case of SoePL this rule is not met. Apart from the interest rate and adjusted consumption, the MRSFE rises very fast in the first 2-5 quarters of forecasting horizon. Next the rise of forecasts errors is rather slow, sometimes one observes a decline in the MRSFE.

3

3.3 Identification of labour market shocks

Many authors consider standard New Keynesian DSGE models (like SoePL-2011) unable to distinguish the wage markup shock from the labour supply shock. In fact these shocks are "unidentified" in double sense, because one may not be able to estimate parameters related to the shocks as well. Analysis of SoePL-2011 specification supports this thesis. The shocks occur in the wage equation exclusively, so, in the general case, one cannot estimate variances and autoregression coefficients (assuming AR(1) stochastic structure) of these shocks unconditionally. Calibration or strong priors imposed on parameters estimated using Bayesian techniques are seldom trustworthy solutions. In this section we neglect the identification of parameters. We assume the parameters are given and focus on the former issue, keeping in mind the equation (3.3).

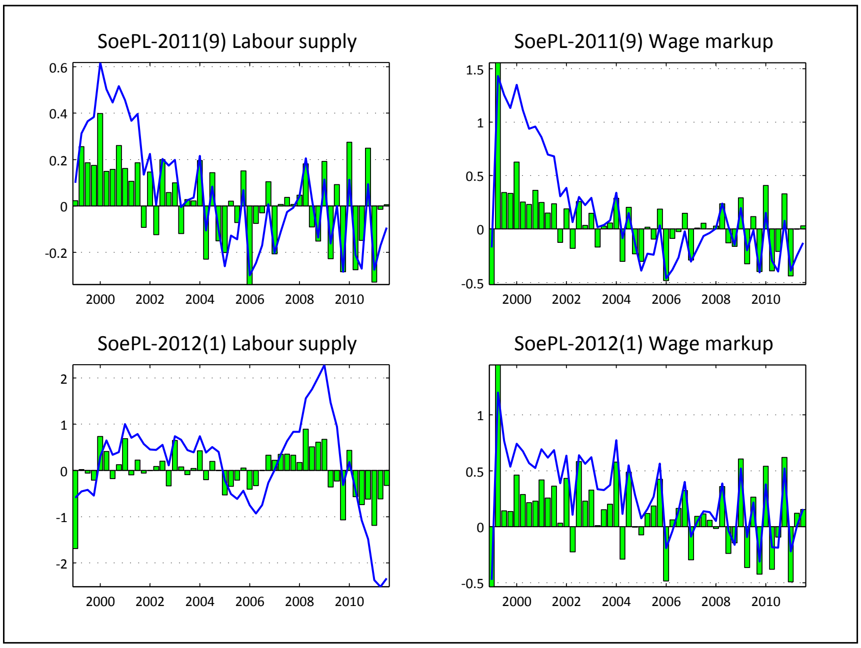
Table 2. Correlations of smoothed (estimated) wages markup and labour supply innovations and disturbances

Case	Model version $\mathscr{M}$	Specification Version $\mathscr{E}$	Parameter's Set $\mathscr{P}$	Additionally Included Observed Variables $\mathscr{D}$	Correlation of Innovations	Correlation of AR(1) Disturbances
1	$\mathscr{M}^{12}$	$\mathscr{E}^{12}$	$\mathscr{P}_{12}$	$\mathscr{D}_{12} = \mathscr{D}_{00} \cup \{\text{Unemp, Emp}\}$	0.492	0.225
2	$\mathscr{M}^{12B}$	$\mathscr{E}^{12}$	$\mathscr{P}_{12}$	$\mathscr{D}_{12A} = \mathscr{D}_{00} \cup \{\text{Emp}\}$	0.866	0.713
3	$\mathscr{M}^{12C}$	$\mathscr{E}^{12}$	$\mathscr{P}_{12}$	$\mathscr{D}_{12C} = \mathscr{D}_{00} \cup \{\text{Unemp}\}$	0.487	0.104
4	$\mathscr{M}^{12D}$	$\mathscr{E}^{12}$	$\mathscr{P}_{12}$	$\mathscr{D}_{00}$	0.932	0.769
5	$\mathscr{M}^{12E}$	$\mathscr{E}^{12}$	$\mathscr{P}_{11}$	$\mathscr{D}_{00}$	0.998	0.989
6	$\mathscr{M}^{12A}$	$\mathscr{E}^{12}$	$\mathscr{P}_{11}$	$\mathscr{D}_{12} = \mathscr{D}_{00} \cup \{\text{Unemp, Emp}\}$	-0.015	-0.321
7	$\mathscr{M}^{11B}$	$\mathscr{E}^{11}$	$\mathscr{P}_{12}$	$\mathscr{D}_{12A} = \mathscr{D}_{00} \cup \{\text{Emp}\}$	0.263	0.386
8	$\mathscr{M}^{11A}$	$\mathscr{E}^{11}$	$\mathscr{P}_{12}$	$\mathscr{D}_{11} = \mathscr{D}_{00} \cup \{\text{Emp(HP)}\}$	0.458	0.341
9	$\mathscr{M}^{11}$	$\mathscr{E}^{11}$	$\mathscr{P}_{11}$	$\mathscr{D}_{11} = \mathscr{D}_{00} \cup \{\text{Emp(HP)}\}$	0.822	0.875

Remark: Unemp — rate of registered unemployment, Emp — growth rate of employment, Emp(HP) — HP filtered level of employment.

To illustrate the problem, correlation coefficients of smoothed (estimated) labour market shocks for several variants of DSGE SoePL model have been computed. A very high correlation indicates inability of the model to discriminate the shocks hence we can check impact of parameters, specification and data — they are considered independent components of a model (see section 3.1). We take into account two variants of SoePL specification. The first one, SoePL-2011, is a standard version of New Keynesian DSGE model ($\mathscr{S}^{11}$). It does not include labour supply equation nor unemployment equation. There is, however, the stochastic wage markup. The second specification, SoePL-2012 ($\mathscr{S}^{12}$), has been described earlier. We also take into account two variants of parameters. The first one has been fitted (estimated conditionally) for SoePL-2011 ($\mathscr{P}_{11}$), the second one, ($\mathscr{P}_{12}$), has been estimated conditionally for the SoePL-2012 specification. In addition, we consider different variants of data set (observables, see also equation (3.6) for details) and apply Kalman smoother to estimate shocks using different sets of observables (for given specification and parameters) — results are collected in Table 2, see also Figure 8 (the plots of some other cases are presented in the Appendix A.5 — see Figure 15).

Figure 8. Estimated labour market innovations (green bars) and disturbances (blue line). Model SoePL-2011 and SoePL-2012, (cases 1, 9, see Table 2).



The cases 1 and 9 are ”regular”. These shocks (innovations and disturbances) have been estimated using the data set, the specification and the parameters specific to those variants. All other cases are ”irregular”, since specification and/or data set and/or parameters do not match up. These irregular cases help us to investigate why one cannot distinguish the wage markup shock from the labour supply preference shock. High correlation coefficients computed for the standard New Keynesian DSGE model (regular case 9) indicate lack of identification of labour supply and wage markup shocks. The inclusion of unemployment equation and data reduces the correlation for SoePL-2012 shocks (regular case 1, and irregular cases 3, and 6). Irregular cases no. 7 and 8 (the model does not include labour supply nor unemployment equations) suggest however that the parameters do matter. Hence, one may identify the shocks in the model like SoePL-2011, if the parameters are calibrated properly. The inclusion of unemployment into model specification and the data allows us to estimate parameters of the model. Given the ”correct” parameter’s set, the labour market shocks can be discriminated. On the other hand, the omission of unemployment data, when labour supply equation is present, leads to lack of identification (irregular cases 2, 4, 5), so the data (observables) do mater as well.

The discussed exercise is to some degree technical. We do not recommend to use irregular cases to explain historical events or to forecast. For example, case no 6 looks very attractive, but it is not a real options because of very poor fit.

3.4 Impulse response functions comparison

The transition from the model SoePL-2011 to SoePL-2012 involves few (basically redundant, but not unimportant) modifications of the specification, but the impulse response functions (IRFs) of a model depend on the specification as well as the estimates of parameters — see equation (3.4). Therefore the IRFs of these models, in most cases, may be (and, in fact, they are) very different — some selected IRFs are presented in Appendix A.6 (see Figure 16–27). Including unemployment rate into our DSGE model we impose some restrictions on parameters (via cross parameters relations and the desire to match an additional time serie), hence the main impact of the model extension focuses on parameters' estimates. For that reason we analyzed the impact of the parameters on IRFs. We wanted to investigate which parameters are responsible for the difference of the IRFs. Table 3 summarizes the results. We limited the investigation to the cases when IRFs exhibited important changes in their shape. The changes in scale are usually the result of the different estimates of shocks' standard deviations.

Table 3. The existence of impact of the parameters on model's IRF differences

Parameters and/or Spec.	IRF of Shock									
	Techn. (TFP) ϵ_t	Techn. inv. υ_t	Cons. pref. ζ_t^c	Lab.sup. pref. ζ_t^h	Wage markup λ_t^w	Home markup λ_t^d	Export markup λ_t^x	Im.cons. markup λ_t^{mc}	Im.inv. markup λ_t^{mi}	Infl. target $\bar{\pi}_t^c$
σ_L	(+)	-	(+)	(+)	(+)	(+)	-	(+)	-	-
ξ_w	-	(+)	-	-	-	-	-	-	-	-
ξ_d	(+)	(+)	(+)	(+)	(+)	-	-	(+)	(+)	-
ξ_{mc}	(+)	-	-	(+)	(+)	-	-	(+)	-	-
η_c	(+)	-	-	-	-	-	-	-	-	-
η_i	(+)	-	-	-	-	-	-	-	-	-
$\tilde{\psi}_a^e$	(+)	-	(+)	(+)	(+)	(+)	-	(+)	-	(+)
$\tilde{\psi}_s^u$	(+)	-	-	-	-	-	-	(+)	-	-
$\tilde{\psi}_s^e$	(+)	-	-	(+)	(+)	(+)	-	(+)	-	(+)
ρ_R	(+)	(+)	(+)	(+)	(+)	(+)	-	(+)	-	-
$\rho_{\bar{\pi}^c}$	(+)	-	-	-	-	-	-	-	-	(+)
ρ_ϵ	(+)	-	-	-	-	-	-	-	-	-
$\mathscr{S}^m$	-	-	-	-	-	-	(+)	-	(+)	-

$\mathscr{S}^m$ — specification of the measurement block

(+) — required change of the parameter/specification to eliminate IFRs differences

The parameters related to labour market (wage Calvo probability ξ^w and inverted labour supply elasticity σ_L) influence seven important IRFs, but to recover old shape of these IRFs one must alter much more parameters. Hence, the introduction of unemployment into DSGE SoePL-2012 is not exclusively responsible for the changes in IRFs. Higher value of interest rate persistence parameter (ρ_R — see Table 4 and equation (A.42) in the Appendix) is a very frequent cause of the IRFs' differences as well. It is a result of new priors — we alter priors of the Taylor rule mainly to improve fit and forecasting accuracy of the interest rate. The higher sensitivity of the interest rate to the foreign (euro area) net assets in euro (parameters $\tilde{\psi}_a^e$) and lower inertia of the exchange rate (parameters $\tilde{\psi}_s^e$, $\tilde{\psi}_s^u$) are also very important. They increased impact of trade balance (in particular the rise/decline of exports and/or imports) on exchange rate which in some cases reversed short term reactions of the real exchange rate to shocks.

Conclusions

Using the classic New Keynesian DSGE models one cannot distinguish wage markup shock from labour supply shock, hence these shocks are unidentified. It is argued that inclusion of observed unemployment in the way proposed by Galı' is a method to solve this problem. Our results support this claim. However, it is not the whole story. To estimate and identify (distinguish) the series of shocks one needs properly calibrated/estimated parameters, in particular parameters specific to labour market: wage rigidity parameters (e.g. Calvo probability), characteristics of shocks (persistences, variance), wage elasticity of labour supply. The additional observed variable (unemployment rate) and the additional restriction defined by the *steady state* unemployment rate help to estimate these parameters. Having properly estimated parameters, one may identify (estimate) these shocks in a DSGE model without observed unemployment, but the precision of the estimates may be poor. On the other hand, a raw (imprecise) calibration of labour market parameters and the presence of observed unemployment in a DSGE model allow to estimate less correlated wage markup and labour supply shock series, but we do not consider it a trustworthy method. We stress the role of parameters' estimates (specific to the economy under consideration) since, in our opinion, a chance to estimate at least some parameters better is the main gain from the extension of the New Keynesian DSGE model.

The *steady state* wage markup and wage elasticity of labour supply define the *steady state* unemployment rate and this relation (restriction) reveals simplicity of the whole approach to the labour market. Such simplification may lead to misspecification, therefore we estimated a set DSGE-VAR models to check how serious the problem is in the case of DSGE SoePL-2012. The results do not support hypothesis of serious misspecification. Restrictions imposed on a simple VAR by DSGE specification are important and relevant. They improve the fit to the data.

Inclusion of observed unemployment rate to New Keynesian DSGE model, in our case transition from DSGE SoePL-2011 to DSGE SoePL-2012, involves minor adjustment of the model's specification (additional labour supply equation and unemployment rate equation) extension of data base, reconsideration of priors, and reestimation of the model's parameters. The extended model has better forecasting properties but — in some cases — very different impulse response functions. Estimated elasticity of labour supply is responsible for much of these differences. DSGE SoePL-2012 is, in fact, a new model.

In spite of apparent drawbacks, simplified description of the labour market in SoePL-2012 organizes and structures analysis of historical decompositions. It helps to identify observed historical events represented by shocks. Therefore, the evolution of unemployment rate, employment level and real wages can be understood better.

A Appendix

A.1 Compact exposition of DSGE SoePL specification

Exogenous technical progress, scaling of growing variables, and stochastic structure of shocks

Exogenous stochastic growth is driven by changes in the level of technology z_t. The growth rate of technology, $\mu_{z,t} \equiv \frac{z_t}{z_{t-1}}$, is governed by the stochastic process:

$$\mu_{z,t} = \left(1-\rho_{\mu_z}\right)\mu_z + \rho_{\mu_z}\mu_{z,t-1} + \varepsilon_{\mu_z,t}, \qquad \varepsilon_{\mu_z,t} \sim N\left(0,\mu_z\sigma_{\mu_z}\right), \qquad \mathrm{E}\,\mu_{z,t} = \mu_z,$$

where ρ_{μ_z} is the persistence coefficient, and μ_z is a long-term growth rate of technology. Beside of that, we assume the existence of a technological trend specific for capital/investment goods $\left(\Psi_t\right)$, whose changes, $\mu_{\Psi,t} \equiv \frac{\Psi_t}{\Psi_{t-1}}$, are governed by the process:

$$\mu_{\Psi,t} = \left(1-\rho_{\mu_\Psi}\right)\mu_\Psi + \rho_{\mu_\Psi}\mu_{\Psi,t-1} + \varepsilon_{\mu_\Psi,t} \qquad \varepsilon_{\mu_\Psi,t} \sim N\left(0,\mu_\Psi\sigma_{\mu_\Psi}\right), \qquad \mathrm{E}\,\mu_{\Psi,t} = \mu_\Psi.$$

The presence of an additional technological trend specific for capital goods, by use of capital as a factor of production, is translated into other macroeconomic categories and extends the neutral technological trend. The common technological trend (z_t^+) for all growing variables, except capital goods, may be presented as:

$$z_t^+ = z_t\Psi_t^{\frac{\varpi}{1-\varpi}}, \qquad \mu_{z^+,t} = \mu_{z,t}\mu_{\Psi,t}^{\frac{\varpi}{1-\varpi}},$$

where ϖ is the share of capital in the production function. The level of technology ($z_t^+\Psi_t$ for capital goods $z_t^+\Psi_t$ and z_t^+ for all the other categories) allows to express the growing variables in a stationary form (usually denoted with small letters), i.e.:

$$y_t \equiv \frac{Y_t}{z_t^+}, \qquad i_t \equiv \frac{I_t}{z_t^+\Psi_t}, \qquad \text{etc.} \tag{A.1}$$

Additionally, nominal variables are stationarized with the use of the level of prices P_t^d, e.g. nominal wages are translated into stationary real wages:

$$w_t \equiv \frac{W_t}{P_t^d z_t^+}. \tag{A.2}$$

At the final stage the whole model may be presented with the use of stationary variables and explicitly determined *steady state*.

