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Inflation fan charts and different dimensions of uncertainty. What if macroeconomic uncertainty is high?

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Contents

Abstract

The paper discusses problems associated with communicating uncertainty by means of 'fan charts', used in many central banks for presenting density forecasts of inflation and other macroeconomic variables. Limitations of fan charts in the case of high macroeconomic uncertainty are shown. Issues related to definition of uncertainty are addressed, stressing the need to distinguish between statistical model errors and uncertainty due to lack of knowledge. Modifications of the standard methods of constructing fan charts are suggested. The proposed approach is based on two distributions, one of which is subjective and describes possible macroeconomic scenarios, while the other describes model errors. Total uncertainty is represented as a mixture distribution or density convolution. The proposed approach, although it is a mix of judgment and statistics, allows preserving information about scenarios and separating in the analysis different types of uncertainties.

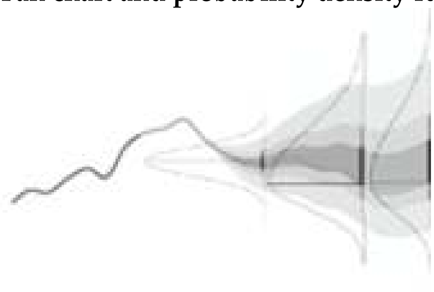
1. Different interpretations of fan charts

The forecasting process involves uncertainty – we use models that are only a simplified picture of reality, we do not have accurate data, we do not know what will happen. Fan charts have been widely used by central banks as a means of visualization of uncertainty. Following the Bank of England and the Sveriges Riksbank, the National Bank of Poland has been publishing fan charts in "Inflation Reports" since 2004.

Fan charts are constructed on the basis of probability density functions (Figure 1).

Figure 1

Fan chart and probability density functions



The method of obtaining distributions, i.e. the knowledge of adopted assumptions and of a kind of uncertainty included in calculations, is essential for a proper interpretation of fan charts. This has become particularly important in the context of the expanding spectrum of models used by central bank since the scope of uncertainty analysis that could be performed is largely determined by the type of a model. Preferences of different forecast teams are also important.

In general, if focus is placed on the chart itself, it is difficult to determine whether possible outcomes, estimated volatility or confidence intervals of point forecasts are presented.

1.1 The original meaning of 'fan chart'

The fan charts ware invented at the Bank of England. The first chart of this type was presented in the *Inflation Report* in February 1996. Earlier, between 1993 and 1995, the Bank of England published its forecasts in the form of charts showing the central path surrounded by the shaded (one color) area with the bounds set by the value of past forecast errors. However, this method of presentation was quickly considered to be unsatisfactory (cf. Britton, Fisher, Whitley 1998) for a number of reasons. The readers of forecasts ignored the information on a potential large error and continued to attach an excessive importance to the central path. The bounds of the shaded area were often misread as indicating upper and lower bounds of the forecast. In addition, there was no information about potential asymmetric deviations.

The fan charts were introduced in order to convey a more precise representation of the Bank's subjective assessment and show the inflationary pressure evolving through time. The statistical errors have been replaced with the probability distributions obtained by mapping a set of possible economic assumptions onto an inflation forecasts. For obvious reasons, only a limited number of assumptions were evaluated and TPN probability distribution was fitted to the results. The limitations related to the assumed form of distribution will be discussed in Chapter 4. At this point, we only wish to stress that originally the term "fan chart" was understood not only as a graph type.

1.2 Fan charts based on statistical errors

Nowadays many types of models used in central banks do not refer to future values of variables, and therefore they do not allow performing scenario analyses, which are the essence of fan charts in their original form. To this category belong not only many simple statistical models but also complex DSGE models. Perhaps this increasing role of such models is the reason for the return to the *ex post* statistical errors. This applies even to the Sveriges Riksbank, which previously attached great importance to indicating the risk of different scenarios, but since 2007, it has started to publish fan charts based on normal distributions with variances determined by historical errors.

Return to the ex post statistical error is difficult to explain especially during the period of the growing of methods opening up new possibilities for uncertainty analysis, in particular methods that use the Bayesian approach and the Kalman filter. Recently, more and more attention is drawn to the need to perform scenario analyses, also in the context of DSGE models.

Historical statistical errors, including the most popular RMSE, are obviously a good starting point for estimating the uncertainty because they account for different sources of errors made in the past. However, the fan chart based on the RMSE indicates only that the forecasters are aware of the previous errors accompanying their point forecasts and that similar errors can affect the currently presented central projection. This information is needed to draw attention to the risk associated with the use of the forecasts. It is however not sufficient from the perspective of decision-makers because it only informs of the average value of the past errors and not of the error at a particular point in time. There is no guarantee that future forecasts will be burdened with the same error. In addition, the RMSE does not provide any information on the structure of errors. It will be different for different classes of models.
On the other hand, the RMSE could be regarded as a quite objective and universal measure of forecast errors, which becomes quite important in view of the diversity of both the models and techniques of estimation, filtering and prediction used in macroeconomic forecasting.

Fan charts based on the RMSE can be used for presenting the prognostic properties of models in different horizons. They can provide a basis for comparison (if the models do not differ too much in the complexity or the degree of exogeneity).

