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Abstract

We investigate the dependence structure between Polish and foreign financial assets, including stocks, bonds and foreign exchange. Our interest is in the importance of global factors for asset valuation and on the strength of financial contagion. We work in the copula framework, which offers a full description of the dependence structure. Importantly, we assess many copula families and pay special attention to the testing procedure thereof. Polish equities, currency and to some extent long-term sovereign bonds exhibit economically significant tail dependence, while short-term bonds appear relatively unaffected. Symmetric tail behaviour characterises the majority of asset pairs, though we also find significant asymmetries in a number of cases, with assets more likely to post large losses when global conditions significantly deteriorate, rather than to gain when they improve.

Keywords: Copulas, Dependence, Tail dependence coefficients, Contagion, Asset classes

JEL: C58, G15

1. Introduction

Two important issues in finance are the degree to which assets are priced locally or globally and differences in dependence between asset prices during crises relative to normal times (Karolyi and Stulz, 2003)). These have serious practical implications both from policy and risk management perspective. However, whereas there is a large body of work on stock market dependence and contagion among developed markets, there is relatively little research on dependence between developed and emerging markets, and fewer still on assets other than equities. In this paper we address these questions taking an emerging market perspective and investigate the dependence structure between financial assets in Poland and abroad, including stocks, bonds and foreign exchange, with particular interest in the dependence in crisis times.

The simple measure of Pearson correlation is likely to be inadequate for this task, as it is appropriate to describe dependence in multivariate normal distribution and to some extent in other elliptical distributions (for details see Embrechts, McNeil and Straumann, 2002), whereas it is well documented that returns on asset prices exhibit fat tails and strongly deviate from normality (Gabraix et al., 2003). A better alternative might be to use concordance measures, such as Spearman's ρ or Kendall's τ, which can capture non-linear relations between any distributions. Still, rank correlations are limited to measuring monotone dependence and do not provide full information on dependence structure, including tails and differences therein. Full dependence structure can in turn be captured with copula functions, which is the method of choice for the present study.

A copula is a function that links together univariate (marginal) distribution functions to form a multivariate distribution function (Sklar, 1959). As each marginal distribution contains all the univariate information on a given variable, and the multivariate distribution contains all the univariate and multivariate information, the functions that link the two contains all the information on the dependence between variables, including the behaviour at the centre of their distributions and in its tails. Copulas have become very popular in financial modelling (for a review see Patton, 2009), as they allow to model separately each marginal distribution and the dependence structure, and thus allow for a far greater flexibility of the multivariate distribution than known multivariate extensions of univariate distributions. This feature is particularly appealing for our study as it enables us to choose from a wide range of marginal distributions and dependence structures without necessarily making strong assumptions about the characteristics of the joint price process for any two assets.

Following a standard *inference function for margins* (IFM) approach proposed by Joe and Hu (1996), we first estimate the parameters of each of the univariate distributions and subsequently estimate the copula functions given the margins. We model univariate distributions using a wide range of ARMA-GARCH model specifications, apply a number of goodness-of-fit tests and choose the one that fits best in order to minimise the impact of misspecification of the margins on the estimated dependence structure. Having obtained the margins, we analyse the dependence structure by testing a number of parametric copula families, using the recent method of Genest and Remillard (2009), which was proven to outperform other goodness-of-fit tests for copula functions. We use daily data from 1 March 2000 to 30 June 2012 obtained from Bloomberg and Reuters.

Given that our interest is in the dependence between variables, particularly in the tails of their distributions and asymmetries therein, our work contributes to studies of financial contagion. Indeed, one of the most common definitions of contagion describes it as a probability of a crisis in one country (or asset) conditional on a crisis in another (for a review of various definitions of contagion see Pericoli and Sbracia, 2003). This is tantamount to tail dependence, which is a copula property. As a result, such an approach has sound mathematical foundations and does not require an ad hoc identification of the crisis periods. The advantage of using copulas in such settings is further underscored by Rodriguez (2007), who shows that findings of unchanged dependence during crises versus normal times (another definition of contagion) based on linear correlation documented in the influential study of Forbes and Rigobon (2002) do not hold once more robust dependence measure is employed.

We contribute to the literature on contagion in the copula context in two ways. First, the vast majority of studies that use copula functions to gain insights into dependence among asset prices concentrate on relations between equity markets only (Jondeau and Rockinger, 2002, Aloui et al., 2011, Christoffersen et al., 2012). Less information is available on dependence between currencies (Patton, 2006, Benediktsdóttir and Scotti, 2009, Dias and Embrechts, 2010) and still less on bonds (Garcia and Tsafack, 2011). Moreover, in contrast with research on equities, these studies focus almost exclusively on developed markets. Second, to our knowledge the dependence between Polish and foreign markets has been studied usually with factor models, multivariate GARCH or VAR framework (Scheicher, 2001, Serwa and Bohl, 2005, Gębka and Serwa, 2007, Li and Majerowska, 2008, Caporale and Spagnolo, 2011, Adam, 2013 and Gijka and Horváth, 2013). While Doman (2011) employs copulas to the study of dependence on the Polish market, he focuses only on the relations within Polish asset classes.

We complement these studies by providing analysis on the dependence and contagion between Polish and global markets for three asset classes – equities, foreign exchange and bonds – using a copula approach.

2. The inference function for margins method

Sklar (1959) has shown that multivariate distribution can be decomposed into marginal distributions and a dependence function between them. This linking function is called a copula. Formally, let H be an n-dimensional distribution function with margins $F_1,\dots,F_n$. Then, there exists a n-copula C such that for all x in $\bar{R}^n$:

$$H(x_1,\dots,x_n) = C(F_1(x_1),\dots,F_n(x_n)) \tag{1}$$

Under an additional assumption that $F_1,\dots,F_n$ are continuous, the copula function is uniquely determined and for any $u \in [0,1]^n$ the following relation holds:

$$C(u_1,\dots,u_n) = H(F_1{}^{-1}(u_1),\dots,F_n{}^{-1}(u_n)) \tag{2}$$

where F^{-1} is the generalised inverse function $F^{-1}(z) = \inf\{x \in R | F(x) \geq z\}$ for all $z \in [0,1]$.

The assumption of continuity proves particularly convenient for the estimation of parametric distributions. Let $\boldsymbol{x} = \{\boldsymbol{x}_1,\dots,\boldsymbol{x}_T\}$ be a sample data matrix, where $\boldsymbol{x}_t = \{x_{1t}, x_{2t},\dots,x_{nt}\}$. If the joint distribution is n times differentiable, the density $h(\boldsymbol{x})$ is equal to the product of marginal densities f_i characterised by parameters $\boldsymbol{\alpha}_i$ and the copula density c with parameter $\boldsymbol{\theta}$ (Patton (2006)):

$$h(\boldsymbol{x}_t,\boldsymbol{\alpha},\boldsymbol{\theta}) = \prod_{i=1}^{n} f_i\,(x_{it},\boldsymbol{\alpha}_i) \cdot c_{\boldsymbol{\theta}}(F_1(x_{1t},\boldsymbol{\alpha}_1),\dots,F_n(x_{nt},\,\boldsymbol{\alpha}_n)) \tag{3}$$

This implies that the joint log-likelihood is the sum of univariate log-likelihoods and the copula log-likelihood:

$$\begin{aligned}\log(h(\boldsymbol{x},\boldsymbol{\alpha},\boldsymbol{\theta})) = &\sum_{t=1}^{T}\sum_{i=1}^{n}\log(f_i\,(x_{it},\boldsymbol{\alpha}_i)) \\ &+ \sum_{t=1}^{T}\log(c_{\boldsymbol{\theta}}(F_1(x_{1t,}\,\boldsymbol{\alpha}_1),\dots,F_n(x_{nt},\,\boldsymbol{\alpha}_n))\end{aligned} \tag{4}$$