Other shocks have common stochastic structure. We define a typical random disturbance (shock) to be referred to later on:

$$\Lambda_t = \left(1-\rho_\Lambda\right)\Lambda + \rho_\Lambda\Lambda_{t-1} + \varepsilon_{\Lambda,t}, \qquad \varepsilon_{\Lambda,t} \sim N\left(0,\Lambda\sigma_\Lambda\right), \qquad \mathrm{E}\,\Lambda_t = \Lambda. \tag{A.3}$$

where Λ is the *steady state* value of the shocks.

Foreign economy

Domestic economy functions in the environment of two foreign economies: the euro area and the rest of the world (dolar area). Interactions with those economies entail exchange of goods and financial flows. There are, therefore, three nominal exchange rates in the model: dollar/zloty, euro/zloty and dollar/euro, denoted respectively by: S_t^u, S_t^e, S_t^x, where $S_t^u = S_t^x S_t^e$. Additionally, we define real exchange rates:

$$x_t^x \equiv \frac{S_t^x P_t^u}{P_t^e}, \qquad x_t^e \equiv \frac{S_t^e P_t^e}{P_t^c}, \qquad x_t^u \equiv \frac{S_t^e S_t^x P_t^u}{P_t^c}, \tag{A.4}$$

where P_t^u is the level of prices in the rest of the world, P_t^e is the level of prices in the euro area, and P_t^c is the level of domestic consumer prices. To real exchange rates, analogically as for the nominal rates, the following applies: $x_t^u = x_t^x x_t^e$.

Producers and suppliers of intermediate goods

There are five markets of intermediate goods: domestic goods, imported consumption goods, imported investment goods, and imported component of export goods, as well as export goods. There are infinitely many agents (continuum determined in the [0,1] interval) suppling heterogeneous intermediate products of a given type that are aggregated into a homogeneous final product representing the production of a given market.

Aggregators

Heterogeneous intermediate products are assembled into final products. There are infinitely many firms (the agents do not consume resources nor generate added value) called aggregators. Each firm operates under perfect competition and uses the same production function. Aggregators purchase heterogeneous intermediate products and transform them into a homogeneous final product (taking the prices of intermediate products and the price of the final product as given).

The production function of the final good in each of the markets O ($O \in \{Y, C^m, I^m, X^m, X\}$) takes the form of the CES function:

$$O_t = \left[\int_0^1 O_{i,t}^{\frac{1}{\lambda_t^o}} di\right]^{\lambda_t^o}, \qquad 1 \leq \lambda_t^o < \infty, \qquad o \in \{d, mc, mi, mx, x\}, \tag{A.5}$$

where O_t is the production of the final good, $O_{i,t}$ is the production by the i-th intermediate goods producer, λ_t^o is the (stochastic) markup in the market o, and o identifies market: domestic products (d), imported consumption goods (mc), imported investment goods (mi), imported component of exported goods (mx), export products (x). Markups specific for each of the markets have stochastic structure given by (A.3).

Profit maximization by an aggregator leads to the demand function for intermediate products of the i-th producer:

$$O_{i,t} = \left[\frac{P_{i,t}^o}{P_t^o}\right]^{-\frac{\lambda_t^o}{1-\lambda_t^o}} O_t, \tag{A.6}$$

where P_t^o is the price of the homogeneous final product in market o, $P_{i,t}^o$ is the price of the intermediate product of the i-th producer. Using equations (A.5) and (A.6) we obtain the equation for the price of the homogeneous final product in a given market:

$$P_t^o = \left[\int_0^1 \left(P_{i,t}^o\right)^{\frac{1}{1-\lambda_t^o}} di\right]^{1-\lambda_t^o}, \qquad o \in \{d, mc, mi, mx, x\}. \tag{A.7}$$

Producers of domestic intermediate goods

The producers of domestic intermediate goods are the sole generators of the GDP. Using the Cobb-Douglas production function, with production technology identical for all of the producers, they use individually determined labour and capital inputs to produce:

$$Y_{i,t} = \epsilon_t \left(z_t H_{i,t}\right)^{1-\varpi} \left(K_{i,t}\right)^{\varpi}, \tag{A.8}$$

where $H_{i,t}$ and $K_{i,t}$ are the inputs of labour (hours) and capital services determined by the i^{th} producer. The total factor productivity shock $\left(\epsilon_t\right)$ has structure given by (A.3). The optimal values for inputs of capital and labour are determined based on the problem of costs minimization:

$$\min_{K_{i,t},H_{i,t}} R_t^{fw} F_t^{\tau} W_t H_{i,t} (1+\tau^s) + R_t^{fk} F_t^{\tau} R_t^k K_{i,t} - \lambda_t P_{i,t} \left[Y_{i,t} - z_t^{1-\varpi} \epsilon_t K_{i,t}^{\varpi} H_{i,t}^{1-\varpi}\right], \tag{A.9}$$

where W_t is the nominal wage, τ^s is the rate of national insurance contribution paid by the employer, R_t^k is the gross nominal rental rate per unit of capital services, λ_t is the Lagrange multiplier. We assume that in each period a fraction of the wage and capital fund, ν^w and ν^k, must be financed with a working capital loan hence the presence of effective gross nominal interest rates, R_t^{fw} and R_t^{fk}, in the cost function, given by:

$$R_t^{fw} \equiv \nu^w R_{t-1} + 1 - \nu^w, \qquad R_t^{fk} \equiv \nu^k R_{t-1} + 1 - \nu^k, \tag{A.10}$$

where R_{t-1} is the gross nominal interest rate. We assume also that the use of labour and capital services involves the use of raw materials (e.g. energy), whose costs are represented by F_t^{τ} function. The $F_t^{\tau}(\cdot,\cdot)$ function, specified explicitly only at the level of log-linearized form, is a linear function of structural shock representing the dynamics of the prices of raw materials (e.g. oil). The solution to the problem of costs minimization (A.9) gives the real marginal cost of the domestic intermediate goods producers:

$$mc_t^d = \frac{1}{\epsilon_t}\left(\frac{1}{\varpi}\right)^{\varpi}\left(\frac{1}{1-\varpi}\right)^{1-\varpi}\left(\bar{r}_t^k R_t^{fk}\right)^{\varpi}\left(w_t R_t^{fw}\right)^{1-\varpi} F_t^{\tau}. \tag{A.11}$$

Prices. The market of domestic intermediate products is characterized with monopolistic competition, which means that manufacturers produce heterogeneous products and may set their prices. At the same time, there are some limitations due to Calvo price setting mechanizm (Calvo (1983)). In every period each manufacturer, with probability $1-\xi_d$, may set the optimal price of its output $P_t^{d,new}$. With probability ξ_d the price cannot be set in the optimal way and it is then indexed to previous inflation (with weight κ_d) and the current inflation target[30] (with weight $1-\kappa_d$):

$$P_{t+1}^d = \left(\pi_t^d\right)^{\kappa_d}\left(\bar{\pi}_{t+1}^c\right)^{1-\kappa_d} P_t^d. \tag{A.12}$$

If a producer is allowed to reoptimize its price, it sets its price to maximize the flow of future profits, assuming that it will not be allowed to reoptimize the price in the future. Thus, the decision-making problem takes the form:

$$\max_{P_t^{d,new}} \mathrm{E}_t \sum_{s=0}^{\infty} \upsilon_{t+s} \left(\beta\xi_d\right)^s \left[\left(\pi_t^d \ldots \pi_{t+s-1}^d\right)^{\kappa_d}\left(\bar{\pi}_{t+1}^c \ldots \bar{\pi}_{t+s}^c\right)^{1-\kappa_d} P_t^{d,new} Y_{i,t+s} - MC_{i,t+s}^d Y_{i,t+s}\right],$$

[30] The inflation target $\left(\bar{\pi}_t^c\right)$ has a stochastic nature and is given by exogenous process defined earlier (see eq. (A.3)). In the *steady state*, inflation target is equal to the *steady state* level of inflation $\left(\bar{\pi}^c \equiv \pi^d\right)$.

where υ_{t+s} is the marginal utility of the households' nominal income[31], β is a discount factor, and MC^d is a nominal marginal cost. When solving the profit maximization problem above, the producer takes into account the demand for their output given by equation (A.6). The solution of the problem takes the form of the Phillips curve for domestic intermediate goods, in which the main inflation determinants become the real marginal costs mc_t^d, given by (A.11), and markup λ_t^d described with the exogenous process (A.3)[32].

Employment. In addition, the manufacturers have to determine of the optimal level of employment E_t, based on the demand for labour services (expressed in hours worked) determined in the process of costs minimization. This process involves Calvo-type rigidities — with probability $(1-\xi_e)$ the producer is allowed to set the level of employment in an optimal way, while with the probability ξ_e the producer cannot change the level of employment. When producer is allowed to reoptimize the level of employment, the decision-making problem takes the form:

$$\min_{E_{i,t}^{new}} \sum_{s=0}^{\infty} \left(\beta \xi_e\right)^s \left(n_i E_{i,t}^{new} - H_{i,t+s}\right)^2, \tag{A.14}$$

where n is the number of hours per employee (equals one when labour demand is expressed in persons rather than hours worked). The solution of the decision-making problem describes the level of employment in the economy.

Suppliers of imported goods

The imported consumption, investment and export component goods make three separate markets of imported products. In each of the markets the importers purchase foreign goods (from the euro area and the rest of the world — we assume fixed geographic structure of the import), and differentiate them. Heterogeneous products are, then, purchased by the aggregators and transformed into homogeneous final products. The monopolistic competition implies that importers may set prices of their products, while the process runs similarly to the case of domestic intermediate goods producers (with specific ξ_o, κ_o and λ^o, $(o \in \{mc, mi, mx\})$ parameters for each market of imported products). Solving the problem of maximization of the importers' profit, we arrive at three Phillips curves in the form compliant with the formula presented in the footnote (see eq. (A.13)).

The marginal costs, with fixed geographic structure of import, take the form:

$$mc_t^o \equiv \frac{S_t^u P_t^u}{P_t^o}\omega^{o,u} + \frac{S_t^e P_t^e}{P_t^o}(1-\omega^{o,u}), \quad o \in \{mc, mi, mx\} \tag{A.15}$$

where $\omega^{mc,u}$, $\omega^{mi,u}$ and $\omega^{mx,u}$ determine the share of the rest of the world in the basket of imported consumption, investment and export goods.

[31]Due to the fact that in each period the profit generated by the firm is transferred to households, the profit in the particular period is weighted with the marginal utility of the households' nominal income.

[32]The log-linearized Phillips curve for domestic intermediate products has the following form:

$$\begin{aligned}\widehat{\pi}_t^o = \frac{\beta\mu}{1+\kappa_o\beta\mu}\left(\widehat{\pi}_{t+1}^o - \widehat{\overline{\pi}}_{t+1}^c\right) + \frac{\kappa_o}{1+\kappa_o\beta\mu}\left(\widehat{\pi}_{t-1}^o - \widehat{\overline{\pi}}_t^c\right) + \frac{1+\kappa_o\beta\mu\rho_\pi}{1+\kappa_o\beta\mu}\widehat{\overline{\pi}}_t^c \\ + \frac{(1-\xi_o\beta\mu)(1-\xi_o)}{\xi_o(1+\kappa_o\beta\mu)}\left(\widehat{\lambda}_t^o + \widehat{mc}_t^o\right).\end{aligned} \tag{A.13}$$

where μ is the *steady state* gross rate growth of nominal money.

The real marginal costs and markups in the markets, described with exogenous processes (A.3), determine inflation for imported consumption goods, investment goods and goods intended for export. The total demand for imported consumption and investment goods depends on the decisions of households, while the demand for import of goods intended for export is determined by the exporters.

Suppliers of exported goods

Exporters operate under monopolistic competition as well. They set prices $P^x_{i,t}$ for heterogeneous export goods $X_{i,t}$. These product are supplied to the world market, hence the prices set by exporters are expressed in dollars. As we assume perfect goods mobility, the price for the euro area market is the same price converted into euro using the USD/EUR exchange rate $P^x_{i,t}S^x_t$. The process of setting the price runs similarly as in the case of domestic goods producers and importers, now with parameters ξ_x, κ_x i λ^x specific for the export market. After solving the problem of profit maximization we obtain the Phillips curve for the export market.

Export good of the i^{th} exporter is produced with the use of domestic products $X^d_{i,t}$ and imported products $X^m_{i,t}$. The production function takes the form:

$$X_{i,t} = \left[\omega_x^{\frac{1}{\eta_{xx}}} \left(X^m_{i,t}\right)^{\frac{\eta_{xx}-1}{\eta_{xx}}} + \left(1-\omega_x\right)^{\frac{1}{\eta_{xx}}} \left(X^d_{i,t}\right)^{\frac{\eta_{xx}-1}{\eta_{xx}}}\right]^{\frac{\eta_{xx}}{\eta_{xx}-1}}, \tag{A.16}$$

where η_{xx} is the elasticity of substitution between domestic and imported products, and ω_x is the share of the imported component. Each exporter must solve the problem of costs minimization:

$$\min_{X^m_{i,t}, X^d_{i,t}} P^{mx}_t X^m_{i,t} + P^d_t X^d_{i,t}, \tag{A.17}$$

where P^{mx}_t is the price of imported component, P^d_t is the price of the domestic component, subject to (A.16). The solution of the problem leads to the marginal costs of exporters:

$$mc^x_t = \frac{1}{S^e_t S^x_t P^x_t}\left[\omega_x \left(P^{mx}_t\right)^{1-\eta_{xx}} + \left(1-\omega_x\right)\left(P^d_t\right)^{1-\eta_{xx}}\right]^{\frac{1}{1-\eta_{xx}}}.$$

The marginal cost together with the markup described with the exogenous process (see (A.3)) is reflected in the Phillips curve for the export market.

Assuming that consumption and investments in the euro area and in the rest of the world are determined based on CES functions with domestic export being one of the inputs, the demand for domestic export on the part of both economies and both types of products is expressed with the following equations:

$$C^{x,o}_t = \left[\frac{P^x_t}{P^o_t}\right]^{-\eta_{f,o}} C^o_t, \qquad I^{x,o}_t = \left[\frac{P^x_t}{P^o_t}\right]^{-\eta_{f,o}} I^o_t, \qquad o \in \{u, e\}$$

where C^e_t $\left(C^u_t\right)$ and I^e_t $\left(I^u_t\right)$ are consumption and investments in the euro area (rest of the world), while $\eta_{f,e}$ $\left(\eta_{f,u}\right)$ is the elasticity of substitution between domestic export products and the euro area (the rest of the world) products. Assuming that the income of foreign economies is entirely divided between consumption and investments, we may express the demand for domestic export as a function of foreign income:

$$X_t \equiv C^{x,e}_t + I^{x,e}_t + C^{x,u}_t + I^{x,u}_t = \left[\frac{P^x_t}{P^u_t}\right]^{-\eta_{f,u}} Y^u_t + \left[\frac{S^x_t P^x_t}{P^e_t}\right]^{-\eta_{f,e}} Y^e_t. \tag{A.18}$$

Thus, the demand for domestic export depends on the relation of export prices and world prices and income (output) abroad.