However, if the decisions do not concern the model selection but monetary policy, usefulness of the RMSE is limited.
(Comparing the models must be remembered that the RMSE applies only to point forecasts, and that its structure may be different for different categories of models.)
Another problem is that the RMSE does not reflect propagation of errors in time. We get a picture of unconditional errors. (Charemza et al. (2009) show how statistical errors can be described taking into account dependence and asymmetry of distributions.)

For certain classes of models, the creation of a fan chart on the basis of analytical predictive distributions derived from the model could be considered natural. A review of methods of constructing prediction intervals for selected types of statistical models can be found for instance in Chatfield (1993). However, in the decision-making context it may also not be the proper choice. Prediction intervals obtained in a traditional way describe the effects of inaccurate estimation of parameters and/or the residual uncertainty. They do not include the uncertainty of exogenous variables. This is to a large extent understandable, since even for simple models it is very difficult (e.g. Feldstein 1997). For example, in deriving predictive distributions for linear regression models – taking into account the uncertainty of parameter estimates and residual uncertainty is a standard. In the case of ARIMA models generally take into account only the residual uncertainty is taken into account– the uncertainty of parameters is ignored, despite known theoretical results (e.g. Fuller, Hasza 1981; de Luna 2000).

Moreover, predictive distributions are accompanied by the assumption of the correctness of the model form. The importance of this assumption is different depending on the degree of the verification of the theory underlying the model. In the case of purely statistical models the uncertainty of the model may be a component of fundamental importance (cf. Chatfield 1995). The remedy may be the Bayesian model averaging (BMA); e.g. Hoeting J., Madigan, Raftery, Volinsky, 1999). BMA has been successfully applied to statistical models of many types. However, it does not lead to the interpretable models, which limits the possibilities of use in decision-making.

Models which support the decision-making process are required to explain both "what has already happened" and "what may happen". A good fit to the data is not sufficient. Furthermore, such models should allow analysis of the implications of observed or anticipated events, and the effects of different decisions. Teams responsible for preparing projections for the purposes of monetary policy face the problem of taking into account the significant sources of uncertainty associated with the assumptions formulated by experts as to the starting point and the external environment. Thus, there is a need to consider uncertainty that results from the incomplete knowledge which can be described probabilistically only by subjective probability distributions, that do not reflect the rules governing random events (as opposed to objective statistical distributions).

2. Uncertainty as a multidimensional concept

As we pointed out in the previous chapter, the nature and scope of the uncertainty portrayed by fan charts could be different. This depends on the type of a model, forecasting methods and the preferences of forecasting teams. The precise definition of uncertainty is needed, but it is difficult without a good typology.

Walker et al. (2003) developed a typology of uncertainty for model–based decision support, which is worth transposing into macroeconomic forecasting. Their typology is universal as it includes and harmonizes many other systematics created for various specific purposes. They suggest describing uncertainty in three dimensions: location, level and nature.

Location of uncertainty

The following locations *can b*e distinguished:

Context – determines what the subject of modeling is, what part of the real world is described by the model and how complete the description is. The context determines the possibilities of using the model for a particular purpose.

The application of the model in the wrong context involves uncertainty of the results.

Model – structure (definitions of variables, relationships between them, assumptions, mathematical algorithms) as well as parameters and computer implementation.

Input area –data that describe the reference system and the external forces that produce changes within the system (scenario and policy variables).

Output area – uncertainty associated with this area is the cumulative effect of uncertainties from the previous areas.

Individual components are propagated according to the logic of the model resulting in the uncertainty of model results.

Level of uncertainty

The following gradation of the level of uncertainty is proposed:

determinism → statistical uncertainty → scenario uncertainty → recognized uncertainty → total ignorance

Determinism corresponds to the ideal situation, i.e. certainty. At the other end there is a situation where we are not even able to determine what we do not know.

Statistical uncertainty is the level at which it is possible to describe the deviation from the true value using statistical formulas. The adoption of this level of uncertainty results from the assumption that the model fairly accurately describes the analyzed phenomenon and that the data used for estimating/calibrating the model are representative of the circumstances in which the model will be applied.

Scenario uncertainty is the level at which we are dealing with the possibility of different values, however, the mechanism leading to these values is not recognized sufficiently enough to be described statistically. Scenarios do not say what will happen but what may happen.

Recognized uncertainty – the level of uncertainty as to the mechanisms and relationships is so high that scenarios cannot be formulated.

Nature of uncertainty

Nature is a dimension for distinguishing the uncertainty arising from the lack of or incomplete knowledge of the analyzed phenomenon from the natural variability inherent in the phenomenon. This distinction is important because different types of uncertainty require different treatment. Walker et al. (2003) use the term *variability* for uncertainty resulting from the variation or randomness and *epistemic* for uncertainty associated with insufficient knowledge.

Econometricians focus their attention on the first dimension, i.e. location, which is also the basis for the classification of forecasts errors (Clements and Hendry (1995) discused this aspect in a great detail). Yet the nature of uncertainty is neglected in the world of their models, though the problem is known to economists thanks to Knight (1921). Knight made an important distinction between measurable uncertainty, which he called risk, and immeasurable uncertainty associated with predictions. By measurability he understood the possibility of describing the phenomenon by means of objective probability. Writing about quantifying the immeasurable uncertainty Knight used the term “subjective probability”.