This form suggests an IFM estimation procedure, consisting of *separate* estimation of the parameters of marginal distributions and then copula parameter conditionally on marginal distributions' parameters fixed, rather than a computationally much more involved, though asymptotically efficient *joint* estimation of parameters for margins and copulas by maximum likelihood (ML). The IFM method was proposed by Joe and Hu (1997) and is commonly applied in similar settings (Patton, 2006, Dias and Embrechts, 2010, Christoffersen et al., 2012), primarily because it is computationally much more effective than the ML method, while the IFM estimator remains asymptotically normal (see Joe, 1997). The IFM method amounts to first, maximising the likelihood for margins $\sum_{t=1}^{T}\sum_{i=1}^{n}\log(f_i\,(x_{it},\boldsymbol{\alpha}_i))$ over $\boldsymbol{\alpha}_i$-s to obtain transformed variable $\hat{u}_{it} = F_i(x_{it},\widehat{\boldsymbol{\alpha}}_{\boldsymbol{i}})$ which is distributed uniformly on a unit interval, and second, maximising the likelihood of the copula function $\sum_{t=1}^{T}\log(c_{\boldsymbol{\theta}}(\hat{u}_{1t},\dots,\hat{u}_{nt})$ over $\boldsymbol{\theta}$. In our application we use the IFM method and limit ourselves to two-dimensional distributions, that is dependence between *pairs* of variables. We use Matlab R2011b, A. Patton's Copula Toolbox and J.P. LeSage's jplv7 toolbox.

2.1. Modelling marginal distributions

Following the IFM method, in the first step we specify parametrically the marginal distributions. To this end, we need an appropriate family of models. We decide to model the data in a broad tradition of GARCH framework which captures most of stylised facts observed in financial data (volatility clustering, asymmetry of gains and losses, thick tails, etc.). In many applications, a simple GARCH(1,1) model seems to be a reasonable approximation of the underlying process' dynamics and complex specification search hardly improves forecasting abilities of the model (Hansen and Lunde, 2001). However, the IFM method requires the marginal distributions to be well-specified and may be non-robust against misspecifications (Kim et al., 2007). Therefore, the right implementation of the method involves allowing for a broad family of models from which the right model will be chosen, as well as using appropriate tests to choose the best alternative from the set of competing models.

Consider the variable of interest x_t. Its conditional mean is parameterised as ARMA(R,M):

$$x_t = C + \sum_{i=1}^{R}\phi_i x_{t-i} + \varepsilon_t + \sum_{j=1}^{M}\theta_j\varepsilon_{t-j} \tag{5}$$

In the models we consider the orders of autoregressive and moving average terms are each limited to three in order to favour more parsimonious representations.

We model the conditional variance of each of the variables either as a pure GARCH(P,Q) or as one of the asymmetric extensions, EGARCH(P,Q) and GJR(P,Q).

The GARCH(P,Q) model is given as:

$$\sigma_t^2 = \kappa + \sum_{i=1}^{P}G_i\sigma_{t-i}^2 + \sum_{j=1}^{Q}A_j\varepsilon_{t-j}^2 \tag{6}$$

with constraints:

$$\sum_{i=1}^{P} G_i + \sum_{j=1}^{Q} A_j < 1$$

$$\kappa > 0$$

$$G_i, A_j \geq 0$$

The GARCH(P,Q) model is symmetric in that it ignores the sign of the error term. It is now, however, a well-known phenomenon that financial variables exhibit asymmetries in response to good and bad news, which traditionally is related to the leverage effect (Black, 1976), or volatility feedback effect (Campbell and Hentschel, 1992). Thus, an appropriate model should allow for asymmetric news impact on conditional volatility, i.e. good news ($\varepsilon_{t-j} > 0$) having different effect than bad news ($\varepsilon_{t-j} < 0$). The two important parameterisations are GJR(P,Q) and EGARCH(P,Q). In the GJR(P,Q), the conditional variance is specified as:

$$\sigma_t^2 = \kappa + \sum_{i=1}^{P} G_i \sigma_{t-i}^2 + \sum_{j=1}^{Q} A_j \varepsilon_{t-j}^2 + \sum_{j=1}^{Q} L_j S_{t-j} \varepsilon_{t-j}^2 \qquad (7)$$

where $S_{t-j} = 1$ if $\varepsilon_{t-j} < 0$ and $S_{t-j} = 0$ otherwise, with constraints:

$$\sum_{i=1}^{P} G_i + \sum_{j=1}^{Q} A_j + \frac{1}{2}\sum_{j=1}^{Q} L_j < 1$$

$$\kappa \geq 0$$

$$G_i, A_j, A_j + L_j \geq 0$$

The conditional variance in the EGARCH(P,Q) parameterisation is given by:

$$\log \sigma_t^2 = \kappa + \sum_{i=1}^{P} G_i \log \sigma_t^2 + \sum_{j=1}^{Q} A_j \left[\frac{|\varepsilon_{t-j}|}{\sigma_{t-j}} - E\left\{ \frac{|\varepsilon_{t-j}|}{\sigma_{t-j}} \right\} \right] + \sum_{j=1}^{Q} L_j \left(\frac{\varepsilon_{t-j}}{\sigma_{t-j}} \right) \qquad (8)$$

We restrain the maximum orders of P and Q to three, analogous to the conditional mean specification. Finally, we allow the error term ε_t in each of the models to follow either normal or *t*-Student's distribution. The models are fitted using standard quasi-maximum likelihood estimation (QMLE) method.

A significant advantage of using the IFM method is that the specifications of margins can be tested using standard diagnostics to ensure that they fit the data well. In the post-estimation analysis, we employ Ljung-Box test for autocorrelation of the standardised residuals and Engle's ARCH test for the presence of the remaining ARCH effects in the residuals ε_t. We also employ the Berkowitz (2001) procedure to test if the hypothetical model's probability integral transform produces observations which are independently and identically distributed $U(0,1)$.

Finally, using the conditional cumulative distribution function of the selected model, we transform our variable of interest x_t into a $U(0,1)$ distributed variable which serves as an input for the second step of the IFM method. In doing this, we calculate

$$\hat{u}_t = F(x_t | I_{t-1}; \hat{\alpha}) \quad (9)$$

which we call *transformed* variable, where I_{t-1} is the information set available at time $t-1$ comprising past realisations of the variable of interest and $\hat{\alpha}$ is the estimated vector of parameters.

2.2. Modelling the dependence structure

The second stage of the IFM method consists of exploring the sole dependence between the two random variables using copula functions.