Households

Households maximize utility yielded from consumption, leisure and cash. Households provide labour and capital services to the producers of domestic intermediate products. In each period households divide their income between domestic and foreign deposits, consumption, investments and purchase/sale of new, installed capital, as well as cover the cost of maintenance of capital that has not been lent to producers. The income of households consists of domestic and foreign deposits plus interest, remuneration for the labour and capital services, as well as profits transferred in the form of dividend. Incomes are adequately taxed, and additionally direct transfers from the state budget are allowed. Moreover, we assume that financial markets are complete. This enables households to acquire state contingent securities making them homogeneous with regard to the possessed resources and incurred expenditures, thanks to which the model may be made operational.

We neglect the two additional dimensions of households proposed by Galí (compare section 1). In our opinion, it has no impact on the set of first order equilibrium conditions of households. The additional dimensions (the labour type indexed by i and the personal disutility of work indexed by j) must be in some way integrated out because the households, indexed by l, (not their members) make the decisions under consideration. The additional dimensions are irrelevant here. The packed (or a reduced form) utility function produces equivalent first order conditions keeping presentation simpler. The utility function of a l-th household takes the form:

$$\underset{t}{\mathrm{E}}\sum_{s=0}^{\infty}\beta^{t+s}\left[\zeta_t^c\ln\left(C_{l,t}-b\,C_{l,t-1}\right)-\tilde{n}_t\,A_L\,\zeta_t^h\,\frac{\left(h_{l,t}\right)^{1+\sigma_L}}{1+\sigma_L}+\zeta_t^q A_q\frac{\left(\frac{Q_{l,t}}{z_t^+P_t}\right)^{1-\sigma_q}}{1-\sigma_q}\right] \tag{A.19}$$

where $C_{l,t}$ is the consumption in period t, $h_{l,t}$ is the supply of labour (hours), $Q_{l,t}$ is cash holdings, σ_L is the inverse of the elasticity of labour supply with respect to wage, σ_q is the elasticity of demand for cash with respect to interest rate. The consumption, leisure and cash holdings are subject to preference shocks (respectively): ζ_t^c, ζ_t^h, ζ_t^q of the form given by (A.3). In addition, following Galí, we define the preference shifter $\tilde{n}$. The idea of shifter is motivated by Jaimovich and Rebelo (2009). Our variant of the shifter is defined as follows:

$$\tilde{n}_t=n_t\left[\left(1+\tau_t^c\right)\gamma_t^{cd}\,\psi_{z^+,t}\right],\qquad\qquad n_t=n_{t-1}^{\left(1-\vartheta_n\right)}\left[\left(1+\tau_t^c\right)\gamma_t^{cd}\,\psi_{z^+,t}\right]^{-\vartheta_n}$$

where: τ_t^c is the consumption tax rate, $\gamma_t^{cd}\equiv\frac{P_t^c}{P_t^d}$ is the ratio of consumer to producer prices, and $\psi_{z^+,t}$ is the marginal utility of income.

Consumption and investment goods purchased by households consist of domestic and imported products:

$$\begin{aligned}C_t&=\left[\left(1-\omega_c\right)^{\frac{1}{\eta_c}}\left(C_t^d\right)^{\frac{\eta_c-1}{\eta_c}}+\omega_c^{\frac{1}{\eta_c}}\left(C_t^m\right)^{\frac{\eta_c-1}{\eta_c}}\right]^{\frac{\eta_c}{\eta_c-1}},\\ I_t&=\left[\left(1-\omega_i\right)^{\frac{1}{\eta_i}}\left(I_t^d\right)^{\frac{\eta_i-1}{\eta_i}}+\omega_i^{\frac{1}{\eta_i}}\left(I_t^m\right)^{\frac{\eta_i-1}{\eta_i}}\right]^{\frac{\eta_i}{\eta_i-1}},\end{aligned} \tag{A.20}$$

where C_t^d $\left(I_t^d\right)$ and C_t^m $\left(I_t^m\right)$ are domestic and import components of consumption (investments), ω_c $\left(\omega_i\right)$ is the share of import in consumption (investments) and η_c $\left(\eta_i\right)$ is the elasticity of substitution between domestic consumption (investment) goods and imported goods.

We assume that aggregation is made in such a way as to maximize the values C_t and I_t subject to budget constraints:

$$P_t^d C_t^d + P_t^{mc} C_t^m = P_t^c C_t, \qquad P^t \frac{I_t^d}{\Psi_t} + P_t^{mi} I_t^m = P_t^i I_t, \tag{A.21}$$

where P_t^d, P_t^{mc} and P_t^{mi} are the prices of the domestic components and imported consumption and investment components. Solving the problems of consumption and investment maximization, we arrive at the equation of demand for domestic and imported components of consumption and investment:

$$C_t^d = (1-\omega_c)\left[\frac{P_t^c}{P_t^d}\right]^{\eta_c} C_t, \qquad C_t^m = \omega_c \left[\frac{P_t^c}{P_t^{mc}}\right]^{\eta_c} C_t,$$

$$I_t^d = (1-\omega_i)\left[\frac{P_t^i \Psi_t}{P_t^d}\right]^{\eta_i} I_t, \qquad I_t^m = \omega_i \left[\frac{P_t^i \Psi_t}{P_t^{mi}}\right]^{\eta_i} \frac{I_t}{\Psi_t}.$$

The prices of final consumption goods $\left(P_t^c\right)$ and investment goods $\left(P_t^i\right)$ are then expressed with the following equations:

$$\begin{aligned} P_t^c &= \left[(1-\omega_c)\left(P_t^d\right)^{1-\eta_c} + \omega_c \left(P^{mc}\right)^{1-\eta_c}\right]^{\frac{1}{1-\eta_c}}, \\ P_t^i \Psi_t &= \left[(1-\omega_i)\left(P_t^d\right)^{1-\eta_i} + \omega_i \left(P_t^{mi}\right)^{1-\eta_i}\right]^{\frac{1}{1-\eta_i}}. \end{aligned} \tag{A.22}$$

The households' physical capital stock $\left(\overline{K}_{j,t+1}\right)$ evolves according to:

$$\overline{K}_{j,t+1} = (1-\delta)\overline{K}_{k,t} + \Upsilon_t F\left(I_{j,t}, I_{j,t-1}\right) + \Delta_{j,t}, \tag{A.23}$$

where δ is the capital depreciation rate, $\Delta_{j,t}$ is the purchase/sale of new, installed capital. Function F is the function of transformation of investment expenditures into physical capital:

$$F\left(I_{j,t}, I_{j,t-1}\right) = \left(1 - \tilde{S}\left(\frac{I_{j,t}}{I_{j,t-1}}\right)\right) I_{j,t}. \tag{A.24}$$

Function $\tilde{S}$ is not explicitly specified, we assume only that:

$$\tilde{S}(x) = \tilde{S}'(x) = 0 \qquad \text{and} \qquad \tilde{S}''(x) \equiv \tilde{S}'' > 0, \qquad x = \mu_z^+ \mu_\Psi. \tag{A.25}$$

This means that full transformation of investments into physical capital takes place when investment expenditures grow at the *steady state* level. In other words, fluctuations in investment expenditures generate costs, which creates the mechanism of smoothening of investment expenditures. An additional factor affecting the effectiveness of transformation of investments into capital goods is the exogenous process Υ_t, called the investment-specific technology shock or effectiveness of transformation of investment into capital. The stochastic structure of the shock is given by (A.3).

The physical capital stock is fully or partially leased to the intermediate goods producers in the form of capital services $K_{j,t}$. With $u_{j,t}$, $\left(u_{j,t} \equiv \frac{K_{j,t}}{\overline{K}_{j,t}}\right)$, we denote utilization rate of capital (in the *steady state* $u = 1$). We assume that incomplete use of the capital resource generates cost for households, depending on the utilization rate — $F^{\tau}_{a,t} a(u_{j,t}) \frac{\overline{K}_{j,t}}{\Psi_t}$. Function $F^{\tau}_{a,t}$ represents a part of the cost depending on the changes in the prices of raw materials and — similarly to the function F^{τ}_{t} — is a function of raw material price shock $\left(\pi^{\text{oil}}_t\right)$ (the solution is based on the work by Christiano et al. (2007a)). Function $a(u_{j,t})$ is not explicitly specified, we assume only that $a(1) = 0$ and $a'' \geq 0$.

The budget constraint of a household takes the form of:

$$
\begin{aligned}
& M_{j,t+1} + S^e_t B^e_{j,t+1} + S^e_t S^x_t B^u_{j,t+1} + P^c_t C_{j,t}\left(1+\tau^c_t\right) + P^i_t I_t \\
& + P^d_t \left(F^{\tau}_{a,t} \frac{a(u_{j,t})}{\Psi_t} \overline{K}_{j,t} + P_{k',t}\Delta_t \right) \\
& = TR_t + D_{j,t} + R_{t-1}\left(M_{j,t} - Q_{j,t}\right) + Q_{j,t} + \left(1-\tau^k_t\right)\Pi_t \\
& + \left(1-\tau^p_t\right) R^k_t u_{j,t} \overline{K}_{j,t} + \left(1-\tau^y_t\right)(1-\tau^w) W_{j,t} h_{j,t} \\
& + R^e_{t-1} \Phi\left(\frac{A^e_{t-1}}{z^+_{t-1}}, \mathrm{E}_t s^e_t s^e_{t-1}, \tilde{\phi}_{t-1} \right) S^e_t B^e_{j,t} + R^u_{t-1} \Phi\left(\frac{A^u_{t-1}}{z^+_{t-1}}, \mathrm{E}_t s^u_t s^u_{t-1}, \tilde{\phi}_{t-1} \right) S^e_t S^x_t B^u_{j,t} \\
& - \tau^k_t \Bigg[\left(R^b_{t-1} - 1\right)\left(M_{j,t} - Q_{j,t}\right) + \left(R^e_{t-1} \Phi\left(\frac{A^e_{t-1}}{z^+_{t-1}}, \mathrm{E}_t s^e_t s^e_{t-1}, \tilde{\phi}_{t-1} \right) - 1 \right) S^e_t B^e_{j,t} \\
& \quad + \left(R^u_{t-1} \Phi\left(\frac{A^u_{t-1}}{z^+_{t-1}}, \mathrm{E}_t s^u_t s^u_{t-1}, \tilde{\phi}_{t-1} \right) - 1 \right) S^e_t S^x_t B^u_{j,t} + B^e_{j,t}\left(S^e_t - S^e_{t-1}\right) \\
& \quad + B^u_{j,t}\left(S^e_t S^x_t - S^e_{t-1} S^x_{t-1}\right) \Bigg] + \tau^p_t P^d_t F^{\tau}_t \frac{a(u_{j,t})}{\Psi_t} \overline{K}_{j,t} + \tau^p_t P_t P_{k',t} \delta \overline{K}_{j,t}
\end{aligned}
\tag{A.26}
$$

where $M_{j,t}$ are domestic financial assets, $B^e_{j,t}$ and $B^u_{j,t}$ are assets denominated in euro and dollar, $P_{k',t}$ is the relative price of capital goods, τ^c_t is the consumption tax rate, τ^k_t is the capital tax rate (on interest from deposits and dividends), τ^p_t is the corporate income tax rate, Π_t are the profits of intermediate goods producers (domestic, exporters and importers), TR_t are lump-sum transfers from state budget, $D_{j,t}$ is the income from state contingent securities, τ^y_t and τ^w are the personal income tax rate and the rate of national insurance contribution paid by an employee.

Foreign assets, $B^e_{j,t}$ and $B^u_{j,t}$, bear interest according to the interest rates for the euro area, R^e_t, and the rest of the world, R^u_t, adjusted for risk premium, see e.g. (Adolfson et al., 2007, page 8) and (Schmitt-Grohé and Uribe, 2003; Engel, 1996):

$$
\Phi\left(\frac{A^o_{t-1}}{z^+_{t-1}}, \mathrm{E}_t s^o_t s^o_{t-1}, \tilde{\phi}_{t-1} \right) \quad \text{where:} \quad A^o_t \equiv \frac{S^o_t B^o_{t+1}}{P_t},\ s^o_t \equiv \frac{S^o_t}{S^o_{t-1}}, \quad \text{and} \quad o \in \{e, u\}
\tag{A.27}
$$

while $\tilde{\phi}_t$ is the risk premium shocks described with stochastic processes given by (A.3). Risk premium for assets in the given currency depends on the position in those assets at the scale of the whole economy, while function Φ is strictly decreasing in A^e_t $\left(A^u_t\right)$. For total foreign assets, $a_t \equiv \frac{A_t}{z^+_t} = \frac{A^e_t}{z^+_t} + \frac{A^u_t}{z^+_t}$, we assume that in the *steady state* they are equal to 0, while foreign assets denominated in euro are positive (then $a^u = -a^e$).

Based on the utility function (A.19), budget constraint (A.26) and the law of motion of capital (A.23) we may formulate an optimization problem and the Lagrange functional in stationary and constant prices terms. Solution to the problem, taking into account usual symmetry argument, as well as some approximations, gives the following conditions:

$$\frac{\zeta_t^c}{c_t - bc_{t-1}\frac{1}{\mu_{z^+,t}}} - \beta b \mathop{\mathrm{E}}_t \frac{\zeta_{t+1}^c}{c_{t+1}\mu_{z^+,t+1} - bc_t} - \psi_{z^+,t}\gamma_t^{cd}\left(1+\tau_t^c\right) = 0, \tag{A.28}$$

$$\frac{\upsilon_t P_t^d p_{k',t}}{\Psi_t} = \omega_t, \tag{A.29}$$

$$\begin{aligned} -\psi_{z^+,t}\gamma_t^{id} + \psi_{z^+,t}p_{k',t}\Upsilon_t F_1\left(i_t, i_{t-1}, \mu_{z^+,t}\mu_{\Psi,t}\right) \\ + \beta \mathop{\mathrm{E}}_t \psi_{z^+,t+1}p_{k',t+1}\Upsilon_{t+1}F_2\left(i_{t+1}, i_t, \mu_{z^+,t+1}\mu_{\Psi,t+1}\right) = 0, \end{aligned} \tag{A.30}$$

$$-\psi_{z^+,t} + \beta \mathop{\mathrm{E}}_t \frac{\psi_{z^+,t+1}}{\pi_{t+1}\mu_{z^+,t+1}}\left[\left(1-\tau_{t+1}^k\right)\left(R_t - 1\right) + 1\right] = 0, \tag{A.31}$$

$$\begin{aligned} -\psi_{z^+,t}p_{k',t} + \beta \mathop{\mathrm{E}}_t \frac{\psi_{z^+,t+1}}{\mu_{z^+,t+1}\mu_{\Psi,t+1}}\Big[\left(1-\tau_{t+1}^p\right)\left(\bar{r}_{t+1}^k u_{t+1} - F_{t+1}^\tau a(u_{t+1})\right) \\ + \tau_{t+1}^p p_{k',t+1}\delta + p_{k',t+1}\left(1-\delta\right)\Big] = 0, \end{aligned} \tag{A.32}$$

$$\bar{r}_t^k = F_{a,t}^\tau a'(u_{j,t}), \tag{A.33}$$

$$\zeta_t^q A_q q_t^{-\sigma_q} - \psi_{z^+,t}\left(1-\tau_t^k\right)\left(R_{t-1}^b - 1\right) = 0, \tag{A.34}$$

$$-\psi_{z^+,t} + \beta \mathop{\mathrm{E}}_t \frac{\psi_{z^+,t+1}}{\mu_{z^+,t+1}\pi_{t+1}}\left[s_{t+1}^e R_t^e \Phi\left(a_t^e, s_{t+1}^e s_t^e, \tilde{\phi}_t\right)\left(1-\tau_{t+1}^k\right) + \tau_{t+1}^k\right] = 0, \tag{A.35}$$

$$-\psi_{z^+,t} + \beta \mathop{\mathrm{E}}_t \frac{\psi_{z^+,t+1}}{\mu_{z^+,t+1}\pi_{t+1}}\left[s_{t+1}^e s_{t+1}^x R_t^u \Phi\left(a_t^u, \mathop{\mathrm{E}}_t s_{t+1}^u s_t^u, \tilde{\phi}_t\right)\left(1-\tau_{t+1}^k\right) + \tau_{t+1}^k\right] = 0. \tag{A.36}$$

We use these conditions to derive (log-linearized) equations for aggregate consumption, investment, exchange rate, money demand (cash and broad money), capital utilization rate, stock of fixed assets, relative prices of fixed assets and income multiplier.