3. Nature of uncertainty (the second dimension) in popular methods of construction fan charts

Using the terminology of Walker et al. (2003), we would say that fan charts were introduced in order to indicate the existence of epistemic uncertainty (the second dimension). Also Blix and Sellin (1999, 2000) from the Bank of Sweden saw the need to include this type of uncertainty. We will briefly recall the method developed by them, since it was very popular for many years and because later in this paper we will propose a new approach, which can be treated as an extension.

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Similarly to the Bank of England, for the description of future inflation (the method was also applied to gdp) two-piece normal distribution $TPN(\mu\ ,\sigma_1,\sigma_2)$ with density given by (1) is used.

$$f\ (x)=\begin{cases}A\exp(-(x-\mu)^2/2\sigma_1^2) & for\ x\le\mu\\ A\exp(-(x-\mu)^2/2\sigma_2^2) & for\ x>\mu\end{cases} \qquad (1)$$

where: $A=\sqrt{2/\pi}(\sigma_1+\sigma_2)^{-1}$.

The parameters $\sigma_1(t)$ and $\sigma_2(t)$ of inflation distribution are determined based on experts opinions on variables Z_j, $j=1,..n$, on which inflation depends.

For each of these variables and forecast horizons experts give:

1) the most probable value $\mu_j(t)$ and the probability $P_j(t)$ of occurrence of values lower than $\mu_j(t)$;
2) the level of uncertainty in comparison with the historical level $\sigma_j(t)=h_j\sigma_{hist}(t)$.

This allows calculating parameters $\sigma_{j,1}(t)$ and $\sigma_{j,2}(t)$ of the distribution $TPN(\mu_j,\sigma_{j,1},\sigma_{j,2})$ describing variable Z_j in the horizon t :

$$\sigma_{j,1}^2(t)=\sigma_j(t)\left[(1-2/\pi)\left(\frac{1-2P_j(t)}{P_j(t)}\right)^2+\left(\frac{1-P_j(t)}{P_j(t)}\right)\right]^{-1} \qquad (2)$$

$$\sigma^2{}_{j,2}(t)=\sigma_j(t)\left[(1-2/\pi)\left(\frac{1-2P_j(t)}{1-P_j(t)}\right)^2+\left(\frac{P_j(t)}{1-P_j(t)}\right)\right]^{-1},$$

Then the corresponding asymmetry parameter $\gamma_j(t)$ defined as the difference between the mean and the mode is determined. In the case of two-piece normal distribution we get:

$$\gamma_j(t) = \sqrt{(2/\pi)}\left(\sigma_{j,2}(t) - \sigma_{j,1}(t)\right) \quad (3)$$

Inflation asymmetry is approximated by a linear combination of parameters $\gamma_j,\ j = 1,..n$:

$$\gamma(t) = \sum_{j=1}^{n} \beta_j(t)\gamma_j(t) \quad (4)$$

3

Elasticities $\{\beta_j\}$ resulting from the model are coefficients.

Positive values of $\gamma(t)$ indicate a greater upside risk for inflation, while negative values – a downside risk.

The knowledge of the asymmetry parameter $\gamma(t)$ allows to find parameters $\sigma_1(t)$ and $\sigma_2(t)$ of the distribution describing the inflation. (First, based on responses of experts and historical errors, inflation variance is calculated.) The resulting distribution is the basis for constructing a fan chart which can indicate the risk of asymmetric deviations from the most likely path of inflation.

Blix and Sellin call the obtained in this way distribution as "partially subjective," emphasizing the fact that the starting point is the objective distributions of historical uncertainty.

4. The scale of uncertainty (the third dimension) – the unsolved problem

Traditional methods of obtaining fan charts are based on the assumption that there is a particular scenario more likely than others. This is reflected in the form of applied distributions – they are unimodal. The uncertainty analysis relates only to deviations from the central path, so it is assumed that the level of uncertainty does not exceed the statistical one. Information about alternative, but less likely scenarios are submitted through the introduction of a large asymmetry. The effectiveness of communication will vary depending on the type of fan chart. We will illustrate this by considering the following hypothetical situation.

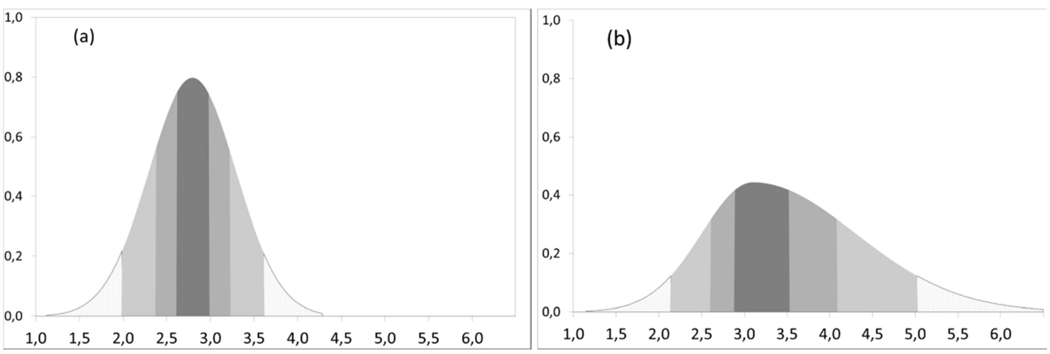
Let us assume that when forecasting inflation, we obtained for two consecutive annual horizons probability densities: $TPN(2.8, 0.5, 0.5)$ for y0+1 and $TPN(3.1, 0.6, 1.2)$ for y0+2. The first function is symmetric (mode=median=mean), while the second one) is characterized by a significant asymmetry.