We chose a set of standard, static, parametric functions, most popular in the literature. They allow a wide range of dependence relations, including asymmetric tail dependence particularly important for investigating contagion effect. Thus, relations ranging from complete independence to dependence of differing grade, also in stress times, can be modelled. -

The definition of contagion which we employ in the present paper can be operationalised with the so-called asymptotic *tail dependence coefficients* introduced by Sibuya (1960) (hereinafter TDC), which, thus, become our measure of contagion. The coefficients describe the propensity of markets to crash or boom together, i.e. they measure the dependence between extreme outcomes of the variables. The upper (lower) TDC is a limiting probability of one variable exceeding (falling behind) a high-order (low-order) quantile, given that the other variable exceeds (falls behind) the same quantile. Formally, if (X, Y) is a vector of continuous random variables with marginal distributions F_x and F_y, respectively, then the upper and lower TDCs are defined as:

$$\lambda_U = \lim_{t \to 1^-} P(Y > F_y{}^{-1}(t) | X > F_x{}^{-1}(t)), \tag{10}$$

and:

$$\lambda_L = \lim_{u \to 0^+} P(Y \leq F_y{}^{-1}(t) | X \leq F_x{}^{-1}(t)). \tag{11}$$

If the upper or lower TDC equals zero, the respective extreme values are independent, otherwise we say that there is dependence between extreme values of the variables considered. Importantly, for the copulas considered in this paper the TDCs are simple functions of copula parameters. The choice of a particular copula may in some cases restrict admissible asymptotic dependence (e.g. Gaussian copula implies asymptotic independence). Table 1 gives an overview of the copulas we employ along with their TDCs. Recall that copula functions are defined on a unitary box, $(u, v) \in [0,1]^2$, where $u = F_x(X)$ and $v = F_y(Y)$ are distributed as $U(0,1)$.

Having obtained a bi-variate pseudo-sample from any two transformed variables of interest as in eq. (9), parameters of the above copulas are obtained by maximising the respective likelihood functions.

2.3. Testing copula functions

The IFM procedure amounts to estimating $\boldsymbol{\theta}$ under the assumption that the copula C linking marginal distributions indeed belongs to a chosen family of copulas C_0, i.e. under $H_0: C \in C_0 = \{C_\theta: \theta \in \Theta\}$. The goodness-of-fit tests, reviewed and compared in Monte Carlo studies by Genest et al. (2009) and Berg (2009), aim at the complementary issue of testing whether H_0 holds. To our knowledge, the cited papers are the latest available and most comprehensive studies of such methods in the literature. The experiments are designed to assess, in a number of different setups, the ability of various goodness-of-fit tests to maintain the nominal levels and their power against a variety of alternatives. The authors investigate the performance of several classes of methods of testing H_0 (based on Rosenblatt’s transform, empirical copula, moment-based etc.), the general conclusion we draw from their work is that the only method that ranks among three best performing in both power studies, is the goodness-of-fit procedure introduced in Genest et al. (2008) (ranking first in Genest et al., 2009 and second in Berg, 2009). It is based on the “empirical copula (a-theoretic information on the dependence structure, to be defined below), it thus belongs to a class of “blanket tests applicable to all copula structures (rather than Gaussian or Clayton only) and free of any strategic choices for their use or parameter fine-tuning. Its implementation involves, however, approximating *p-values* for testing H_0 with a bootstrap procedure.

Table 1. Copula functions and their characteristics

Copula name	$C(u,v)$	λ_L	λ_U
Normal	$\Phi_\rho(\Phi^{-1}(u),\Phi^{-1}(v))$, where Φ_ρ is the bivariate standardised Gaussian cdf with Pearson's correlation ρ and Φ^{-1} is the inverse of the univariate standardised Gaussian cdf	0	
Clayton	$(u^{-\alpha}+v^{-\alpha}-1)^{-1/\alpha}$ $\alpha>0$	$2^{-1/\alpha}$	0
Rotated Clayton	$u+v-1+C(1-u,1-v)$, where C is Clayton copula	0	$2^{-1/\alpha}$
Plackett	$((1+(\theta-1)(u+v))-\sqrt{(1+(\theta-1)(u+v))^2-4uv\theta(\theta-1)})/(2(\theta-1))$, for $0<\theta\neq 1$, uv, for $\theta=1$	0	
Frank	$\frac{1}{\alpha}\ln(1+\frac{(e^{\alpha u}-1)(e^{\alpha v}-1)}{(e^{\alpha}-1)})$ $\alpha\neq 0$	0	
Gumbel	$\exp(-(-\ln u)^\alpha+(-\ln v)^\alpha)^{1/\alpha})$, $\alpha>1$	0	$2-2^{1/\alpha}$
Rotated Gumbel	$u+v-1+C(1-u,1-v)$, where C is Gumbel copula	$2-2^{1/\alpha}$	0
***t*-Student**	$t_{v,r}\left(t_v^{-1}(u),t_v^{-1}(v)\right)$, where $t_{v,r}$ is the bivariate t-Student's cdf with parameter r and degrees of freedom v and t_v^{-1} is the inverse of the univariate t-Student's cdf with v degrees of freedom	$2t_{v+1}(-\sqrt{\frac{(v+1)(1-r)}{(1+r)}})$	
Symmetrised Joe-Clayton	$0.5(C_{\tau^U,\tau^L}(u,v)+u+v-1+C_{\tau^L,\tau^U}(1-u,1-v))$, where $C_{\tau^U,\tau^L}(u,v)=1-\left\{[(1-(1-u)^\kappa)^{-\gamma}+[(1-(1-v)^\kappa)^{-\gamma}-1]]^{-1/\gamma}\right\}^{1/\kappa}$, for $\kappa=1/\log_2(2-\tau^U)$, $\gamma=-1/\log_2(\tau^L)$, and $\tau^U,\tau^L\in(0,1)$	τ^L	τ^U
Independence copula	uv	0	

The idea is to compare the distance between the “empirical copula with the estimated parametric one. To assess whether the distance is significantly different from zero, a bootstrap procedure is implemented. As the input, the goodness-of-fit test takes the maximally invariant with respect to continuous, strictly increasing transformations of the components of bivariate distribution statistic, i.e. ranks obtained from the pseudo-sample $\{(\hat{u}_t, \hat{v}_t)\}_{t=1}^{T}$. The information on dependence comprised in the pseudo-sample is summarised in the “empirical copula C_T

$$C_T(u,v) = \frac{1}{T}\sum_{t=1}^{T} \mathbf{1}(U_t \leq u, V_t \leq v), \tag{12}$$

for $u, v \in [0,1]$, where U_t is obtained by dividing the rank of $\hat{u}_t$ (in a set $\{\hat{u}_t\}_{t=1}^{T}$) by $(T+1)$, and V_t - by dividing the respective rank of $\hat{v}_t$. The test statistics is based on the empirical process $\mathbb{C}_T = \sqrt{T}(C_T - C_{\hat{\theta}})$, and it is given by the Cramer-von Mises statistic

$$S_T = \int\limits_{[0,1]^2} \mathbb{C}_T(u,v)^2 dC_T(u,v), \tag{13}$$

whose large values imply the rejection of H_0. Asymptotic *p-values* could in theory be deduced from the limiting distribution of the above statistic. However, as the asymptotic behaviour of the empirical process depends on the family of copulas under the composite H_0 and on the unknown true parameter θ, whose estimate is used in $\mathbb{C}_T$ instead, the only viable way to execute statistical test is to resort to specially adapted parametric bootstrap procedure. It consists of the following steps:

1) Compute C_T and estimate $\hat{\theta}$
2) Compute $\hat{S}_T = \sum_{t=1}^{T}[C_T(\hat{u}_t, \hat{v}_t) - C_{\hat{\theta}}\ (\hat{u}_t, \hat{v}_t)]^2$
3) For a large B repeat the following for $b = 1, \dots, B$:
    a. Generate a random sample from the distribution $C_{\hat{\theta}}$
    b. Using the random sample compute $C_T^{(b)}$ and estimate $\hat{\theta}^{(b)}$
    c. Compute $S_T^{(b)}$ analogously to 2)
4) Approximate *p-value* with $p = \frac{1}{B}\sum_{b=1}^{B} \mathbf{1}(S_T^{(b)} > \hat{S}_T)$.