Wages. The labour market is characterised by monopolistic competition. Households provide heterogeneous labour services $h_{l,t}$ and the unions (representing workers) set wages $W_{l,t}$ taking into account wealth/consumption of households. We again use packed/reduced form utility function, since the decision problem solved by unions (to optimize wages) focuses on the households utility. Aggregate demand for labour services H_t, set by the domestic intermediate goods producers, is defined as follows:

$$H_t = \left[\int_0^1 \left(h_{l,t}\right)^{\frac{1}{\lambda_t^w}} dl\right]^{\lambda_t^w}, \qquad 1 \le \lambda_t^w < \infty,$$

where wage markup is described with an exogenous process of the form given by (A.3). The process of wage setting runs similarly to the process of price setting by the producers (Calvo model) — in each period, with probability $1-\xi_w$, a household (union) may set optimal wage; with probability ξ_w wage cannot be reoptimized, it may only be indexed to previous inflation of consumer prices (with weight κ_w), the current value of the inflation target (with weight $1-\kappa_w$) and the current technology growth:

$$W_{l,t+1}=\left(\pi_t^c\right)^{\kappa_w}\left(\overline{\pi}_{t+1}^c\right)^{1-\kappa_w}\mu_{z^+,t+1}W_{l,t}.$$

When a household is allowed to set the wage in an optimal way, it maximizes the difference between the utility of income on account of wage and disutility of leisure reduction:

$$\max_{W_t^{new}}\mathop{\mathrm{E}}_t\sum_{s=0}^{\infty}\left(\beta\xi_w\right)^s\left[-\zeta_{t+s}^h A_L\tilde{n}_{t+s}\frac{\left(h_{l,t+s}\right)^{1+\sigma_L}}{1+\sigma_L}+\upsilon_{t+s}\left(1-\tau_{t+s}^y\right)\left(1-\tau^w\right)\times\right.$$
$$\left.\times\left(\left(\pi_t^c\ldots\pi_{t+s-1}^c\right)^{\kappa_w}\left(\overline{\pi}_{t+1}^c\ldots\overline{\pi}_{t+s}^c\right)^{1-\kappa_w}\left(\mu_{z^+,t+1}\ldots\mu_{z^+,t+s}\right)W_t^{new}\right)h_{l,t+s}\right].$$

The first order condition of the above decision-making problem leads to the equation of real wage in economy. The log-linearized basic variant of real wage $\widehat{\widehat{w}}_t$ equation is as follows:

$$\begin{aligned}\widehat{\widehat{w}}_t=&\frac{b_w\,\xi_w}{\left[b_w\left(1+\beta\,\xi_w^2\right)-\lambda^w\,\sigma_L\right]}\left[\left(\widehat{\widehat{w}}_{t-1}+\beta\mathop{\mathrm{E}}_t\widehat{\widehat{w}}_{t+1}\right)+\kappa_w\left(\widehat{\pi}_{t-1}^c-\beta\widehat{\pi}_t^c\right)\right.\\&\left.+\left(1-\kappa_w\right)\left(\widehat{\overline{\pi}}_t^c-\beta\mathop{\mathrm{E}}_t\widehat{\overline{\pi}}_{t+1}^c\right)-\left(\widehat{\pi}_t-\beta\mathop{\mathrm{E}}_t\widehat{\pi}_{t+1}\right)\right]\\&+\frac{\lambda^w-1}{\left[b_w\left(1+\beta\,\xi_w^2\right)-\lambda^w\,\sigma_L\right]}\left[\sigma_L\widehat{H}_t^d+\widehat{\zeta}_t^h+\widehat{n}_t+\widehat{\lambda}_t^w+\widehat{\gamma}_t^{cd}+\frac{\tau_c}{1+\tau^c}\widehat{\tau}_t^c+\frac{\tau_y}{1-\tau^y}\widehat{\tau}_t^y\right]\end{aligned}\tag{A.37}$$

Labour supply. To derive aggregate labour supply, we assume that each household faces given wage (set by unions) and given demand for each type of labour services (set by firms given wages). The household composed of infinite members (marked by pair (i,j), where i is the labour type and j is personal disutility to work) send to work members with lowest disutility from work. Again we use the packed/reduced form utility function:

$$\max\mathop{\mathrm{E}}_t\sum_{s=0}^{\infty}\beta^s\left[-\zeta_{t+s}^h A_L\tilde{n}_{t+s}\frac{\left(h_{l,t+s}^s\right)^{1+\sigma_L}}{1+\sigma_L}+\psi_{z^+,t+s}\left(1-\tau_{t+s}^y\right)\left(1-\tau^w\right)\overline{w}_{t+s}h_{l,t+s}^s\right]\tag{A.38}$$

Solving this problem under assumption of symmetry, we receive the following log-linearized form of aggregate labour supply:

$$\widehat{H}_t^s=\frac{1}{\sigma_L}\left[\left(\widehat{\overline{w}}_t-\widehat{\gamma}_t^{cd}-\frac{\tau^y}{(1-\tau^y)}\widehat{\tau}_t^y-\frac{\tau^c}{1+\tau^c}\widehat{\tau}_t^c\right)-\left(\widehat{\zeta}_t^h+\widehat{n}_t\right)\right]\tag{A.39}$$

The equation is an enhanced version of aggregate labour supply function (1.5) taken from Galí's papers. The packed version of utility function used to derive our labour supply function does not distort the result.

Unemployment. We define the unemployment rates in the following way:

$$un_t^g = \frac{H_t^s/n - H_t^d/n}{H_t^s/n} = \frac{E_t^s - E_t^d}{E_t^s} \quad \text{and} \quad un_t = \frac{\tilde{E}_t^s - \tilde{E}_t}{\tilde{E}_t^s} \approx \frac{H_t^s - \tilde{E}_t}{H_t^s} \tag{A.40}$$

where n is the number of hours worked per employee and E_t^s, E_t^d are the full-time employment (supply and demand). The first rate is casted to conditions of the model (we call it the model's unemployment rate), the second (called the observed unemployment rate) refers to observed variables: $\tilde{E}_t$ is observed employment and $\tilde{E}_t^s$ is the labour force (or number of persons looking for a job given the wage), however we assume $H_t^s = \tilde{E}_t^s$ and $n = 1$ (see equation (A.14) and comments below this formula), to use the approximation. Notice, that *steady state* of these rates is the same:

$$un^g = un = 1 - \left(\frac{1}{\lambda^w}\right)^{\frac{1}{\sigma_L}} \tag{A.41}$$

and

$$un_t = un_t^g - \frac{\tilde{E}_t - H_t^d}{H_t^s} = un_t^g + dun_t.$$

The equation (A.41) indicates that the deep parameters λ^w, σ_L are not independent.

Behaviour of other agents

The SoePL model explicitly considers the existence of two additional agents — the central bank and the government. These agents have not been assigned any autonomous object functions. It is only assumed that the purpose of the central bank is to control price dynamics, and the only instrument the bank has at its disposal is the interest rate. The government manages budget funds, i.e. charges taxes from which expenditures are financed.

Central bank. The central bank conducts strategy of direct inflation targeting and follows a simplified Taylor type interest rate rule. The rule is defined in log-linearized form:

$$\begin{aligned}\widehat{R}_t = \rho_R \widehat{R}_{t-1} + \left(1-\rho_R\right)\left[r_{\overline{\pi}^c}\widehat{\overline{\pi}}_t^c + r_\pi\left(\widehat{\pi}_{t-1}^c - \widehat{\overline{\pi}}_t^c\right) + r_y \widehat{y}_{t-1} + r_x \widehat{x}_{t-1}^{eu}\right] \\ + r_{\Delta\pi}\Delta\widehat{\pi}_t^c + r_{\Delta y}\Delta\widehat{y}_t + \epsilon_{R,t},\end{aligned} \tag{A.42}$$

where $\widehat{x}_t^{ue} = \widehat{x}_t^u + \left(1-\gamma^{xux}\right)\widehat{x}_t^x$ is the real effective interest rate. In the current version of the model we assume that $r_{\overline{\pi}^c} \equiv 1$, therefore, $\widehat{\overline{\pi}}_t^c$ shall be interpreted as the perception of the policy of the central bank (inflation target) by the agents. The disturbance of the interest rate (monetary policy, monetary disturbance) $\epsilon_{R,t}$ is defined as innovations.

Government. The government manages the state income redistribution. The government expenditures and three taxes (the consumption tax τ_t^c, the income tax τ_t^y and the corporate income tax τ_t^p) form an external SVAR model (a kind of reduced form model of fiscal block). The SVAR model is estimated separately and the results (approximation of processes that drive government expenditures and effective taxes' rate fluctuations) are included into the main DSGE model. The government budget constraint is taken into account forming macroeconomic market clearing conditions and resource (real and nominal) constraints. We present the constraint in the next section.

Macroeconomic balance conditions

Below we present the main components of the system of macroeconomic balance conditions: account of profits (a part of total income of households and the state budget), aggregated households incomes and expenditures balance, the state budget balance, the balance of banking sector. The aggregate of these balances defines foreign net assets, hence it replaces usual balance of payment (it fact the balance of payment is expressed in slightly different form).

Profits in the economy

The domestic intermediate goods producers, importers and exporters transfer their profits to households where the profits are taxed with the dividend tax. Total profit in the economy is the sum of the profits generated in particular sectors of the economy: $\Pi_t = \Pi_t^d + \Pi_t^x + \Pi_t^m$. We follow the suggestion of Christiano et al. (2007b, page 26–28) and approximate the profits generated by firms computing the difference between the marginal cost and the actual price.

Domestic products manufacturers In the case of intermediate products manufacturers we have:

$$\Pi_t^d = \int_0^1 P_{jt}^d Y_{jt} \, \mathrm{d} j - P_t^d mc_t^d \left(\int_0^1 Y_{jt} \, \mathrm{d} j \right) = P_t^d Y_t - P_t^d mc_t^d \left(Y_t \left(\frac{\bar{P}_t^d}{P_t^d} \right)^{\frac{\lambda_t^d}{1-\lambda_t^d}} \right).$$

The value $\left(\frac{\bar{P}_t^d}{P_t^d} \right)^{\frac{\lambda_t^d}{1-\lambda_t^d}}$ is the allocation inefficiency. Such expression may be approximated with the function of markup $f(\lambda_t^d, \ldots) \equiv f_t^d$, however, in the case of log-linearization it is also justified to treat the price relations as equal one. Taking into account the above, profits at macro scale may be estimated as:

$$\Pi_t^d = P_t^d Y_t - P_t^d mc_t^d \left(Y_t f_t^d \right). \tag{A.43}$$

Profits in export Assuming, further, that profits in export (calculated in domestic currency) are subject to domestic tax, omitting the existence of fixed costs, we have:

$$\Pi_t^x = P_t^x S_t^e S_t^x \left(C_t^x + I_t^x \right) \left(1 - mc_t^x f_t^x \right), \tag{A.44}$$

where f_t^x is defined as f_t^d.

Profits in import Out of the two possible methods of defining profits (on the micro level with further aggregation or with direct reference to the macro scale), we have used the macroeconomic convention:

$$\Pi_t^m = P_t^{mc} C_t^m + P_t^{mi} I_t^m + P_t^{mx} X_t^m - S_t^e S_t^x P_t^\star \left(C_t^m + I_t^m + X_t^m \right).$$

Income and expenditures of households

The aggregated version of the budget condition of households is as follows:

$$\begin{aligned}
&\left(1-\tau_t^k\right)\left(R_{t-1}-1\right)\left(M_t-Q_t\right)\\
&+\left(1-\tau_t^k\right)R_{t-1}^e\Phi\left(a_{t-1}^e,s_t^e,s_{t-1}^e,\tilde{\phi}_{t-1}^e\right)S_t^e B_t^e+\tau_t^k S_{t-1}^e B_t^e\\
&+\left(1-\tau_t^k\right)R_{t-1}^u\Phi\left(a_{t-1}^u,s_t^x,s_t^e,s_{t-1}^x,s_{t-1}^e,\tilde{\phi}_{t-1}^u\right)S_t^e S_t^x B_t^u+\tau_t^k S_{t-1}^e S_{t-1}^x B_t^u\\
&+\left(1-\tau_t^k\right)\Pi_t+\left(1-\tau_t^p\right)\left(R_t^k u_t \overline{K}_t-\frac{1}{\Psi_t}P_t F_{a,t}^{\tau}a\left(u_t\right)\overline{K}_t\right)+\tau_t^p P_t P_{k',t}\delta\overline{K}_t\\
&+\left(1-\tau_t^y\right)(1-\tau^w)W_t H_t+M_t+TR_t+D_t\\
&-M_{t+1}-S_t^e B_{t+1}^e-S_t^e S_t^x B_{t+1}^u-\left(1+\tau_t^c\right)P_t^c C_t-P_t^i I_t-P_t P_{k',t}\Delta_t=0.
\end{aligned}\tag{A.45}$$

State budget

The term state budget or ”government budget” means here, approximately, the sector of public finance and a fragment of the financial sector specializing in pension insurance.

$$\begin{aligned}
P_t G_t+\left(TR_t+D_t\right)=R_{t-1}&\left(M_{t+1}-M_t\right)+\tau_t^c P_t^c C_t+\left(\tau^w+\tau^s+\tau_t^y\,(1-\tau^w)\right)W_t H_t\\
&+\tau_t^k\Big[\Pi_t+\left(R_{t-1}-1\right)\left(M_t-Q_t\right)\\
&\quad+\left(R_{t-1}^e\Phi^e(a_{t-1}^e,\ldots,\tilde{\phi}_{t-1}^e)-1\right)S_t^e B_t^e+B_t^e\left(S_t^e-S_{t-1}^e\right)\\
&\quad+\left(R_{t-1}^u\Phi^u(a_{t-1}^u,\ldots,\tilde{\phi}_{t-1}^u)-1\right)S_t^u B_t^u+B_t^u\left(S_t^u-S_{t-1}^u\right)\Big]\\
&+\tau_t^p\left[R_t^k u_t\overline{K}_t-\frac{1}{\Psi_t}P_t F_{a,t}^{\tau}a\left(u_t\right)\overline{K}_t-P_t P_{k',t}\delta\overline{K}_t\right].
\end{aligned}\tag{A.46}$$

Monety market clearing

As in the original version of the SOE model, we define the broad money dynamics as:

$$\mu_t\equiv\frac{M_{t+1}}{M_t}=\frac{m_{t+1}}{m_t}\mu_{z^+,t}\,\pi_t^d.\tag{A.47}$$

We notice that in the *steady state* the following applies:

$$\pi^d=\frac{\mu}{\mu_{z^+}}.$$

The banking system must provide financing to firms with working capital loans. Therefore, we have the dependence:

$$\nu_t^k\,\mathrm{F}_t^{\tau}\,R_t^k K_t+\nu_t^w\,\mathrm{F}_t^{\tau}\,(1+\tau^s)\,W_t H_t=M_{t+1}-Q_t,\tag{A.48}$$

where $M_{t+1}=\mu_t M_t$.