Knowing the density function we can determine intervals with given probabilities. Figure 2 shows two popular ways of creating bands in fan charts.

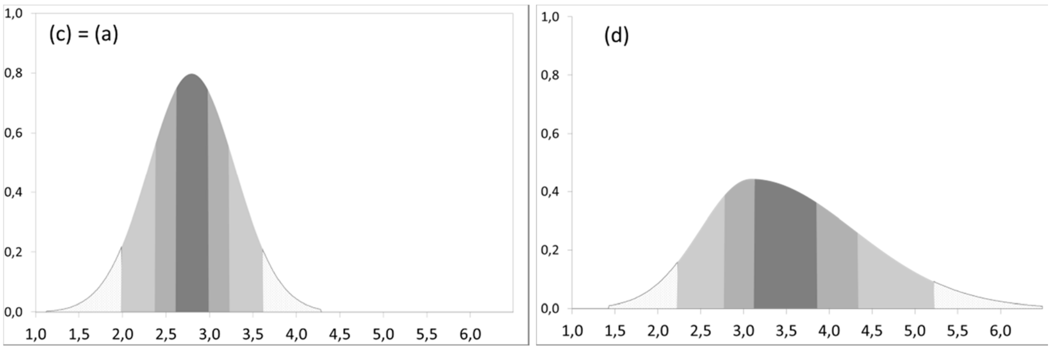
Figure 2

Different ways of building prediction intervals

a, b - bands built around the dominant



c, d - bands built around the median



Figures 2a and 2b show the intervals of probability 0.3, 0.6, 0.9 which are built around the mode, figures 2c and 2d – around the median.

4

For asymmetric density function describing inflation in horizon y0 +1 in both cases we get the same intervals. For horizon y0+2, because of the asymmetry, the boundaries of the intervals are different what is clearly seen when we compare the "darkest" intervals in Figure 2b and 2d. The interval, which is shown in Figure 2b has the property of being the shortest among all intervals of the probability 0.3. The values of density function at its ends are equal. The intervals created in this way are the basis of fan charts which we will call "modal". Modal fan charts are used by the Bank of England.

The middle band in chart 2d is an interval set by the 0.35-quantile and 0.65-quantile, therefore the median divides it into two intervals of equal probability. Such fan charts will be named as "quantile". Their advantage is that they allow comparing the probability of values higher and lower than the central path. In the case of modal fan charts, only the sum of the probability of ranges of the same color is known. Quantile fan chart are used, for example, in the Sveriges Riksbank and National Bank of Poland.

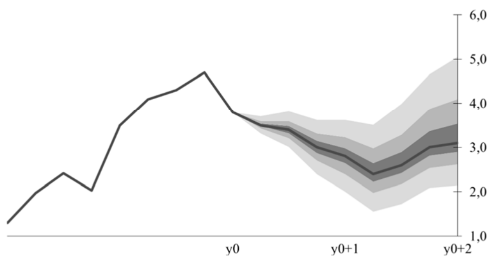
Modal and quantile fan charts, created on the basis of the discussed density functions are presented in Figures 3a-3b (standard deviations for quarters 1-3 in subsequent years were obtained by interpolation, assuming a steady increase in uncertainty over the year).

Both types of fan charts clearly indicate a risk of higher values. The quantile chart allows conclude that the probability of higher values for the two-year horizon is about twice as high. In the modal fan chart the central path is always located in the darkest area. It may be omitted (so that readers of forecasts do not attach much attention to the point forecasts) without a fear of loss of information concerning asymmetric risk. The situation is different in the case of the quantile fan charts, as shown in figure 3c.

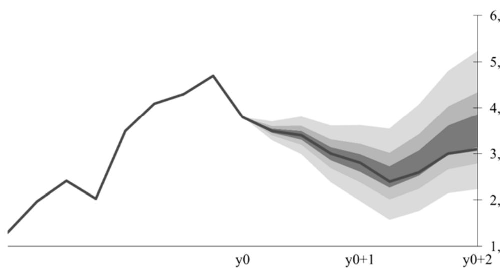
Figure 3

Visualizing asymmetry by different types of fan charts

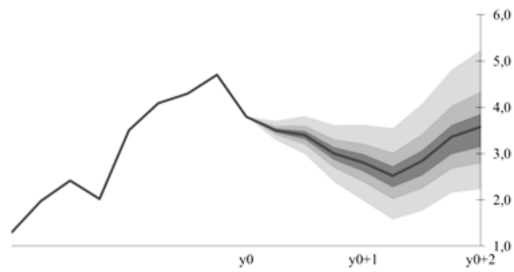
(a) modal fan chart with, cp= mode



(b) quantile fan chart, cp=mode



(c) quantile fan chart, cp=mean



Although fan chart 3c differs from 3b only in the type of central path, a strong asymmetry is not visible. There is another problem – if the probability distribution describes the degree of belief, the presentation of the mean without the information on the mode, i.e. the hypothesis considered to be the most probable, seems to be a significant drawback.