The final question, if the above goodness-of-fit test admits more than one copula, concerns the choice of one particular function for further analysis. Following Heilpern (2007), we chose the parametric copula with the lowest distance to the “empirical copula, as measured by $\hat{S}_T$. Then, we compute the TDCs.

3. The data on Polish and global financial instruments

Our data set is comprised of foreign and Polish variables. The foreign variables include stock index and sovereign bond yields in the United States (SP500, US 2Y, US 10Y), Germany (DAX, DE 2Y, DE 10Y), the CBOE Market Volatility Index, which measures the implied volatility of the S&P500 index options and is commonly used as a market risk-aversion and uncertainty indicator (VIX), the euro-dollar exchange rate, quoted as a US dollar price of the euro (EUR/USD), Deutsche Bank G10 Currency Future Harvest Index reflecting the return from carry trades in G10 currencies (FX CARRY), EURO STOXX Banks Index, a banking sub-index for the EMU banks with the largest capitalisation (EU BANKS) and three emerging market composites, including MSCI Emerging Markets Index, a free float-adjusted market capitalisation index measuring equity market performance in the global emerging markets (MSCI), sovereign bond spreads relative to the US (EMBI) and JPMorgan Emerging Markets Currency Index (FX EM). For Poland, the variables include main stock market index (WIG), banking sub-index (WIG BANKS), short and long-term sovereign bond yields (PL 2Y, PL 10Y) and the main foreign exchange rate with respect to the euro (EUR/PLN), quoted as Polish zloty price of the euro.

We use daily data from 1 March 2000 to 30 June 2012 which are obtained from Bloomberg and Reuters in the following way. First, daily data for all series are imported from Bloomberg. Second, whenever Bloomberg data are unavailable, we substitute them with Reuters data. We transform the original data in levels into rates of return between two consecutive trading days or into changes in yields (all sovereign bonds, percentage points) and spreads (EMBI, basis points) as appropriate, obtaining 2893 observations.

Table 2 presents descriptive statistics of the dataset. All variables exhibit high kurtosis accompanied by often high absolute skewness, which leads to strong rejection of normality by the Jarque-Bera test. ARCH effects are present in all of the variables and all but two show autocorrelation. These characteristics justify and motivate the use of ARMA-GARCH models for marginal distributions.

Table 3 reports unconditional dependence for pairs of variables based on Spearman's ρ. Transformation into ranks makes it a valid measure of monotone dependence without stringent distributional assumption that have been shown to be violated in Table 2. The variables are paired according to their class, with an exception of VIX, which is used for all asset classes, being a frequently used proxy for global factors relevant for all markets in similar studies, including

Table 2. Descriptive statistics.

	Mean	Min	Max	St. dev.	Skew	Kurtosis	ARCH(10)	Q(20)	J-B
EUR/PLN	2,1E-05	-0.045	0.055	0.007	0.442	8.525	415.80*	53.94*	3774*
PL 2Y	-4,2E-04	-1.42	0.987	0.106	-0.685	29.021	310.04*	101.20*	81846*
PL 10Y	-2,1E-04	-0.844	0.865	0.087	-0.060	22.494	374.19*	112.15*	45809*
WIG	2.1E-04	-0.102	0.061	0.014	-0.383	6.367	223.736*	39.49*	1438*
WIG BANKS	3,5E-04	-0.225	0.089	0.018	-0.565	13.695	186.73*	54.34*	969*
VIX	-2,07E-05	-0.351	0.496	0.064	0.677	6.930	235.78*	89.97*	2083*
EUR/USD	8,6E-05	-0.038	0.039	0.007	-0.034	5.284	177.23*	19.10	630*
FX EM	-5,2E-05	-0.059	0.063	0.005	-0.454	35.886	771.44*	20.65	130463*
FX CARRY	2.0E-04	-0.078	0.054	0.007	-0.963	13.907	448.08*	38.80*	14786*
DE 2Y	-0.002	-0.390	0.331	0.051	0.083	8.120	130.17*	48.17*	3163*
US 2Y	-0.002	-0.565	0.473	0.066	-0.235	9.815	267.08*	65.18*	5626*
DE 10Y	-0.001	-0.226	0.251	0.047	0.162	4.731	180.79*	30.30***	374*
US 10Y	-0.002	-0.473	0.428	0.069	0.158	5.309	154.39*	47.16*	654*
EMBI	-0.097	-86.30	97.83	9.514	0.570	16.929	784.45*	118.18*	2354*
DAX	-8.0E-05	-0.098	0.135	0.017	0.087	8.270	485.15*	53.05*	3351*
SP500	-1.7E-05	-0.095	0.104	0.014	-0.120	10.082	844.06*	70.30*	6052*
MSCI	2.0E-04	-0.114	0.129	0.014	-0.493	12.986	556.75*	122.185*	12136*
EU BANKS	-4.6E-04	-0.140	0.178	0.021	0.042	10.713	337.26*	53.06*	7172*

Notes: The table displays sample statistics for daily returns, changes in yields (all bonds) or spreads (EMBI) between 1 March 2000 and 30 June 2012, spanning 2893 observations for each series. ARCH(10), Q(20) and J-B are the Lagrange multiplier test of no ARCH effects up to 10 lags, the Ljung-Box statistics of no serial correlation up to 20 lags and the Jarque-Bera test for normality of distribution. *, ** and *** denote statistical significance at 1%, 5% and 10% level, respectively.

emerging markets (Pan and Singleton, 2008). The general pattern is that Polish equities appear to be highly dependent on global factors that affect valuation on core markets and other emerging markets. This relation appears considerably weaker for foreign exchange and still weaker in the case of bonds. However, it should be kept in mind that concordance measure captures average dependence across the whole distribution and does not provide information about possibly different behaviour in the tails or asymmetries.

One potential issue for copula analysis is the negative dependence between some pairs. There are copula functions that do not allow negative dependence (e.g. Clayton, Gumbel), yet they may still be very good at capturing the dependence between transformed variables. For example, the Gumbel copula does exhibit upper tail dependence and no lower tail dependence. This is a plausible relation between stocks in Poland and (inverted) VIX – there could be higher

Table 3. Spearman's ρ coefficients

EUR/PLN	**VIX**	**EUR/USD**	**FX CARRY**	**FX EM**
	0.227	0.032	-0.040	-0.141
PL 2Y	**VIX**	**DE 2Y**	**US 2Y**	**EMBI**
	0.066	0.066	0.040	0.088
PL 10Y	**VIX**	**DE 10Y**	**US 10Y**	**EMBI**
	0.075	0.084	0.052	0.107
WIG	**VIX**	**DAX**	**SP500**	**MSCI**
	-0.251	0.500	0.316	0.536
WIG BANKS	**VIX**	**DAX**	**SP500**	**MSCI**
	-0.228	0.462	0.295	0.456

Notes: The table reports Spearman's ρ calculated between daily returns (or changes) of Polish and foreign variables between 1 March 2000 and 30 June 2012. The Spearman's ρ coefficient between WIG BANKS and EU BANKS is 0.456.

propensity for WIG to fall when VIX increases significantly (bad news) than for WIG to increase when VIX falls (good news), consistent with the leverage effect for margins. Even though the asymmetric behaviour could perfectly reflect the Gumbel copula, we would not see it, as this copula family does not allow negative dependence. To allow this possibility, we transform the foreign variables by taking an inverse of it, i.e. $1/z$, z being the original variable in levels, and use this variable to estimate model for margins. This essentially changes the sign in rates of return of the series, reverses the rank of observations and simply changes the dependence between variables from negative to positive without any other changes in its characteristics, so the interpretation of the results is straightforward. We use this approach for the three pairs that exhibit relatively high negative dependence (WIG-VIX, WIG Banks – VIX and EUR/PLN-FX EM). As an alternative, one could estimate the model on the original data and then transform the $\hat{u}_t$ series into $1 - \hat{u}_t$ before using it for copula estimation.