Balance of payment

Merging the households budget constraint (A.45) and the state budget, we arrive at the formula, which upon simplification takes the following form:

$$
\begin{aligned}
P_t^g G_t + P_t^i I_t + P_t^c C_t + S_t^e B_{t+1}^e + S_t^e S_t^x B_{t+1}^u &= \Pi_t + (1+\tau^s) W_t H_t \\
&+ \left(R_t^k u_t - \frac{1}{\Psi_t} P_t F_a^\tau a(u_t) \right) \overline{K}_t + (R_{t-1} - 1)(M_{t+1} - Q_t) \\
&+ R_{t-1}^e \Phi(a_{t-1}^e, ..., \tilde{\phi}_{t-1}) S_t^e B_t^e + R_{t-1}^u \Phi(a_{t-1}^u, ..., \tilde{\phi}_{t-1}) S_t^e S_t^x B_t^u .
\end{aligned}
$$

Taking into account balance of the banking sector, we obtain:

$$
\begin{aligned}
P_t^g G_t + P_t^i I_t + P_t^c C_t + S_t^e B_{t+1}^e + S_t^e S_t^x B_{t+1}^u &= \Pi_t + (1+\tau^s) \left[\mathrm{F}_t^\tau \left(R_t^{fw} - 1 \right) + 1 \right] W_t H_t \\
&+ \left[\left[\mathrm{F}_t^\tau \left(R_t^{fk} - 1 \right) + 1 \right] R_t^k u_t - \frac{1}{\Psi_t} P_t \, \mathrm{F}_{a,t}^\tau \, a(u_t) \right] \overline{K}_t \\
&+ R_{t-1}^e \Phi(a_{t-1}^e, ..., \tilde{\phi}_{t-1}) S_t^e B_t^e + R_{t-1}^u \Phi(a_{t-1}^u, ..., \tilde{\phi}_{t-1}) S_t^e S_t^x B_t^u .
\end{aligned}
\tag{A.49}
$$

The above equation shows that expenditures calculated at the macro level of economy (consumer expenditures, investment expenditures, government expenditures, new foreign deposits (net foreign assets), and capital adjustment are financed from the profits, income from labour, income from capital and revenues from (mature) foreign deposits. Net foreign assets amount in total to $a_t = a_t^e + a_t^u$.

The aggregate resource constraint

The starting point for further considerations is the formula in which all of the components are expressed at fixed prices. In resources constraint we omit the factor characterising the ineffectiveness of allocation (the effect of Calvo price settings). The obtained inequality has the form:

$$
G_t + C_t^d + \frac{1}{\Psi_t} \left[I_t^d + F_{a,t}^\tau \, a(u_t) \overline{K}_t \right] + X_t^d \le \varepsilon_t \, z_t^{1-\varpi} K_t^\varpi H_t^{1-\varpi} \tag{A.50}
$$

A.2 Parameters of DSGE SoePL-2012

Table 4. Estimates of selected parameters of SoePL-2011 (the second regime) and SoePL-2012 models

Para-meters	Prior Distribution						Optimization Results			
	Type		Mean		Std.dev./DF		Mode		Std. Er./Hessian/	
	2011	2012	2011	2012	2011	2012	2011	2012	2011	2012
ξ_w	Beta	Beta	0.60	0.53	0.13	0.07	0.46	0.66	0.10	0.07
ξ_d	Beta	Beta	0.60	0.65	0.13	0.13	0.78	0.85	0.03	0.03
ξ_{mc}	Beta	Beta	0.60	0.65	0.13	0.13	0.74	0.79	0.05	0.04
ξ_{mi}	Beta	Beta	0.60	0.65	0.13	0.13	0.64	0.66	0.08	0.07
ξ_{mx}	Beta	Beta	0.60	0.65	0.13	0.13	0.37	0.33	0.08	0.06
ξ_x	Beta	Beta	0.60	0.65	0.13	0.13	0.58	0.62	0.06	0.06
ξ_e	Beta	Beta	0.60	0.65	0.13	0.13	0.71	0.67	0.04	0.04
κ_w	Beta	Beta	0.40	0.40	0.14	0.14	0.37	0.29	0.13	0.11
κ_d	Beta	Beta	0.40	0.40	0.14	0.14	0.18	0.18	0.07	0.07
κ_{mc}	Beta	Beta	0.40	0.40	0.14	0.14	0.30	0.24	0.13	0.10
κ_{mi}	Beta	Beta	0.40	0.40	0.14	0.14	0.20	0.18	0.09	0.08
κ_{mx}	Beta	Beta	0.40	0.40	0.14	0.14	0.26	0.28	0.11	0.12
κ_x	Beta	Beta	0.40	0.40	0.14	0.14	0.15	0.16	0.07	0.07
$\tilde{S}'$	TNor	TNor	7.00	8.00	0.50	0.40	6.95	8.10	0.50	0.40
b	Beta	Beta	0.65	0.63	0.10	0.10	0.78	0.78	0.06	0.06
σ_L	fixed	TNor	–	2.98	–	0.17	1.00	2.86	–	0.19
ϑ_n	–	Beta	–	0.30	–	0.13	–	0.16	–	0.05
ν^{τ}	TNor	TNor	0.70	0.47	0.13	0.10	0.15	0.03	0.10	0.03
$\nu^{a\tau}$	TNor	TNor	0.25	0.20	0.13	0.15	0.12	0.10	0.06	0.05
$\tilde{\phi}_a^u$	InvG	InvG	0.50	0.45	2.00	2.00	0.59	0.61	0.22	0.28
$\tilde{\phi}_a^e$	InvG	InvG	0.25	0.45	2.00	2.00	0.24	0.39	0.07	0.12
$\tilde{\phi}_s^u$	Beta	Beta	0.33	0.20	0.15	0.15	0.28	0.17	0.12	0.11
$\tilde{\phi}_s^e$	Beta	Beta	0.33	0.20	0.15	0.15	0.21	0.13	0.08	0.09
ρ_{ϵ}	Beta	Beta	0.70	0.60	0.13	0.13	0.73	0.75	0.09	0.06
ρ_{Υ}	Beta	Beta	0.70	0.60	0.13	0.13	0.69	0.55	0.07	0.07
$\rho_{\tilde{z}^{\star}}$	Beta	Beta	0.90	0.90	0.13	0.13	0.98	0.96	0.02	0.06
ρ_{μ_z}	Beta	Beta	0.60	0.50	0.13	0.13	0.61	0.53	0.12	0.12
$\rho_{\mu_{\Psi}}$	Beta	Beta	0.60	0.50	0.13	0.13	0.52	0.41	0.12	0.13
ρ_{ζ^c}	Beta	Beta	0.70	0.60	0.13	0.13	0.44	0.47	0.12	0.13
ρ_{ζ^h}	Beta	Beta	0.70	0.60	0.13	0.13	0.57	0.80	0.14	0.07
$\rho_{\lambda^{mc}}$	Beta	Beta	0.80	0.75	0.10	0.10	0.64	0.51	0.12	0.10
$\rho_{\lambda^{mi}}$	Beta	Beta	0.80	0.75	0.10	0.10	0.63	0.59	0.14	0.14
$\rho_{\lambda^{mx}}$	Beta	Beta	0.85	0.80	0.10	0.10	0.95	0.98	0.03	0.01
ρ_{λ^x}	Beta	Beta	0.85	0.70	0.10	0.10	0.82	0.74	0.07	0.08
ρ_{λ^w}	Beta	Beta	0.67	0.60	0.10	0.13	0.64	0.52	0.12	0.15
$\rho_{\tilde{\phi}^e}$	Beta	Beta	0.67	0.75	0.10	0.13	0.57	0.79	0.08	0.08
$\rho_{\nu^{oil}}$	Beta	Beta	0.67	0.67	0.25	0.25	0.33	0.39	0.11	0.12
σ_{ϵ}	InvG	InvG	1.00	1.00	2.00	2.00	2.22	2.14	0.47	0.49
σ_{Υ}	InvG	InvG	1.00	1.00	2.00	2.00	1.05	1.16	0.15	0.17
$\sigma_{\tilde{z}^{\star}}$	InvG	InvG	1.00	1.00	2.00	2.00	0.45	0.46	0.07	0.07
σ_{μ_z}	InvG	InvG	0.75	0.75	2.00	2.00	0.36	0.38	0.05	0.06
$\sigma_{\mu_{\Psi}}$	InvG	InvG	0.75	0.75	2.00	2.00	0.55	0.67	0.14	0.21
σ_{ζ^c}	InvG	InvG	0.75	1.00	2.00	2.00	0.67	0.72	0.09	0.10
σ_{ζ^h}	InvG	InvG	0.75	1.00	2.00	2.00	0.40	0.67	0.07	0.09
σ_{λ^d}	InvG	InvG	1.00	1.00	2.00	2.00	0.96	0.92	0.11	0.10
$\sigma_{\lambda^{mc}}$	InvG	InvG	1.00	1.00	2.00	2.00	0.65	0.55	0.15	0.11
$\sigma_{\lambda^{mi}}$	InvG	InvG	1.00	1.00	2.00	2.00	1.86	1.64	0.55	0.35
$\sigma_{\lambda^{mx}}$	InvG	InvG	1.00	5.00	2.00	2.00	9.79	9.75	3.47	2.93
σ_{λ^x}	InvG	InvG	1.00	1.00	2.00	2.00	2.11	1.94	0.52	0.46
σ_{λ^w}	InvG	InvG	1.00	1.00	2.00	2.00	0.46	0.47	0.07	0.07

See next page

Parameters	Prior Distribution						Optimization Results			
	Type		Mean		Std.dev./DF		Mode		Std. Er./Hessian/	
	2011	2012	2011	2012	2011	2012	2011	2012	2011	2012
σ_R	InvG	InvG	0.28	0.33	2.00	2.00	0.19	0.23	0.03	0.03
$\sigma_{\tilde{\pi}^c}$	InvG	InvG	0.25	0.25	2.00	2.00	0.37	0.16	0.18	0.04
$\sigma_{\tilde{\phi}}$	InvG	InvG	2.00	1.00	2.00	2.00	2.10	2.04	0.43	0.58
$\sigma_{y^{oil}}$	InvG	InvG	5.00	5.00	2.00	2.00	13.59	13.68	1.27	1.33
ρ_R	Beta	Beta	0.80	0.65	0.09	0.13	0.84	0.88	0.03	0.03
r_π	TNor	TNor	1.70	1.85	0.15	0.14	1.87	1.92	0.12	0.14
$r_{\Delta\pi}$	TNor	TNor	0.30	0.30	0.07	0.07	0.22	0.21	0.04	0.04
r_y	Norm	Norm	0.13	0.13	0.07	0.07	0.07	0.13	0.06	0.06
$r_{\Delta y}$	TNor	TNor	0.13	0.17	0.07	0.07	0.10	0.14	0.03	0.04
λ^w	TNor	TNor	1.15	1.18	0.07	0.07	1.19	1.41	0.07	0.07
λ^d	TNor	TNor	1.20	1.20	0.07	0.07	1.29	1.22	0.06	0.06
λ^{mc}	TNor	TNor	1.20	1.20	0.07	0.07	1.27	1.27	0.07	0.05
λ^{mi}	TNor	TNor	1.20	1.20	0.07	0.07	1.24	1.28	0.07	0.06
η_c	InvG	InvG	4.00	4.75	2.00	2.00	3.14	3.67	0.37	0.46
η_i	InvG	InvG	4.00	4.75	2.00	2.00	3.58	4.79	0.64	1.14
η_{xx}	InvG	InvG	4.00	4.75	2.00	2.00	4.13	5.77	0.86	1.57
η_{fu}	InvG	InvG	3.00	3.00	2.00	2.00	3.02	3.14	0.52	0.53
η_{fe}	InvG	InvG	2.00	2.00	2.00	2.00	1.92	1.96	0.13	0.15
μ_z	TNor	TNor	1.01	1.01	0.00	0.00	1.01	1.01	0.00	0.00
μ_Ψ	TNor	TNor	1.01	1.01	0.00	0.00	1.01	1.01	0.00	0.00

Norm — Normal, TNor — Truncated normal, InvG — Inverted Gamma

Table 5. Steady state of the DSGE SoePL-2011 and SoePL-2012

Variable	Value [%]	
	SoePL-2011	SoePL-2012
Growth rate of real GDP (annual)	4.1	4.1
Growth rate of real consumption (annual)	4.1	4.1
Growth rate of real exports and imports (annual)	4.1	4.1
Growth rate of real investment (annual)	5.3	5.2
Growth rate of real wages	4.1	3.4†
CPI inflation (annual)	2.6	2.5
Interest rate (unadjusted)	7.1	7.1
Interest rate before 2008	5.7†	5.3†
Interest rate after 2008	5.7†	4.5†
Unemployment rate	–‡	11.3
Growth rate of real US GDP (annual)	2.9†	2.3†
Growth rate of real euro area GDP (annual)	2.0†	2.1†
Growth rate of US GDP deflator	2.3†	2.0†
Growth rate of euro area GDP deflator	2.3†	2.0†
Interest rate of US dollar, before 2008 (annual)	4.6†	3.2†
Interest rate of US dollar, after 2008 (annual)	4.6†	2.6†
Interest rate of euro, before 2008 (annual)	3.7†	3.2†
Interest rate of euro, after 2008 (annual)	3.7†	2.6†

† Adjusted, the method was described in section 2.3.1

‡ Value implied by SoePL-2012 specification (eq. (2.7)) and SoePL-2011 parameters is 16.0

A.3 Low and high labour supply elasticity — IRFs comparison

Figure 9. Impulse response function of wage markup shock — low (green line) and high (blue line) labour elasticity

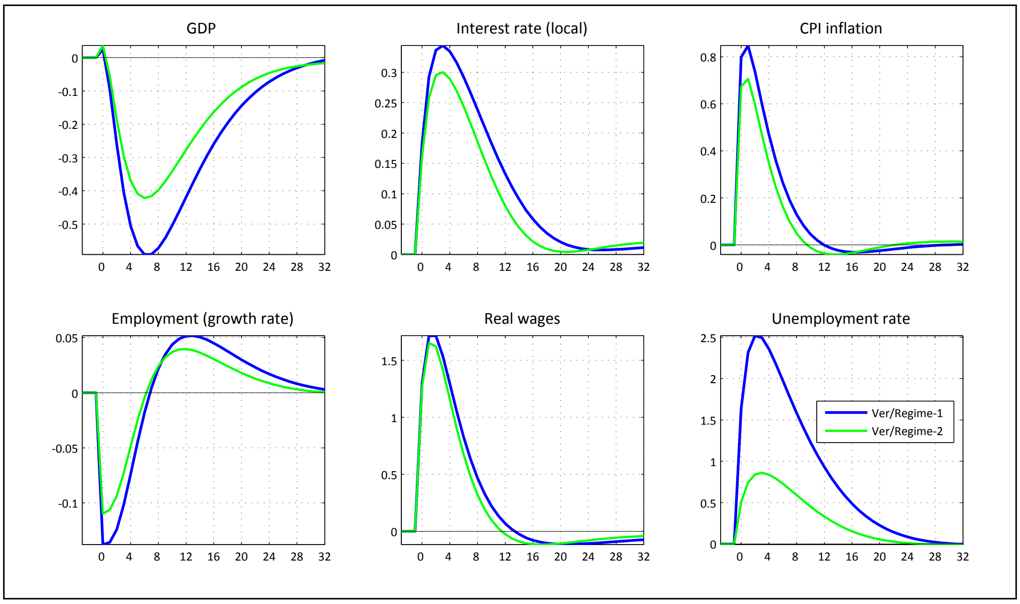


Figure 10. Impulse response function of labour supply preferences shock — low (green line) and high (blue line) labour elasticity