On the other hand, displaying the most likely path when the distribution is strongly asymmetric may lead to inconsistent communication. A central forecast could be found outside the central band of the quantile fan chart.

Summing up:

In periods of relative stability, when uncertainty is low and can be classified as statistical in the third dimension, local analysis of the uncertainty (around the central path) may be sufficient regardless of which measure of central tendency – mode, median or mean - will be adopted. It fails when we are dealing with a large asymmetry or if one dominant scenario cannot be assumed at all.

5. Mixing of uncertainties of a different nature

When fan charts are based on distributions of past forecast errors (either directly or obtained by modifications) there is a problem with the assessment of contribution of particular components of uncertainty. Recipients do not receive information how much of variance which determines the span of the fan is due to imperfect models, and how much is the result of input data uncertainty. Moreover, high proportion of symmetrical statistical errors may distort the message on asymmetric deviation from the central path.

We face a similar problem when simulations are used. Simulations allow taking into account many sources of uncertainty simultaneously, but they can also lead to mixing of different effects. The lack of identifiability of the uncertainty components makes the received distributions difficult to interpret. Paradoxically, the desire to obtain a full picture of uncertainty may lead to a reduction of the informational value of forecasts and their usefulness in the decision-making process.

In many publications attention is drawn more and more frequently to the need for a separate propagation of variability and uncertainty of knowledge (e.g. Hoffman and Hammonds 1994; Frey, Burmaster, 1999; Wu, Tsang 2004).

In the case of macroeconomic forecasts, it seems very important to separate the effects of uncertainty of the model itself from the uncertainty associated with the model input area, due to their different nature and different kind of probability distributions used for describing the uncertainty of both areas. In the first case we can rely on objective distributions of historical errors and residuals. We may also use bootstrap techniques.

In the second case, in order to describe the uncertainty of knowledge concerning the input area, it is necessary to use subjective distribution.

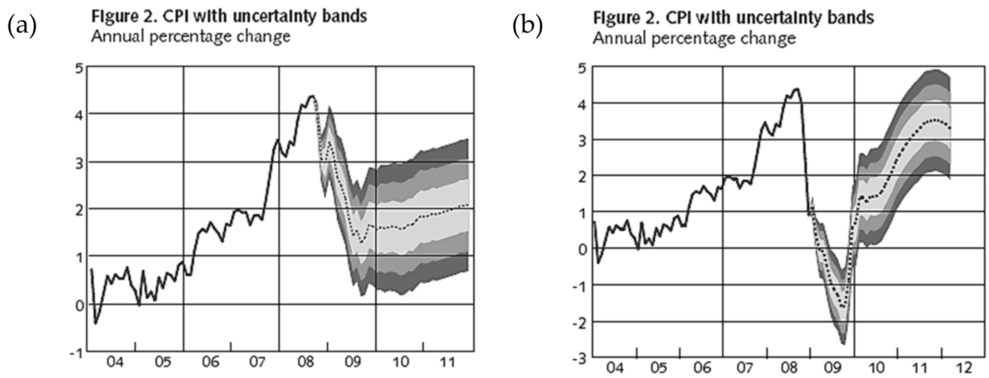
6. Modifications allowing to reflect scenario uncertainty

Nowadays fan charts illustrate, in most cases, the error of the central path (point prediction). The change of the assumptions for the main scenario, for instance, adoption of other initial values or exogenous variables changes the course of the central path and causes a shift of a fan somewhat like "tied" to it.

Suggestive examples of this effect can be fan charts presented in two consecutive "Monetary Policy Reports" published by Sveriges Riksbank in 2008:3 Figure and in 2009:1 (Figure . The consequences of changes in the central path when the fan chart is based on historical errors

Figure 4

The consequences of changes in the central path when the fan chart is based on historical errors



Źródła : a - „Monetary Policy Report 2008:3"; b - "Monetary Policy Report February 2009"; S veriges Riksbank

The comparison of these graphs leads to the conclusion that in the period between reports the level of uncertainty was much higher than statistical and could not be properly reflected.

To avoid such situations we are going to propose a new approach. It could be considered as a generalization of the methods currently used. The proposed modifications enable to provide information about scenarios which are important from the perspective of the decision-making process.

Unlike to the standard methods, we will not assume the existence of a scenario which is much more probable than the others, nor the form of a resulting distribution. Construction of a fan chart will require two probability distributions for each forecast horizon. The first distribution is subjective and reflects experts' opinions on various economic scenarios. Weights assigned to scenarios will imply a distribution of output variables obtained from the model. This can be a discrete distribution (if the number of scenarios is specified) or a continuous one if a density function is matched to the results.