4. Univariate GARCH models – empirical results

Prior to choosing the models for marginals, we estimate a broad set of ARMA-GARCH models. The selection process is as follows. First, we use the standard goodness-of-fit tests described in Section 2.1 to discard the models with misspecifications. It turns out that we are able to find more than one model for each variable of interest which passes all of the tests. Ljung-Box, Engle's ARCH and standard Berkowitz tests are all passed at the 5% level of significance (for lags in appropriate tests see Table 2). Therefore, to choose the right specification from the set of candidates we use the Akaike Information Criterion (AIC). For each foreign and local variable, we select the model with the lowest AIC value.

The final results of the GARCH fitting procedure are reported in Tables 7a-7b in Annex, Table 4 below is its shortened version. No single model (autoregressive or moving average) dominates the conditional means which are predominantly some versions of ARMA. However, a clear AR structure is confirmed for the EUR/PLN, while a constant only is chosen for the conditional mean of FX CARRY. Interestingly, for all but one series the best fit is obtained by using the EGARCH model with *t*-Student's error terms for the conditional variance equation. For PL BANKS, the GJR-GARCH is chosen. Therefore, the choice to include asymmetric models in the set of candidates proves right. The leverage terms are mostly significant, indicating that the response of volatility to shocks of positive and negative sign is different in the variables. The yields of government bonds, where the leverage terms tend to be insignificant constitute a notable exception. There is no obvious tendency of a certain lag length structure chosen by the AIC, although more models manifest two or more lags in the variance equation. We also note the sum of autoregressive and moving average parameters in the variance equation is often close to 1. It indicates that volatility exhibits high level of persistence with large changes followed by other large changes and small changes followed by other small changes.

With the chosen model, we construct a transformed variable as in eq. (9), which is used for copula analysis (see Figure 1 in Annex).

Table 4. Results for the marginal distributions (short view).

	Conditional Mean	Conditional Variance
EUR/PLN	AR(3)	EGARCH(3,3)
EUR/USD	ARMA(2,3)	EGARCH(3,3)
PL 2Y	ARMA(3,3)	EGARCH(2,3)
PL 10Y	ARMA(2,3)	EGARCH(3,3)
DE 2Y	ARMA(3,3)	EGARCH(3,3)
DE 10Y	ARMA(2,2)	EGARCH(1,3)
US 2Y	ARMA(1,3)	EGARCH(2,3)
US 10Y	ARMA(3,3)	EGARCH(3,3)
EMBI	ARMA(3,2)	EGARCH(2,2)
WIG	ARMA(3,3)	EGARCH(2,2)
PL BANKS	ARMA(3,2)	GJR(2,1)
DAX	ARMA(2,2)	EGARCH(2,3)
EU BANKS	ARMA(2,1)	EGARCH(2,3)
SP500	ARMA(1,1)	EGARCH(1,3)
MSCI	ARMA(2,3)	EGARCH(2,2)
VIX	ARMA(2,2)	EGARCH(2,3)
VIX(-1)	ARMA(2,2)	EGARCH(2,3)
FX CARRY	Const. only	EGARCH(3,3)
FX EM (-1)	ARMA(2,1)	EGARCH(3,3)

Note: All models have *t*-Student's error terms

5. Dependence and contagion – empirical results

Table below for each analysed pair of variables presents copulas that pass the goodness-of-fit test and the copula that is chosen as the best description of the dependence between them. The most often allowed copula is *t*-Student, followed by Gaussian and Symmetrised Joe-Clayton. However, it is worth noting that for a large number of pairs the Gaussian copula is rejected as a satisfactory description of the dependence function. In our view, this is a further argument against a common practice of using the linear correlation coefficient as a universal measure of dependence between variables, particularly financial ones. In pair-wise comparisons, it turns out that some pairs of variables, e.g. PL 10Y and US 10Y yields, can be described by any of the considered copula, while others (i.e. Polish and Eurozone banks' shares) – by none.

Table 5. Goodness-of-fit test results and the choice of the copula.

		Normal	Clayton	Rotated Clayton	Plackett	Frank	Gumbel	Rotated Gumbel	*t*-Student	Symm. Joe-Clayton	Indepen-dence copula
EUR/PLN	**VIX**			0.275 (0.231)			1.154 (0.655)			**0.044 0.117 (0.485)**	
EUR/PLN	**EUR/USD**								**0.030 3.822 (0.072)**		
EUR/PLN	**FX CARRY**	-0.049 (0.255)	0.000 (0.226)	0.000 (0.231)	0.907 (0.753)	-0.193 (0.769)	1.000 (0.248)	1.000 (0.244)	-0.041 13.464 (0.588)		**no param. (0.319)**
EUR/PLN	**FX EM(-1)**	**0.120 (0.067)**		0.132 (0.942)			1.070 (0.920)		0.116 15.491 (0.078)	0.000 0.034 (0.899)	
PL 2Y	**VIX**	0.060 (0.223)			**1.208 (0.314)**	0.373 (0.286)		1.042 (0.090)	0.060 62.029 (0.246)	0.003 0.000 (0.182)	
PL 2Y	**DE2Y**	0.077 (0.135)	0.084 (0.095)		1.238 (0.770)	0.416 (0.075)	1.039 (0.083)	1.048 (0.204)	0.074 18.449 (0.230)	**0.003 0.001 (0.148)**	
PL 2Y	**US2Y**	0.048 (0.230)	0.061 (0.513)		1.138 (0.158)	0.257 (0.168)	1.018 (0.058)	1.032 (0.557)	0.047 36.293 (0.220)	**0.002 0.000 (0.386)**	
PL 2Y	**EMBI**	0.094 (0.129)		0.106 (0.484)			1.057 (0.675)		**0.094 18.720 (0.199)**	0.000 0.021 (0.592)	
PL 10Y	**VIX**	0.075 (0.060)		0.076 (0.170)	**1.265 (0.104)**	0.462 (0.099)	1.034 (0.094)		0.075 31.804 (0.059)	0.000 0.004 (0.184)	
PL 10Y	**DE10Y**			0.153 (0.145)			1.082 (0.097)			**0.000 0.053 (0.083)**	
PL 10Y	**US10Y**	0.071 (0.447)	0.071 (0.118)	0.082 (0.208)	1.235 (0.261)	0.406 (0.268)	1.046 (0.389)	1.044 (0.365)	0.071 12.127 (0.753)	**0.000 0.008 (0.247)**	
PL 10Y	**EMBI**			0.144 (0.110)			1.076 (0.167)			**0.001 0.038 (0.066)**	

Notes: The table continues overleaf, see notes on the next page.

Table 5 (continued). Goodness-of-fit test results and the choice of the copula.