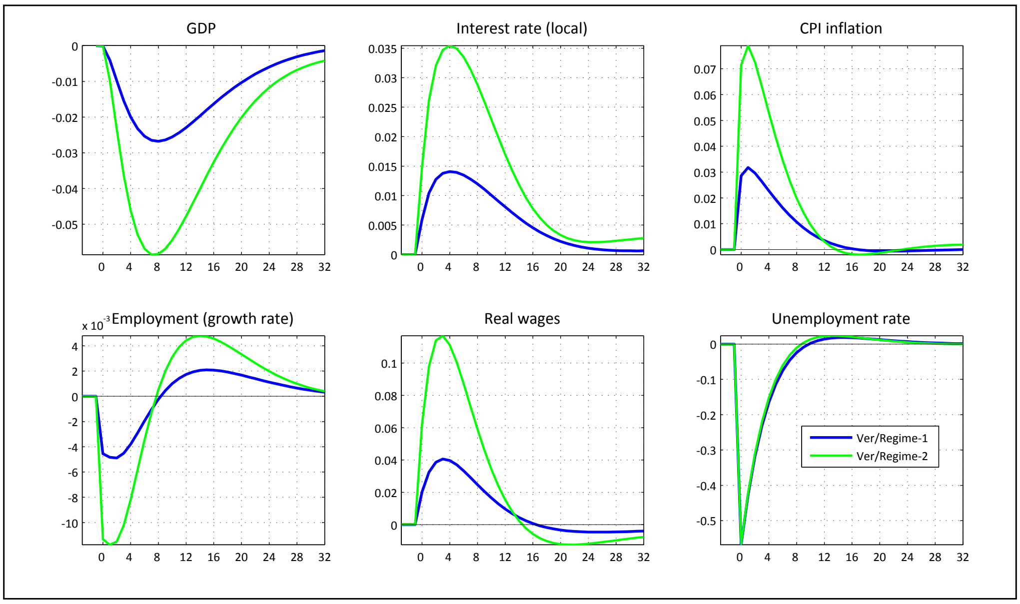


Figure 11. Impulse response function of TFP shock — low (green line) and high (blue line) labour supply elasticity

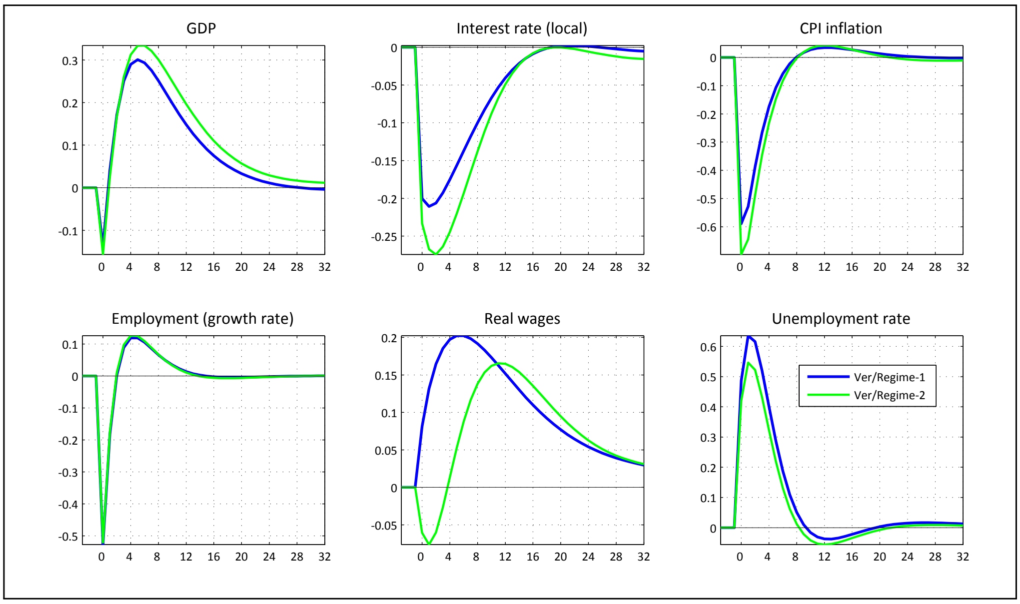
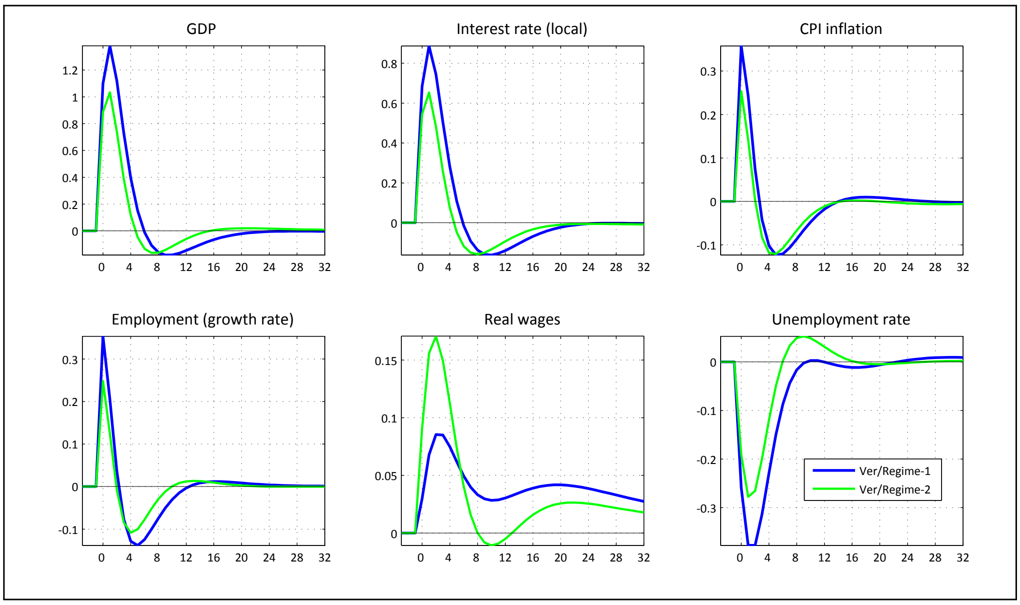


Figure 12. Impulse response function of risk premium shock — low (green line) and high (blue line) labour elasticity



A.4 DSGE SoePL-2012 and SoePL-2012 — Rolling forecasts

Observed variables presented in this section (but unemployment rate and interest rate) are annual growth rates [%].

Figure 13. SoePL-2011 — Rolling *ex post* forecasts.

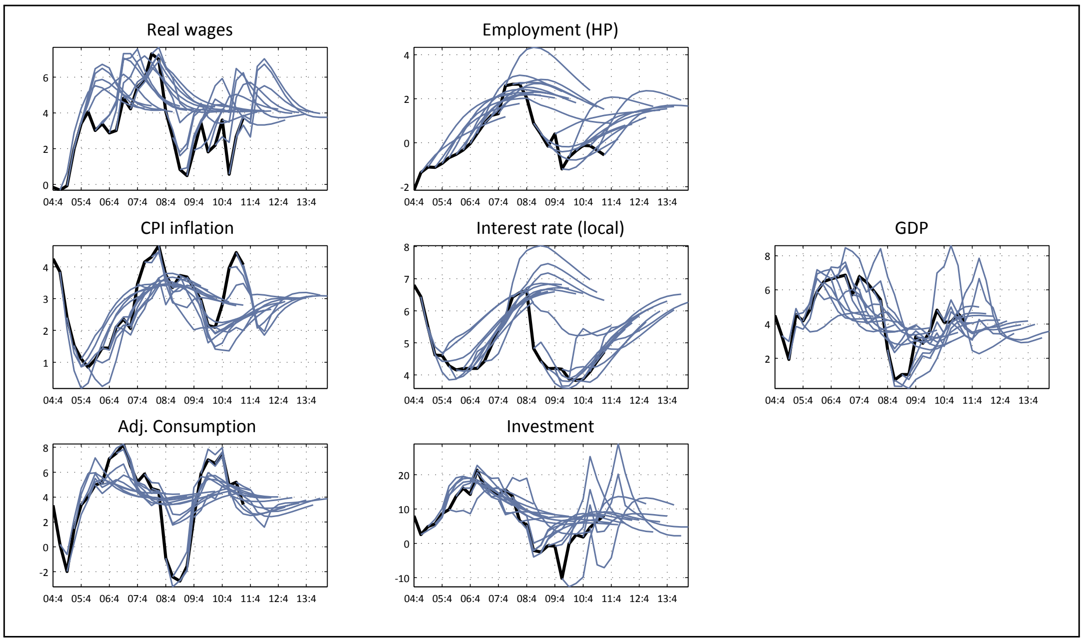
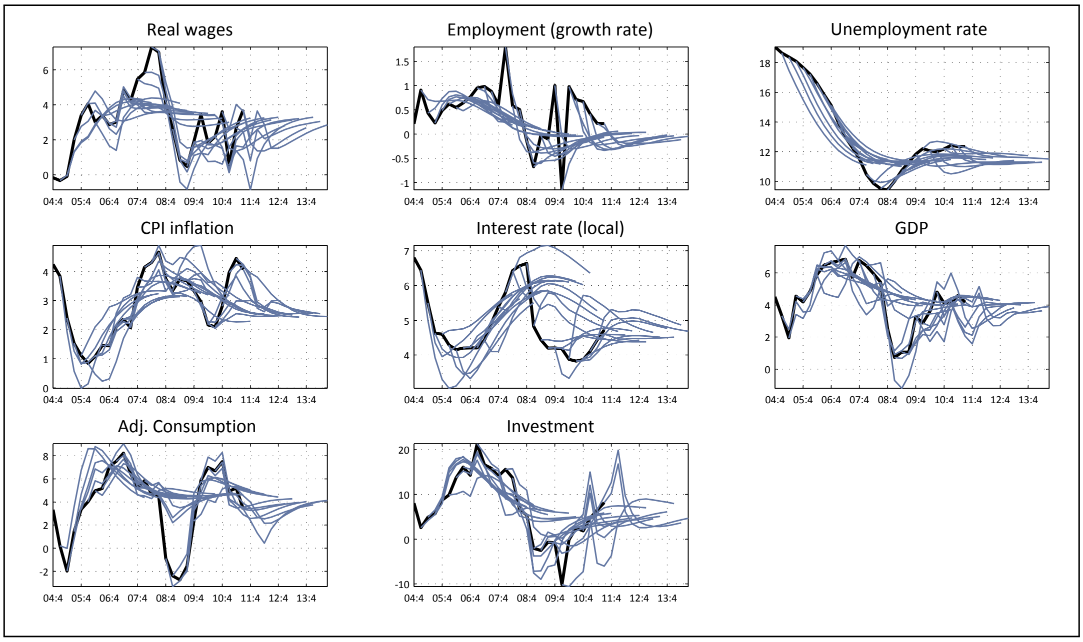
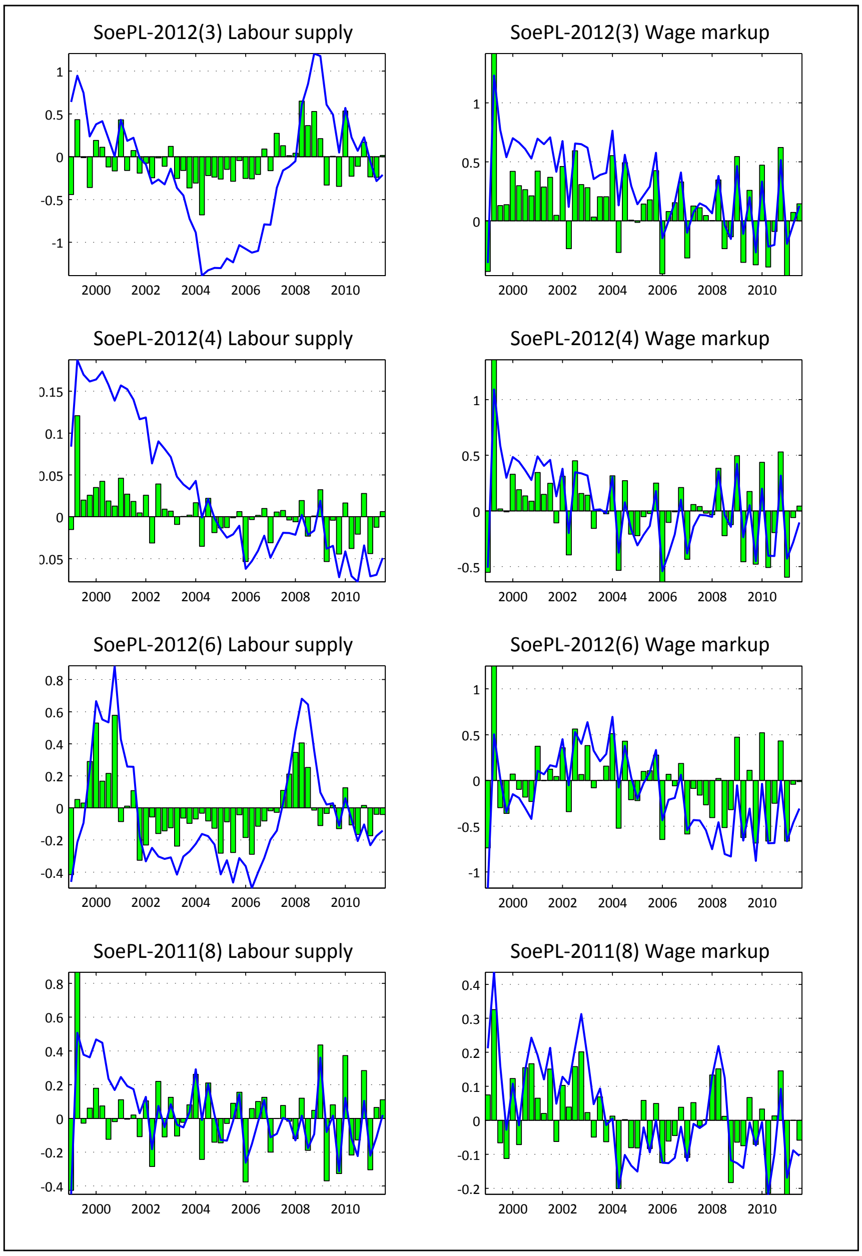


Figure 14. SoePL-2012 — Rolling *ex post* forecasts.



A.5 Estimated labour market shocks — Irregular cases

Figure 15. Estimated labour market innovations (green bars) and disturbances (blue line). for irregular variants of DSGE SoePL-2011 and SoePL-2012 models, (cases 3, 4, 6, and 8, see Table 2).