The concept of a scenario does not need to be limited to different values of exogenous variables or the starting point. It may also relate to deviations from relationships that were previously observed. The second distribution is used for describing the uncertainty of prediction tools (uncertainty of the model – the structure, parameters and adequacy in a given context). It could be created on the basis of a historical statistical error (cleared from assumption errors) or on the basis of residuals of estimated equations.

The case of a discrete subjective distribution

Let $\{x_i(t)\}_{i=1...N}$ be the set of paths (point forecasts) corresponding to possible economic scenarios $\{s_i\}_{i=1...N}$ (horizon description t will be omitted for simplicity).

We will denote chances of the realization of scenario s_i and so the outcome x_i by p_i.

Let us treat the model as a kind of a measure with the help of which indirect measurements of x_i are made, i.e. direct measurements of inputs (scenario s_i) are processed according to the model equations (algorithm by which the measure operates) into the output x_i.

To take into account the measurement error, the deterministic forecast x_i should be replaced by random variable $X_i = x_i + \Delta X_i$, where ΔX_i is the error.

The accuracy of the model is its feature and can be assessed with the use of statistical methods. The effects of the model error propagation for subsequent horizons could be, for instance, determined by stochastic simulations.

We will focus on a fixed forecast horizon. The error refers equally to each scenario, so we will assume that ΔX_i ($i = 1,2,...N$) are random variables with identical distribution, which we denote by $g(x)$. Then the distribution $g_i(x)$ of X_i will be described by:

$$g_i(x) = g(x - x_i) \tag{5}$$

Knowing distributions p and g we can obtain the distribution which will be a probabilistic description of the following situation: we choose a scenario with probability p_i; then, taking into account a random error, we obtain the value of forecasted variable. Such a hierarchical method leads to a mixture distribution. We denote its density function by f:

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$$f(x) = \sum_i^S p_i g(x - x_i) \tag{6}$$

The case of a continuous subjective distribution

A continuous counterpart of the density (6) is

$$f(x) = \int_{-\infty}^{+\infty} p(s) g(x - s) ds \tag{7}$$

It can be seen that density $f(x)$ is a convolution $p * g$ and thus the density of the distribution of the sum of two independent random variables with distributions p and g. It is consistent with our previous interpretation of the density f as a distribution describing the result of the measurement of unknown variable X characterized by the subjective distribution p when the measurement error err is described by distribution g. The measurement result is then the sum $X + err$ with the distribution described by the convolution of the densities.

For the error described by the normal distribution we will obtain:

$$f(x) = (1/\sqrt{2\pi\sigma_g^2}) \int_{-\infty}^{+\infty} p(s)\exp(-(x-s)^2 / 2\sigma_g^2) ds \tag{8}$$

Using continuous density distribution $p(s)$ seems reasonable, for instance, in the following cases:

1. The triangular distribution can be adopted when it is possible to determine the most probable value, the minimum and the maximum.
2. The two-piece normal distribution is useful when we can specify the most probable value and probability of higher or lower values.
3. The partially uniform distribution can be applied when it is possible to specify quantiles and thus obtain intervals of a given probability.

6

The case of the *TPN* subjective distribution

Let us consider a situation when $p = TPN(\mu_p, \sigma_{1p}, \sigma_{2p})$ and $g = N(0, \sigma_g)$.

The normal distribution is a special case of two-piece normal distribution, thus:

$$p * g = TPN(\mu_p, \sigma_{1p}, \sigma_{2p}) * TPN(0, \sigma_g, \sigma_g) \tag{9}$$

We will use the results described in Garvin and McClean (1997) concerning the form and parameters of distribution of the sum of independent random variables with two-piece normal distributions.

Firstly: it can be assumed that the convolution of two-piece normal distributions belongs to a class of two-piece normal distributions.

Secondly: the parameters of this distribution can be determined on the basis of the moments generating function which is the product of component moments generating functions.

Thirdly: it can be shown that the third central moment of the sum of two variables with the two-piece normal distribution is equal to the sum of the component third central moments:

$$\overline{m}_3(p*g) = \overline{m}_3(p) + \overline{m}_3(r) = \overline{m}_3(p) \quad (10)$$

The form of the moment generating function and its successive derivatives could be found in Garvin and McClean (1997).

In our case:

the first moment round 0:

$$m_1(p*g) = E(X+err) = E(X) = \mu_p + \sqrt{(2/\pi)}(\sigma_{2p} - \sigma_{1p}) \quad (11)$$

the third central moment:

$$\overline{m}_3(p*g) = \overline{m}_3(p) = \sqrt{(2/\pi)}(\sigma_{2p} - \sigma_{1p})\left[\left(\frac{4}{\pi} - 1\right)(\sigma_{2p} - \sigma_{1p})^2 + \sigma_{2p}\sigma_{1p}\right] \quad (12)$$

and:

$$Var(X+err) = \sigma_p^2 + \sigma_r^2 \quad (13)$$

6

Knowing the third central moment of the convolution we can calculate the skewness:

$$\frac{\overline{m}_3(p*g)}{\sigma_{p*g}^3}. \quad (14)$$

Next we will use the fact that Pearson's coefficient for the two-piece normal distribution $TPN(\mu, \sigma_1, \sigma_2)$ with variance σ^2 is equal to:

$$\sqrt{(2/\pi)}(\sigma_2 - \sigma_1)/\sigma \quad (15)$$

Both measures of skewness differ significantly, only for $\sigma_2 >> 2\sigma_1$ or $\sigma_1 >> 2\sigma_2$ (e.g. Garvin and McClean 1997). As a result of equating them we obtain the following equation:

$$\sqrt{(2/\pi)}(\sigma_2 - \sigma_1) = \frac{\overline{m}_3}{\sigma^2} \quad (16)$$