		Normal	Clayton	Rotated Clayton	Plackett	Frank	Gumbel	Rotated Gumbel	*t*-Student	Symm. Joe-Clayton	Indepen-dence copula
WIG	VIX(-1)							1.174 (0.432)		**0.135** **0.049** **(0.095)**	
WIG	DAX								**0.512** **9.522** **(0.084)**		
WIG	SP500	0.351 (0.187)							0.349 13.007 (0.339)	**0.144** **0.191** **(0.167)**	
WIG	MSCI								**0.551** **9.919** **(0.146)**		
WIG BANKS	VIX(-1)	0.242 (0.073)							**0.241** **18.424** **(0.066)**		
WIG BANKS	DAX	0.463 (0.126)							**0.467** **11.626** **(0.524)**		
WIG BANKS	SP500	0.319 (0.294)							**0.319** **15.323** **(0.281)**		
WIG BANKS	MSCI	0.491 (0.087)							**0.487** **11.887** **(0.252)**		
WIG BANKS	EU BANKS										

Notes: Each cell contains parameter estimates (in case of *t*-Student – the first number is correlation parameter and the second is the degree of freedom, in the case of Symmetrised Joe-Clayton – τ^L and τ^U, respectively) and goodness-of-fit test's *p-value* (in parentheses). A cell in bold and underlined denotes the copula that additionally is $\hat{S}_T$-closest to the "empirical copula for a given pair. An empty cell denotes a case of *p-value* lower than 0.05 and rejection of the copula class.

A striking observation is that for a number of pairs many copulas, with often opposite properties (e.g. Gumbel and its rotated variant), are admitted as a satisfactory description of the dependence structure. It thus seems that with the data available, the goodness-of-fit alone provides rather weak guidance to choosing the copula, and the second criterion, the $\hat{S}_T$ distance to "empirical copula eq. (12), needed to be introduced. This was essential for drawing conclusions, as for example in many cases the two copulas admitted displayed entirely opposite tail behaviour – tail independence in the case of Gaussian copula and tail dependence in the case of *t*-Student. Our conjecture is that if several copulas are admitted, these cases likely correspond to a weak dependence between the variables, and thus the estimated copula parameters imply in fact that the copula is *close* to independence. Indeed, for most of these cases the Spearman's rank correlation coefficient computed on a pseudo-sample of transformed (as in eq. 9) rates of return is close to zero (Spearman's ρ-s of the transformed variables are virtually the same as in Table 3). A notable exception to the above pattern is the EUR/PLN and EUR/USD pair, with the single *t*-Student copula allowed by the goodness-of-fit test and low Spearman's ρ of only 3%. To explain this fact, note that this is the only copula whose density has some mass in all four corners of the unit square, which is precisely an empirical observation we can make by inspecting the scatter plot of the pseudo-sample of this pair of currencies (see Figure 2 in Annex). On the other hand, the relationship between Polish and Eurozone banks' shares, although quite strong (with Spearman's ρ of 44%) must have some highly nonstandard shape, since all the copulas fitted fail the goodness-of-fit test.

The choice of a single copula that best describes the dependence between the variables, as explained in Section 2.3, is based on the distance $\hat{S}_T$. Table 5 above shows that two copulas clearly stand out as the most widely occurring dependence structure – it is the *t*-Student and Symmetrised Joe-Clayton copulas (actually, the Gaussian copula appears only once). Thus, in most cases copulas that display tail dependence were chosen, though in some – of rather low degree.

In the next step, for each pair of variables we compute TDCs using the formulas in Table 1 for the chosen copula. The respective coefficients are presented in the table below. Each cell of the table reports lower and upper TDCs. In cases when the chosen copula does not allow tail dependence (i.e. $\lambda_L = 0$ or $\lambda_U = 0$ by assumption), it prints "0; if the copula allows tail dependence but the computed TDC is low, it prints "0.00, while in cases when the estimated TDC is equal to or higher than 0.05 – the level which we consider economically significant – it is in bold.

Table 6. Estimated TDCs.

EUR/PLN	**VIX**	**EUR/USD**	**FX CARRY**	**FX EM(-1)**
	0.04 / **0.12**	**0.09 / 0.09**	0.00 / 0	0 / 0
PL 2Y	**VIX**	**DE 2Y**	**US 2Y**	**EMBI**
	0 / 0	0.00 / 0.00	0.00 / 0.00	0.00 / 0.00
PL 10Y	**VIX**	**DE 10Y**	**US 10Y**	**EMBI**
	0 / 0	0.00 / **0.05**	0.00 / 0.01	0.00 / 0.04
WIG	**VIX(-1)**	**DAX**	**SP500**	**MSCI**
	0.14 / 0.05	**0.09 / 0.09**	**0.14 / 0.19**	**0.10 / 0.10**
WIG BANKS	**VIX(-1)**	**DAX**	**SP500**	**MSCI**
	0.00 / 0.00	**0.05 / 0.05**	0.01 / 0.01	**0.06 / 0.06**

Notes: Each cell contains λ_L / λ_U, 0 denotes no tail dependence implied by the chosen copula class, 0.00 an estimated TDC up to two decimal places, in bold – values larger than or equal to 0.05. No copula was allowed for WIG Banks and EU Banks, therefore TDCs are not computed.

Two broad observations can be made based on the above results. First, asset classes differ greatly in their susceptibility to contagion, and second, the responses to upturns and downturns in global markets are often asymmetric.

Equities appear most prone to contagion from all international markets under consideration. The developments on two key stock exchanges we are considering – German and American – both affect Warsaw, though the influence of the US market is more pronounced. The situation on emerging markets translates into Poland (with 10% probability), as could be expected. Importantly, sudden changes in uncertainty and risk premia on core markets, as elucidated by the VIX, have an asymmetric bearing on the Polish stocks. A sudden increase in VIX has a 14% chance of spilling to the Warsaw stock exchange, while a reverse situation has only 5% chance to lift the WIG. Perhaps surprisingly, Polish banks' stocks behaviour differs from the broad WSE index. The contagion from global markets to Polish banks is more limited and related primarily to situation in Germany and other emerging markets. The regional factor – German stock exchange – can have both positive and negative spill-over effects on Polish banks, albeit with a modest probability of 5%. It may be a reflection of the financial linkages of the banking sector with their Eurozone partners, whose situation is proxied by the DAX index. The above tail dependence coefficients are similar to the average probabilities found by Christoffersen et al. (2012) for a group of emerging and developed economies in the same period, and are in the lower range of estimates reported by Aloui et al. (2011) among big emerging economies and the US stock market. These two studies differ with regard to asymmetry between the lower and upper tail dependence – Christoffersen et al. (2012) find a lower tail dependence to be considerably higher compared to upper tail dependence, whereas Aloui et al. (2011) find no such asymmetry. For Polish stocks, both patterns emerge and – depending on the choice of foreign asset – the Polish stock market exhibits symmetric or asymmetric tail dependence.

The Polish zloty is in some respects similar to equities. Just as a sudden increase in VIX weights on Polish stocks, it also depreciates the zloty with 12% probability, while a positive sentiment has only 4% chance of strengthening the Polish currency against the euro. The above pattern is consistent with the notion of crash risk that is prevalent for emerging market currencies. However, in the period under review, the Polish zloty did not fall (or rise) together with other emerging currencies or carry trade index in G10 currencies. Interestingly, although the overall dependence between EUR/PLN and EUR/USD is low (with Spearman's ρ of 3%), there is 9% chance that they experience extreme changes together, both in the upper and lower tail. Low dependence in tranquil times should therefore not lead one to false complacency, as the strong dependence may reveal itself in stress times. It also underscores the usefulness of copulas as a description of full dependence structure. Compared to tail dependence among the G10 currencies reported by Benediktsdóttir and Scotti (2009), the respective probabilities of the EUR/PLN versus the VIX or EUR/USD are relatively low, though certainly not negligible. The literature provides mixed results concerning asymmetry – Patton (2006) and Benediktsdottir and Scotti (2009) do find it, whereas Dias and Embrechts (2010) report *t*-Student copula model to provide the best fit. Our results suggest that both patterns are present for the zloty. Thus, similarly to equities, when considering potential dependence structure one should allow both for symmetric and asymmetric behaviour in the tails.