A.6 Comparison of the impulse response functions: SoePL-2011 vs. SoePL-2012

Figure 16. Impulse response function of technology shock — SoePL-2011 (green line) vs. SoePL-2012 (blue line)

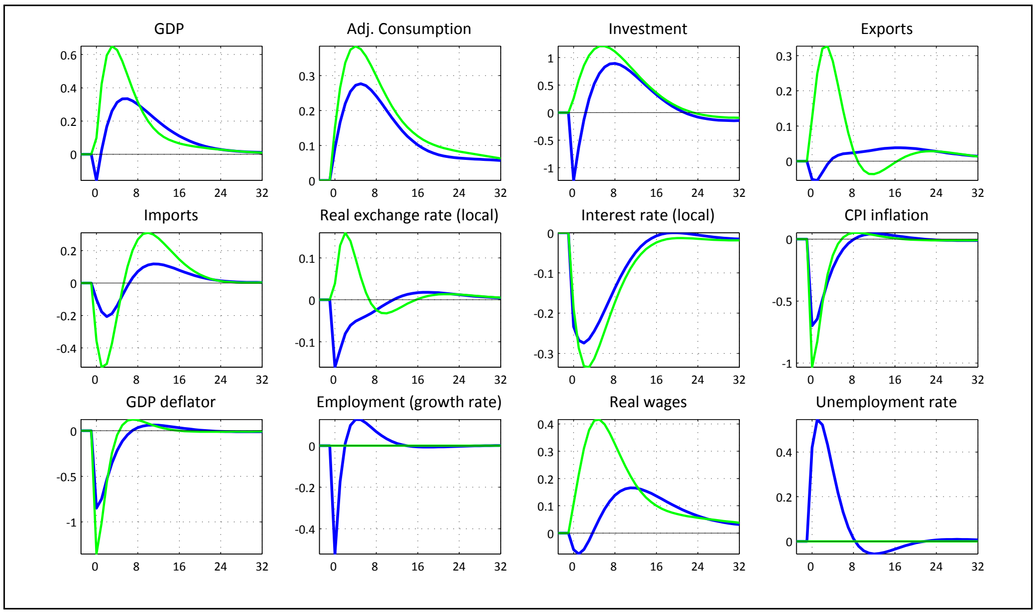


Figure 17. Impulse response function of investment technology shock — SoePL-2011 (green line) vs. SoePL-2012 (blue line)

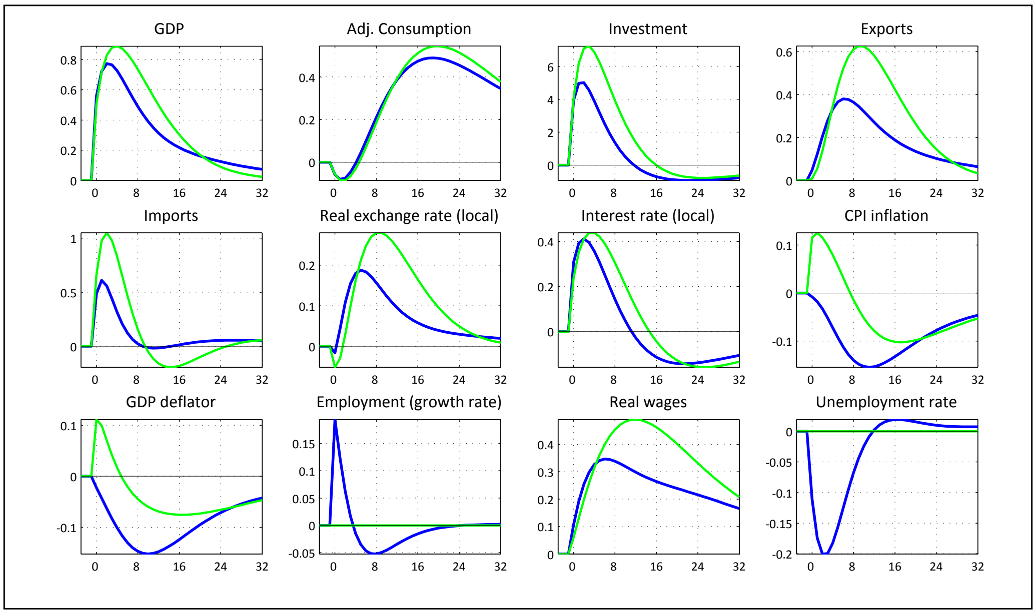


Figure 18. Impulse response function of consumption preferences shock — SoePL-2011 (green line) vs. SoePL-2012 (blue line)

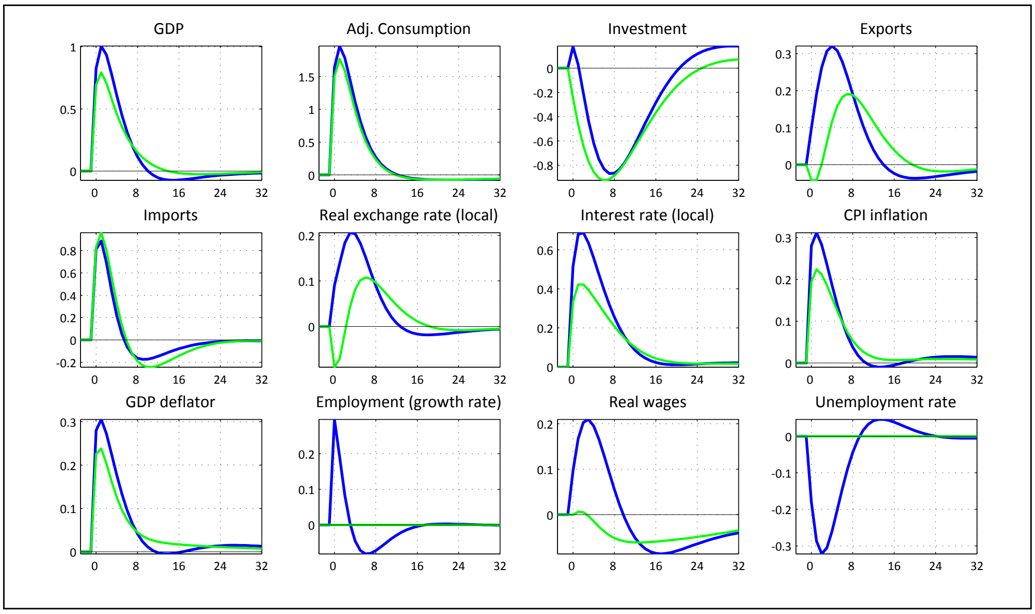


Figure 19. Impulse response function of labour supply preferences shock — SoePL-2011 (green line) vs. SoePL-2012 (blue line)

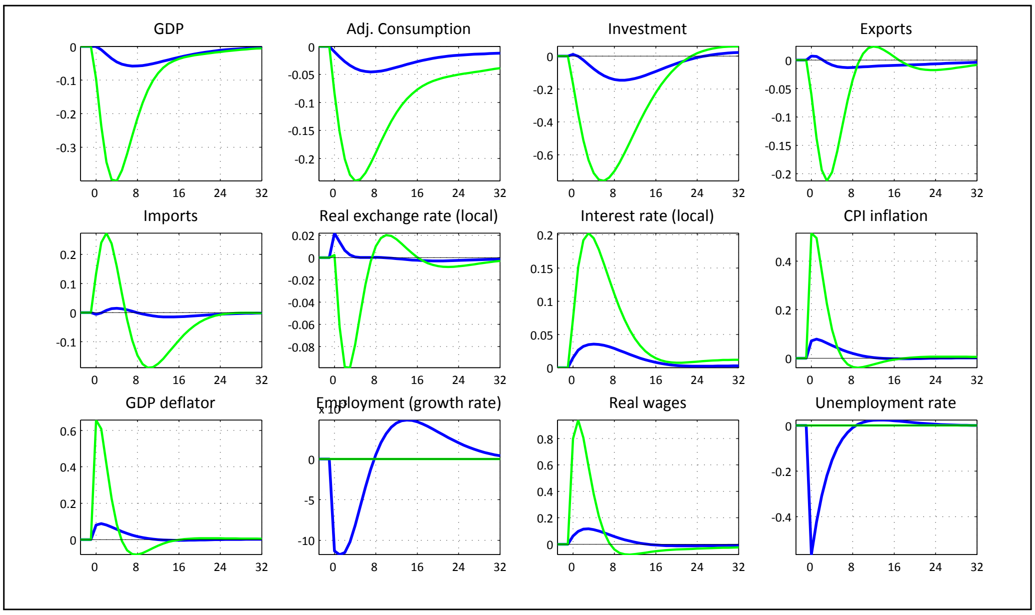


Figure 20. Impulse response function of wage markup shock — SoePL-2011 (green line) vs. SoePL-2012 (blue line)

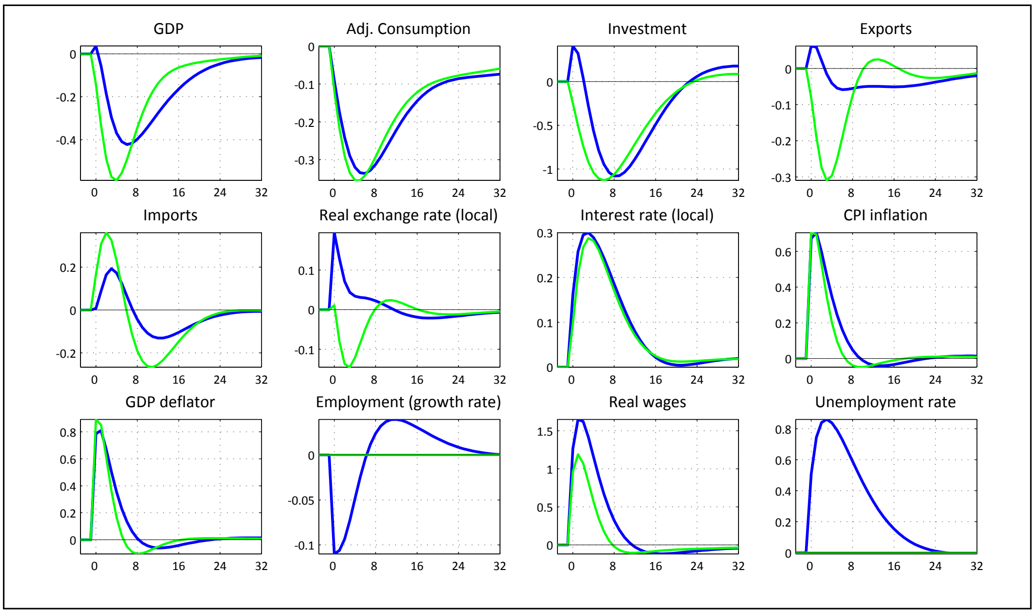


Figure 21. Impulse response function of domestic products markup shock — SoePL-2011 (green line) vs. SoePL-2012 (blue line)

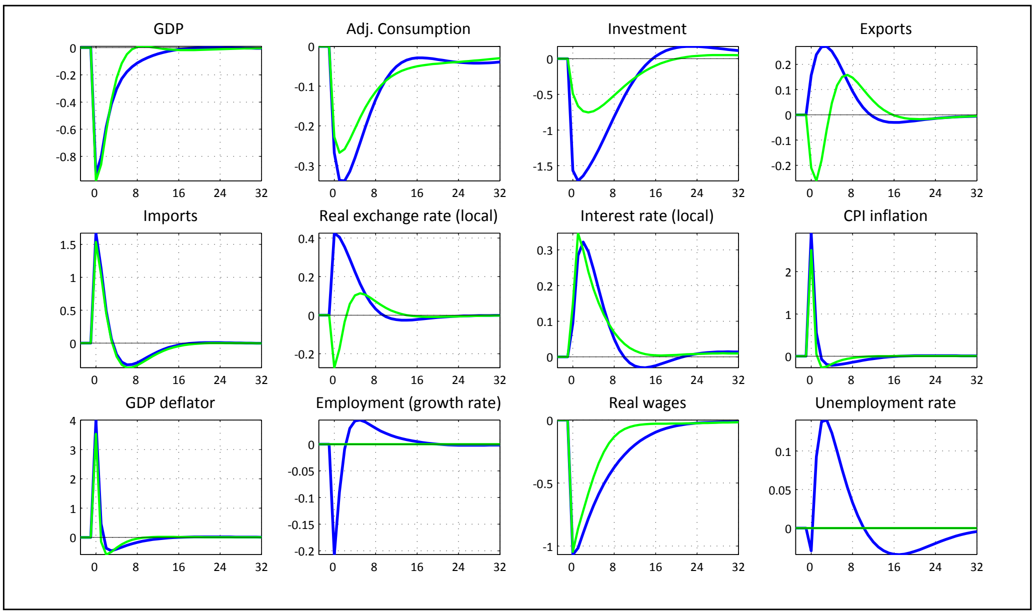


Figure 22. Impulse response function of imported consumption products markup shock — SoePL-2011 (green line) vs. SoePL-2012 (blue line)

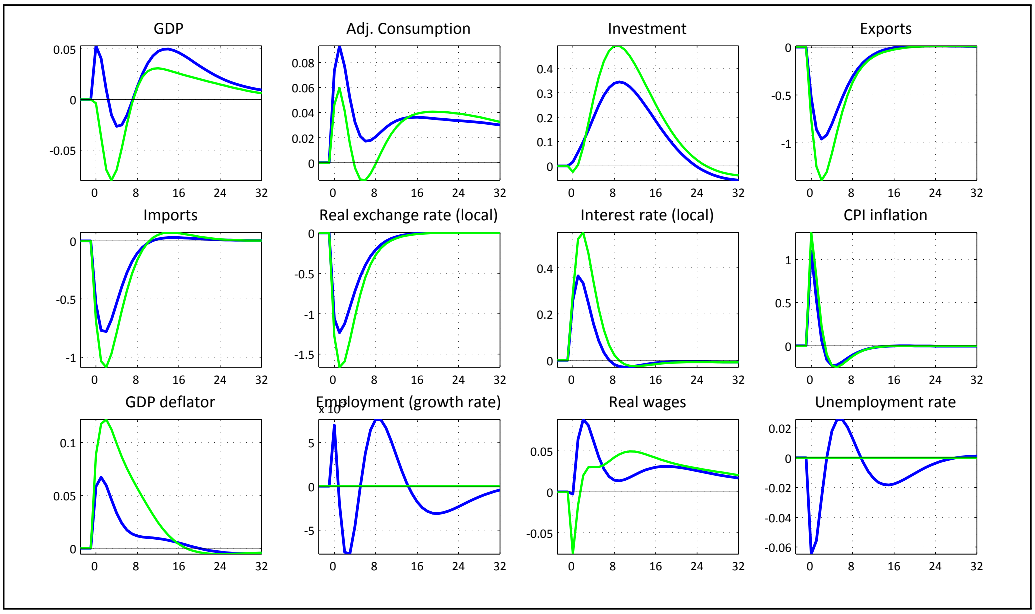


Figure 23. Impulse response function of imported investment products markup shock — SoePL-2011 (green line) vs. SoePL-2012 (blue line)

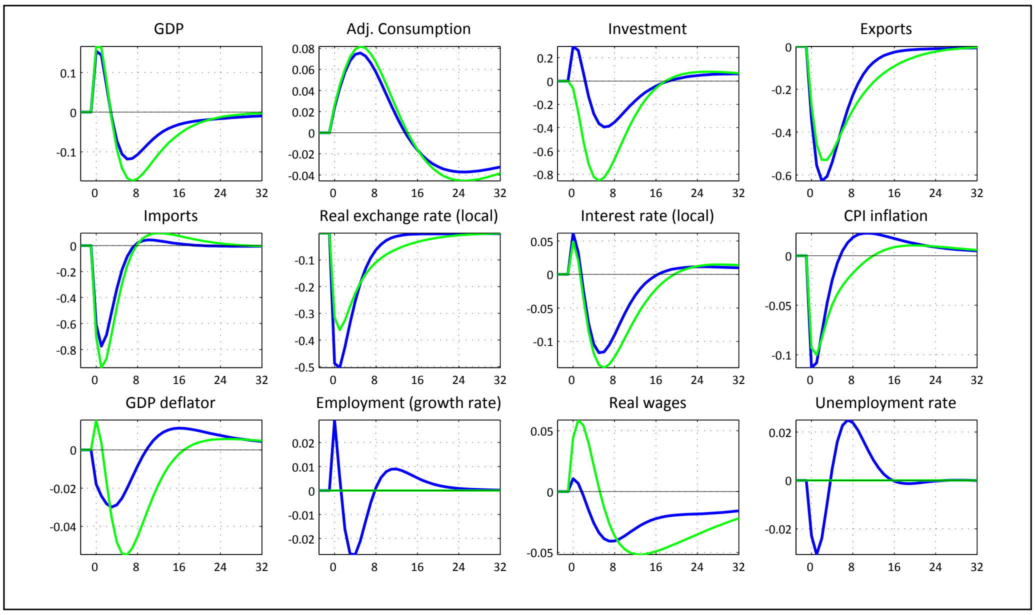


Figure 24. Impulse response function of export markup shock — SoePL-2011 (green line) vs. SoePL-2012 (blue line)