Knowing the variance of the convolution and its third central moment, unknown parameters $\sigma_{p*g,1}$ and $\sigma_{p*g,2}$ could be obtained by solving the following system of equations:

$$\begin{cases} \sqrt{(2/\pi)}(\sigma_{p*g,,2} - \sigma_{p*g,1}) = \dfrac{\overline{m}_3(p*g)}{\sigma_p^2 + \sigma_g^2} \\ \sigma_p{}^2 + \sigma_g{}^2 = (1 - 2/\pi)(\sigma_{p*g,2} - \sigma_{p*g,1})^2 + \sigma_{p*g,1}\sigma_{p*g,2} \end{cases} \tag{17}$$

The mode of the convolution $p*g$ will then be equal to:

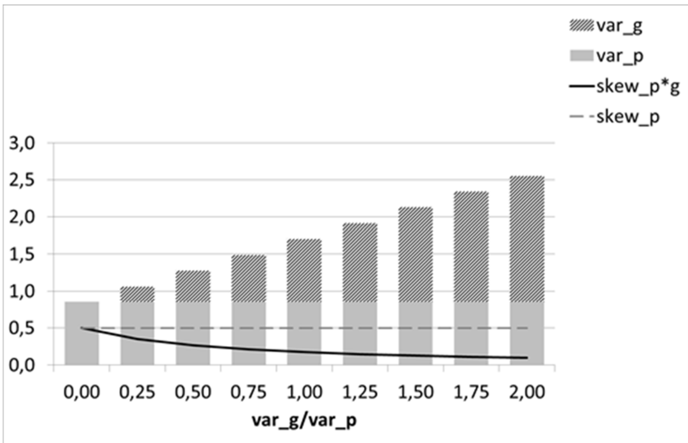
$$\mu_{p*g} = \overline{\mu}_{p*g} - \sqrt{(2/\pi)}\left[\sigma_{p*g,2}(t) - \sigma_{p*g,1}(t)\right] = \overline{\mu}_p - \sqrt{(2/\pi)}\left[\sigma_{p*g2}(t) - \sigma_{p*g,1}(t)\right] \tag{18}$$

So the mean $\overline{\mu}_{p*g}$ is equal to $\overline{\mu}_p$, but the modes are different.

It is worth noting that the relationship between the error of prediction tools and the variance of the distribution describing the uncertainty of scenarios is very important. Figure 5 shows how the skewness of the convolution $p*g$ changes depending on the relationship between σ_g^2 and σ_p^2.

Figures 5

Influence of the model error on skewness



The following distributions were assumed:

$p = TPN(3.1, 0.6, 1.2)$ (cf. Fig.2)

$g = N(0, k * \text{var\_}p)$

where: $k = \text{var\_}g / \text{var\_}p$

var – variance;

skew – skewness defined on the basis of moments

Figure 6 shows the effect of model error on the mode of convolution of p*g for the same distributions.

Figures 6

Influence of the model error on the mode

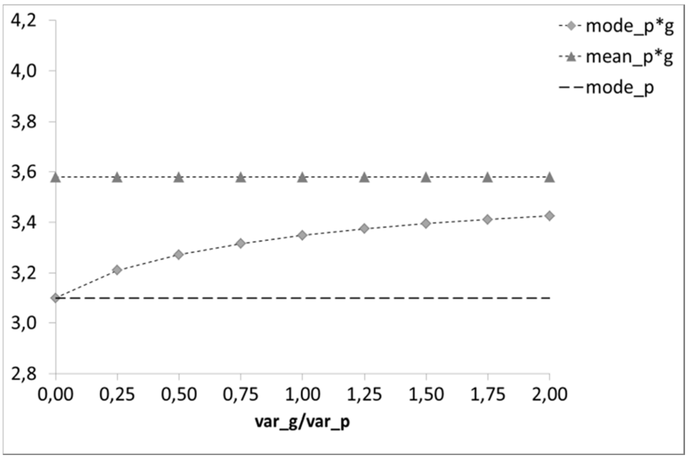


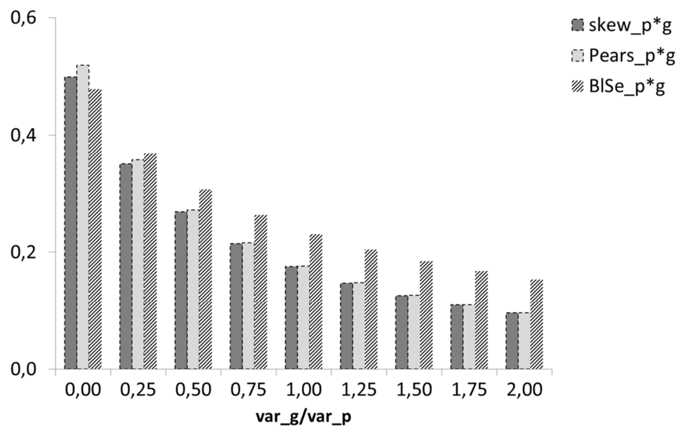
Figure 7 is an illustration of the problems associated with defining the asymmetry as the difference between the mean and mode. It explains why we rejected this measure in the proposed method.