Polish bonds differ from equities and foreign exchange in that they do exhibit very limited contagion from foreign markets. PL 2Y do not seem to be affected by any of the external factors considered. Even if a copula admits some dependence, which is a case for all factors other than VIX, the computed TDC is close to zero. This may be due to the fact that the yields on the short term bonds are generally determined mainly by the expectations about future *local* interest rates. In the period under review, monetary policy in Poland operated under inflation targeting framework, freely floating interest rates and not particularly open economy – a mix that probably contributed to interest rates primarily reflecting domestic conditions. For 10-year bonds, the contagion is however visible, though relatively weak compared to other asset classes and limited to Germany (and to some extent other emerging markets) – rapidly rising yields on DE 10Y spill over to analogous bonds in Poland with 5% chance. This can be due to higher risk premia embedded in longer-term bonds, particularly credit risk that has been found to co-move with global factor for a number of emerging economies, as documented by Longstaff et al. (2011). Compared to the high degree of tail co-movement between bonds on economically connected developed markets, found by Garcia and Tsafack (2011) to be usually much above 50%, the results suggest a much lower degree of contagion.

It has to be acknowledged that choosing one copula from the allowable set (defined by the goodness-of-fit test) and making inferences based solely on this particular copula risks ignoring potentially useful information contained in the copulas that have passed the goodness-of-fit test, though with a worse fit. Also, note that we have not tested formally whether the distances of the various copulas from the “empirical copula eq. (12) are *significantly* different. It is well possible that the difference between the lowest distance and the second-lowest is actually statistically insignificant. As a cross-check then, for each pair of indices we computed mean TDCs including copulas admitted by the goodness-of-fit test. The results are qualitatively very similar, with the exception of WIG and SP500 pair, where the non-rejection of the Gaussian copula lowered the mean TDC to $\bar{\lambda}_L = 0.05$ and $\bar{\lambda}_U = 0.07$, as compared to $\lambda_L = 0.14$ and $\lambda_U = 0.19$ implied by the chosen Symmetrised Joe-Clayton copula. Though still economically significant, the evidence of high degrees of (positive as well as negative) contagion from US to Polish stocks needs to be treated with caution.

6. Conclusions

Our findings suggest that Polish equities, currency and long-term sovereign bonds are to a different extent susceptible to contagion from global markets, whereas short-term Polish bonds are not. Equities appear most prone to co-move with global risk and foreign stock markets in the tails as well as in the bulk of the distribution. The zloty is also susceptible to changes in risk on a global market, though the importance of developments in other currencies appears relatively smaller. As far as long-term bonds are considered, the contagion from global markets is relatively weak. It appears likely that lack of contagion in short-term bonds is a reflection of monetary policy independence and this may be also partly responsible for subdued dependence on the longer horizons.

Even though Polish assets are susceptible to contagion, there may still exist benefits from international diversification. Though the tail dependence between Polish equities and foreign markets appears stronger than between foreign exchange and bonds, it remains lower compared to big emerging economies and weaker still compared to dependence found between the developed markets themselves. The diversification benefits for the zloty, which exhibits more limited co-movement with other emerging currencies and was actually independent from carry trades in G10 currencies, appear even higher, though not as high as in the case of Polish bonds.

We find the *t*-Student copula to provide the best fit for the largest number of pairs in all three asset classes. Although the Gaussian copula cannot be rejected in a number of occasions, its fit is inferior to that of the *t*-Student, underscoring the importance of tail behaviour. While the above two copulas are symmetric, this does not necessarily mean that asymmetries in tails are irrelevant – quite the opposite, as the Symmetrised Joe-Clayton copula with substantial differences in tail behaviour was found for some pairs. The asymmetry is particularly visible for Polish equities and the zloty is much more likely to lose in value when global conditions (VIX) deteriorate, rather than to gain when they improve. This feature, together with asymmetric distribution of returns, is consistent with the crash risk embedded in Polish assets.

Our findings underscore not only the importance of global, common factors in asset valuation, but also of flexible modelling approach. The co-movement is sometimes limited to tails, while the dependence in other regions of the distribution may be weak, so the inference based on measures that do not account for the whole dependence structure can be misleading. Unfortunately, even the best available goodness-of-fit test for copula models appear to have problems with indicating the single correct dependence structure and has to be complemented with other measures (i.e. based on distance to empirical copula). The co-existence of symmetric and asymmetric dependence structure further suggests that a wide range of competing models is advisable. In this respect, we acknowledge the limitation to time invariant copulas is a serious restriction on the dependence relations, as the data cannot speak on the issue of dependence evolution, even though we thoroughly test the static dependence structure specification. Hence, allowing for dynamic copula is among the lines of future research.

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Annex

Table 7a. Results for the marginal distributions

	EUR/PLN	EUR/USD	PL 2Y	PL 10Y	DE 2Y	DE 10Y	US 2Y	US 10Y	EMBI
Conditional Mean	**AR(3)**	**ARMA(2,3)**	**ARMA(3,3)**	**ARMA(2,3)**	**ARMA(3,3)**	**ARMA(2,2)**	**ARMA(1,3)**	**ARMA(3,3)**	**ARMA(3,2)**
C	-2.4E-04 (1.0E-04)	1,4E-04 (8,1E-05)		-6,5E-06 (2,0E-05)		-0,004 (0,002)		-0,005 (0,003)	-0,696 (0,245)
ϕ_1	-0,047 (0,018)	1,208 (0,011)	0,671 (0,007)	0,142 (0,253)	0,144 (0,167)	-0,884 (0,086)	0,998 (0,002)	-0,693 (0,215)	-0,605 (0,019)
ϕ_2	-0,009 (0,018)	-0,971 (0,011)	-0,681 (0,004)	0,842 (0,251)	0,172 (0,160)	-0,841 (0,073)		-0,669 (0,176)	-0,951 (0,013)
ϕ_3	-0,047 (0,018)		0,986 (0,007)		0,682 (0,121)			0,147 (0,197)	0,072 (0,019)
θ_1		-1,238 (0,019)	-0,666 (0,009)	-0,057 (0,254)	-0,085 (0,165)	0,922 (0,084)	-1,080 (0,018)	0,635 (0,212)	0,679 (0,001)
θ_2		1,012 (0,022)	0,685 (0,005)	-0,859 (0,222)	-0,203 (0,148)	0,854 (0,071)	0,033 (0,027)	0,574 (0,175)	1,000 (0,001)
θ_3		-0,020 (0,016)	-0,976 (0,009)	-0,060 (0,034)	-0,699 (0,119)		0,052 (0,019)	-0,233 (0,190)	
Conditional Variance	**EGARCH(3,3)**	**EGARCH(3,3)**	**EGARCH(2,3)**	**EGARCH(3,3)**	**EGARCH(3,3)**	**EGARCH(1,3)**	**EGARCH(2,3)**	**EGARCH(3,3)**	**EGARCH(2,2)**
κ	-0,749 (0,169)	-0,082 (0,036)	-0,008 (0,005)	-0,011 (0,016)	-0,203 (0,077)	-0,045 (0,019)	-0,001 (0,002)	-0,243 (0,086)	3,0E-04 (5,1E-04)
G_1	-0,717 (0,005)	0,457 (0,131)	1,505 (0,118)	1,346 (1,956)	-0,672 (0,130)	0,993 (0,003)	1,831 (0,098)	-0,930 (0,006)	1,844 (0,035)