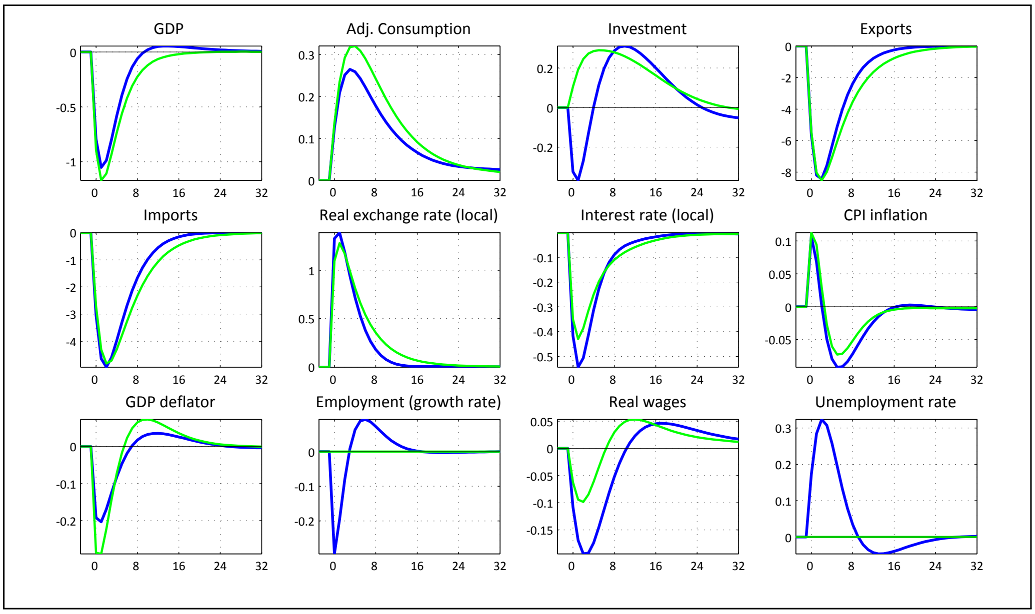


Figure 25. Impulse response function of risk premium shock — SoePL-2011 (green line) vs. SoePL-2012 (blue line)

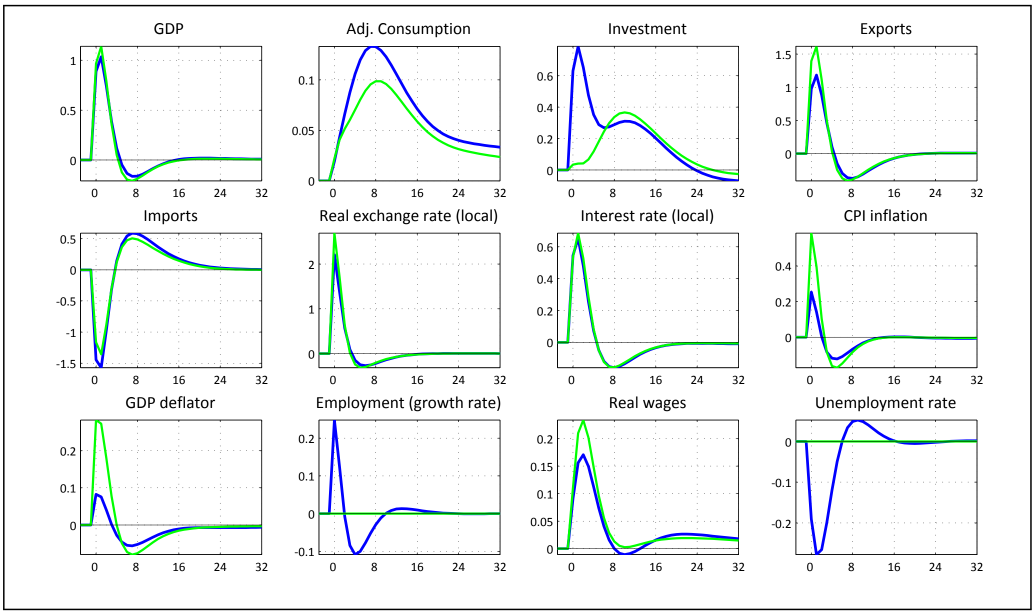


Figure 26. Impulse response function of interest rate shock — SoePL-2011 (green line) vs. SoePL-2012 (blue line)

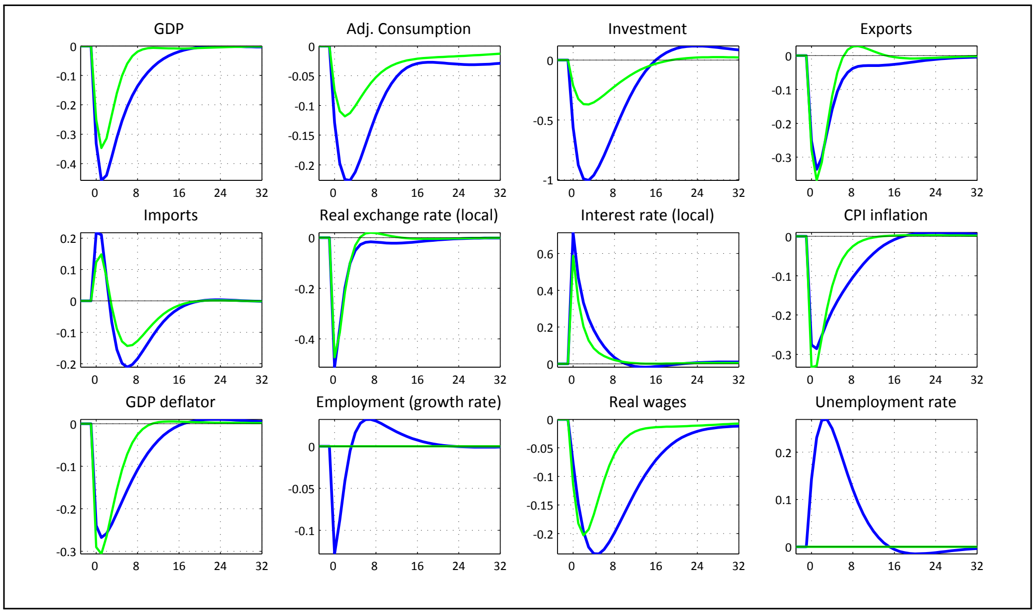
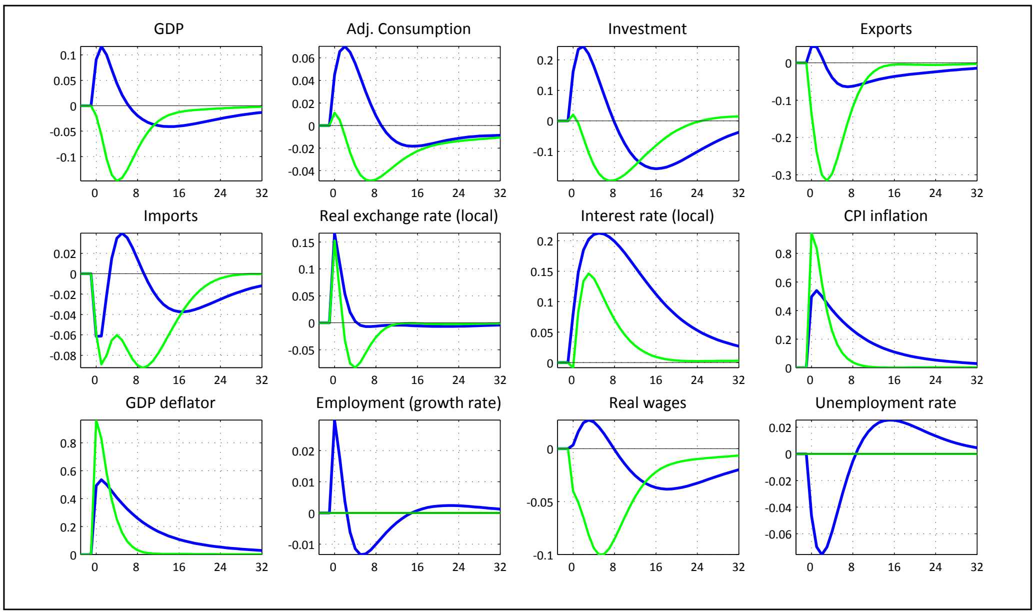


Figure 27. Impulse response function of inflation target shock — SoePL-2011 (green line) vs. SoePL-2012 (blue line)



References

ADOLFSON, M., LASÉEN, S., LINDÉ, J., AND VILLANI, M. 2007. Evaluating An Estimated New Keynesian Small Open Economy Model. *Sveriges Riksbank Working Paper Series* 203.

CALVO, G. 1983. Staggered Contracts in a Utility-Maximising Framework. *Journal of Monetary Economics* 12:383–398.

CHARI, V. V., KEHOE, P. J., AND MCGRRATAN, E. R. 2009. New Keynesian Models: Not Yet Useful for Policy Analysis. *American Economic Journal: Macroeconomics* 1:242–266.

CHRISTIANO, L. 2011. Comments on Galí, Smets and Wouters, Unemployment in an Estimated New Keynesian Model. Unpublished Manuscript.

CHRISTIANO, L., EICHENBAUM, M., AND EVANS, C. 2005. Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy. *The Journal of Political Economy* 113:1–45.

CHRISTIANO, L., MOTTO, R., AND ROSTAGNO, M. 2007a. Financial Factors in Business Cycles. Manuscript. URL http://faculty.wcas.northwestern.edu/~lchrist/course.

CHRISTIANO, L., MOTTO, R., AND ROSTAGNO, M. 2007b. Technical Appendix for Financial Factors in Business Cycles. Manuscript. URL http://faculty.wcas.northwestern.edu/~lchrist/course.

CHRISTIANO, L., TRABANDT, M., AND WALENTIN, K. 2010a. Involuntary Unemployment and the Business Cycle. *Sveriges Riksbank Working Paper Series* 238.

CHRISTIANO, L., TRABANDT, M., AND WALENTINE, K. 2010b. DSGE Model for Monetary Policy Analysis. *NBER Working Paper Series* 16074.

CHRISTOFFEL, K., KUESTER, K., AND LINZERT, T. 2009. The Role of Labor Markets for Euro Area Monetary Policy. *European Economic Review* 53(8):908–936.

DEL NEGRO, M. AND SCHORFHEIDE, F. 2004. Priors from General Equilibrium Models for VARS. *Internetional Economic Review* 45(2):643–673.

DEL NEGRO, M. AND SCHORFHEIDE, F. 2006. How Good Is What You Have Got? DGSE-VAR as a Toolkit for Evaluating DSGE Models. *FED Atlanta Economic Review* pp. 21–37.

DEL NEGRO, M., SCHORFHEIDE, F., SMETS, F., AND WOUTERS, R. 2007. On the Fit the New Keynesian Models. *Journal of Business and Economic Statistics* 25(2):123–162.

ENGEL, C. 1996. The Forward Discount Anomaly and the Risk Premium. A Survey of Recent Evidence. *Journal of Empirical Finance* 3:123–192.

ERCEG, C. J., HENDERSON, D. W., AND LEVIN, A. T. 2000. Optimal Monetary Policy with Staggered Wage and Price Contracts. *Journal of Monetary Economics* 46:281–313.

FERNÁNDEZ-VILLAVERDE, J. AND RUBIO-RAMÍREZ, J. F. 2004. Comparing Dynamic Equilibrium Models to Data: A Bayesian Approach. *Journal of Econometrics* 123:153–187.

FERNÁNDEZ-VILLAVERDE, J., RUBIO-RAMÍREZ, J. F., AND SARGENT, T. J. 2005. *A*, *B*, *C*'s (and *D*) for Understanding VARs. May.

GALÍ, J. 2011a. Monetary Policy and Unemployment, pp. 487–544. *In* B. M. Friedman and M. Woodford (eds.), Handbook of Monetary Economics, volume 3A, chapter 10. Elsevier B.V.

GALÍ, J. 2011b. The Return of the Wage Phillips Curve. *Journal of the European Economic Association* 9(3):436–461.

GALÍ, J. 2011c. Unemployment Fluctuations and Stabilization Policies. A New Keynesian Pespective. The MIT Press.

GALÍ, J., SMETS, F., AND WOUTERS, R. 2011. Unemployment in an Estimated New Keynesian Model. NBER Macroeconomics Annual Conference 2011 Cambridge MA, National Bureau of Economic Research.

GERTLER, M., SALA, L., AND TRIGARI, A. 2008. An Estimated Monetary DSGE Model with Unemployment and Staggered Nominal Wage Bargaining. *Journal of Money, Credit and Banking* 40:1713–1764.

GOURIEROUX, C., MONFORT, A., AND RENAULT, E. 1993. Indirect Inference. *Journal of Applied Econometrics* 8:S85–S118.

GRABEK, G., KŁOS, B., AND KOLOCH, G. 2011. SOE-PL.2009 — An Estimated Dynamic Stochastic General Equilibrium Model for Policy Analysis and Forecasting. *NBP Working Papers* 83. URL http://www.nbp.pl.

GRABEK, G., KŁOS, B., AND UTZIG-LENARCZYK, G. 2007. SOE-PL — Model DSGE małej otwartej gospodarki estymowany na danych polskich. Metodologia, specyfikacja, wyniki estymacji i pierwsze zastosowania. *Materiały i Studia NBP* 217. URL http://www.nbp.pl.

GRABOWSKA-LUSIŃSKA, I. AND OKÓLSKI, M. 2008. Migracaja z polski po 1 maja 2004 r.: jej intensywność i kierunki geograficzne oraz alokacja. Working Paper 33/91, Centre of Migration Research, Faculty of Economic Sciences, University of Warsaw. URL http://www.migracja.uw.edu.pl.

JAIMOVICH, N. AND REBELO, S. 2009. Can news about the future drive the business cycle? *American Economic Review* 99(4):1097–1118.

JUSTINIANO, A., PRIMICERI, G. E., AND TAMBALOTTI, A. 2011. Is There a Trade-Off Between Inflation and Output Stabilization? Working Paper 17071, National Bureau of Economic Research. URL http://www.nber.org/papers/w17071.

KEANE, M. P. AND ROGERSON, R. 2011. Reconciling Micro and Macro Labor Supply Elasticities: A Structural Perspective. *NBER Working Paper* 17430.

MIODUSZEWSKA, M. 2008. Najnowsze migracje z polski w świetle danych badania aktywności ekonomicznej ludności. Working Paper 36/94, Centre of Migration Research, Faculty of Economic Sciences, University of Warsaw. URL http://www.migracja.uw.edu.pl.

MORTENSEN, D. T. AND PISSARIDES, C. A. 1994. Job Creation and Job Destruction in the Theory of Unemployment. *Review of Economic Studies* 61:397–415.

SCHMITT-GROHÉ, S. AND URIBE, M. 2003. Closing Small Open Economy Models. *Journal of International Economics* 61:163–185.

SMETS, F. AND WOUTERS, R. 2007. Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach. *American Economic Review* 97:686–606.

STRZELECKI, P., WYSZYŃSKI, R., AND SACZUK, K. 2009. Zjawisko chomikowania pracy w polskich przedsiębiorstwach po okresie transformacji. *Bank i Kredyt* 40:77–104.

WALSH, C. E. 2005. Labor Market Search, Sticky Prices, and Interest Rate Policies. *Review of Economic Dynamics* 8:829–849.

WARNE, A. 2012. YADA Manual — Computational Details. EBC, Manuscript. URL http://www.texlips.net/yada/index.html.

Note: The Internet addresses inform about the place from which the documents have been downloaded. We cannot guarantee that the aforementioned pages still exist and/or still provide the cited materials.