6

With a relatively large model error the distribution becomes symmetric. Without the information about the distribution describing scenarios, its usefulness in decision-making may be low. It is, therefore, necessary to present the information coming from all distributions: p , g and $p*g$.

Figure 7

Comparison of various measures of asymmetry



skew – skewness defined on the basis of moments;
Pears – Pearson coefficient;
BISe – asymmetry measured as the difference between the mean and the mode;

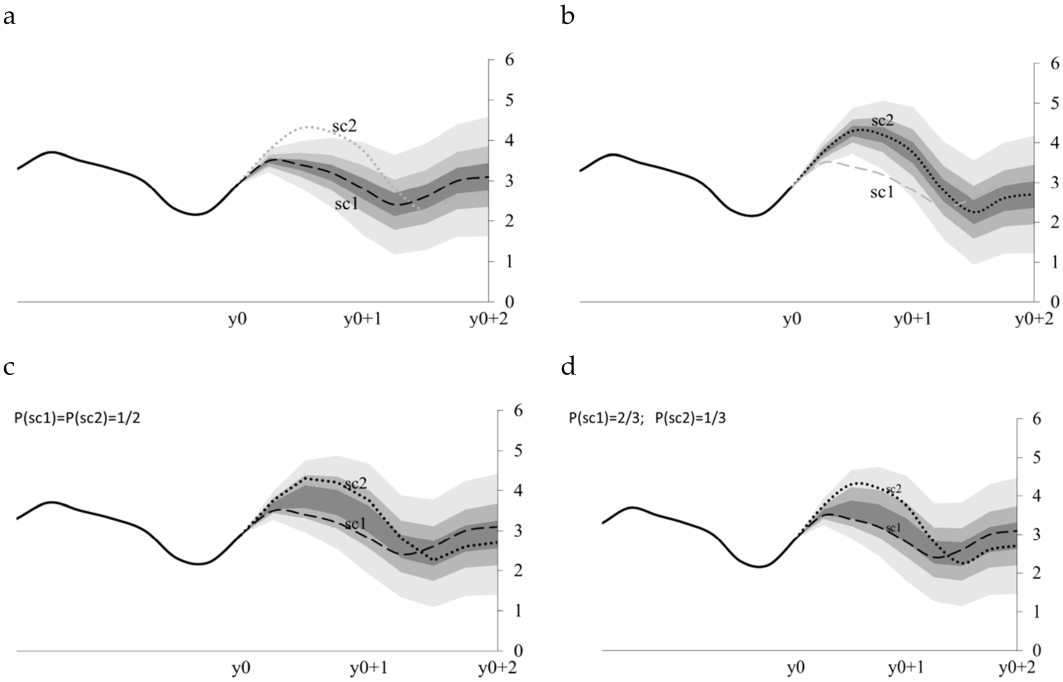
Illustration of the proposed method – the case of two scenarios

Let us assume that two significantly different scenarios are considered.

Using the standard approach, two different fan charts should be shown. They are presented in figures 8 a-b. Each of the fan charts was built around a different central path corresponding to a different scenario. The alternative paths are beyond the 90- percent bounds and theirs uncertainties are not included.

The fan charts shown in figures 8c and 8d are the results of the application of the new method. In the first case both scenarios are equally probable; in the second case the probability of scenario sc1 is twice as high.

Figure 8
Comparison of the standard method (a,b) and the modified method



The traceability of scenarios has been preserved. The paths corresponding to the both scenarios are within the 90-percent bounds. It would be also possible to quantify the likelihood with a greater accuracy without fear of inconsistency of the communication.

Concluding Remarks

Information about uncertainty associated with application of a particular model or forecasting method is undoubtedly very important from the point of view of both readers and creators of forecasts. It allows recipients to assess to what extent they can trust the presented forecasts and it helps forecasting teams to improve their models. However, fan charts based only on statistical errors can be of little use in policy-making and explaining decisions.

The proper response to the uncertainty requires the probability distributions to describe not only past statistical errors, but also, or even above all, the possible states of the economy and to allow illustrating the consequences of the realization of different macroeconomic scenarios. This is particularly important when the uncertainty about the future is high.

The growing popularity of fan charts - treating them as a universal tool for presenting macroeconomic forecasts - requires accurate determining what is meant by "uncertainty". It is necessary to take into account not only the sources of uncertainty but also its nature and scale and to adjust methods of presenting uncertainty to the needs of different groups of recipients.

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