(the table continues overleaf)

G_2	0,670 (0,008)	0,912 (0,056)	-0,507 (0,117)	-0,038 (3,414)	0,852 (0,044)		-0,831 (0,097)	0,910 (0,008)	-0,844 (0,035)
G_3	0,973 (0,005)	-0,377 (0,121)		-0,310 (1,462)	0,787 (0,111)			0,977 (0,006)	
A_1	0,157 (0,020)	-0,206 (0,050)	0,399 (0,050)	0,403 (0,049)	0,156 (0,038)	-0,001 (0,049)	0,148 (0,050)	0,125 (0,018)	0,199 (0,034)
A_2	0,242 (0,034)	0,050 (0,024)	-0,338 (0,047)	-0,224 (0,798)	0,159 (0,040)	-0,032 (0,064)	-0,064 (0,097)	0,215 (0,033)	-0,194 (0,033)
A_3	0,111 (0,021)	0,250 (0,056)		-0,133 (0,721)	0,085 (0,047)	0,126 (0,046)	-0,068 (0,054)	0,099 (0,018)	
L_1	0,047 (0,013)	-0,038 (0,029)	0,064 (0,033)	0,061 (0,033)	-0,037 (0,024)	0,009 (0,030)	0,049 (0,033)	0,005 (0,012)	0,142 (0,021)
L_2	0,079 (0,020)	0,044 (0,022)	-0,056 (0,033)	-0,023 (0,130)	-0,034 (0,019)	0,014 (0,043)	-0,070 (0,063)	-0,027 (0,018)	-0,135 (0,020)
L_3	0,052 (0,012)	-0,007 (0,029)		-0,034 (0,122)	0,013 (0,025)	-0,036 (0,030)	0,020 (0,035)	-0,036 (0,010)	
t-Student's degrees of freedom	7,476 (0,877)	8,260 (1,177)	3,627 (0,301)	3,924 (0,314)	5,367 (0,540)	9,111 (1,482)	6,317 (0,657)	10,851 (1,923)	6,115 (0,564)
AIC	-2,1E+04	-2,1E+04	-7894,804	-8602,841	-9798,696	-9906,941	-8784,208	-7687,686	2,0E+04

Notes: The table reports ML estimates for the univariate ARMA-GARCH models of the marginal distributions. All models have *t*-Student's error terms. Standard errors are in brackets.

Table 7b. Results for the marginal distributions (continued)

	WIG	PL BANKS	DAX	EU BANKS	SP500	MSCI	VIX	VIX(-1)	FX CARRY	FX EM (-1)
Conditional Mean	**ARMA(3,3)**	**ARMA(3,2)**	**ARMA(2,2)**	**ARMA(2,1)**	**ARMA(1,1)**	**ARMA(2,3)**	**ARMA(2,2)**	**ARMA(2,2)**	**Const. only**	**ARMA(2,1)**
C	9,2E-06 (1,2E-05)		5,2E-04 (4,2E-04)	8,1E-05 (7,0E-05)	1,4E-04 (7,2E-05)	0,001 (4,1E-04)	-0,001 (3,8E-04)	0,001 (3,8E-04)	5,4E-04 (8,1E-05)	
ϕ_1	-0,297 (0,101)	-1,007 (0,152)	-0,099 (0,033)	0,754 (0,157)	0,572 (0,138)	0,111 (0,009)	-0,184 (0,079)	-0,179 (0,083)		1,077 (0,020)
ϕ_2	0,494 (0,083)	-0,679 (0,144)	-0,948 (0,032)	-0,059 (0,018)		-0,979 (0,009)	0,652 (0,069)	0,648 (0,072)		-0,082 (0,019)
ϕ_3	0,785 (0,091)	-0,009 (0,025)								
θ_1	0,340 (0,095)	1,057 (0,151)	0,089 (0,038)	-0,724 (0,157)	-0,635 (0,132)	0,076 (0,020)	0,091 (0,070)	0,087 (0,073)		-0,990 (0,008)
θ_2	-0,495 (0,075)	0,705 (0,152)	0,933 (0,037)			0,959 (0,009)	-0,768 (0,063)	-0,764 (0,066)		
θ_3	-0,814 (0,084)					0,197 (0,018)				
Conditional Variance	EGARCH(2,2)	GJR(2,1)	EGARCH(2,3)	EGARCH(2,3)	EGARCH(1,3)	EGARCH(2,2)	EGARCH(2,3)	EGARCH(2,3)	EGARCH(3,3)	EGARCH(3,3)
κ	-0,077 (0,026)	5,4E-06 (1,7E-06)	-0,066 (0,022)	-0,022 (0,013)	-0,123 (0,022)	-0,035 (0,012)	-0,019 (0,011)	-0,022 (0,013)	-0,115 (0,051)	-0,031 (0,020)
G_1	1,911 (0,168)	0,183 (0,090)	1,463 (0,133)	1,704 (0,132)	0,987 (0,002)	1,783 (0,043)	1,729 (0,101)	1,704 (0,113)	0,458 (0,158)	1,247 (0,215)
G_2	-1,359 (0,295)	0,704 (0,088)	-0,470 (0,131)	-0,707 (0,130)		-0,787 (0,042)	-0,732 (0,100)	-0,707 (0,111)	0,993 (0,003)	0,177 (0,363)

(the table continues overleaf)

G_3	0,439								-0,462	-0,427
	(0,142)								(0,156)	(0,166)
A_1	-0,019	0,056	-0,107	-0,057	-0,199	0,047	0,116	0,115	0,137	0,306
	(0,032)	(0,015)	(0,041)	(0,046)	(0,043)	(0,031)	(0,053)	(0,053)	(0,043)	(0,030)
A_2	0,093		0,228	0,220	0,265	-0,015	-0,089	-0,085	0,069	-0,267
	(0,039)		(0,074)	(0,083)	(0,066)	(0,033)	(0,099)	(0,099)	(0,028)	(0,088)
A_3			-0,058	-0,126	0,051		-0,016	-0,018	-0,041	-0,004
			(0,058)	(0,053)	(0,048)		(0,055)	(0,055)	(0,050)	(0,072)
L_1	-0,098	0,087	-0,255	-0,212	-0,243	-0,205	0,101	-0,101	-0,187	0,083
	(0,024)	(0,022)	(0,027)	(0,030)	(0,031)	(0,023)	(0,035)	(0,035)	(0,031)	(0,027)
L_2	0,077		0,137	0,190	-0,020	0,198	-0,012	0,008	0,001	0,018
	(0,023)		(0,072)	(0,071)	(0,044)	(0,022)	(0,068)	(0,068)	(0,012)	(0,048)
L_3			0,064	-0,002	0,153		-0,071	0,073	0,160	-0,096
			(0,046)	(0,043)	(0,031)		(0,038)	(0,038)	(0,030)	(0,035)
t-Student's degrees of freedom	7,606	6,814	13,398	8,926	9,704	8,262	6,324	6,294	5,347	5,670
	(1,000)	(0,816)	(2,521)	(1,396)	(1,575)	(0,968)	(0,657)	(0,654)	(0,565)	(0,364)
AIC	-1,7E+04	-1,6E+04	-1,7E+04	-1,6E+04	-1,8E+04	-1,8E+04	-8338,925	-8338,545	-2,2E+04	-2,4E+04

Notes: The table reports ML estimates for the univariate ARMA-GARCH models of the marginal distributions. All models have *t*-Student's error terms, except for PL BANKS following GJR-GARCH process with *t*-Student's error term. Standard errors are in brackets.

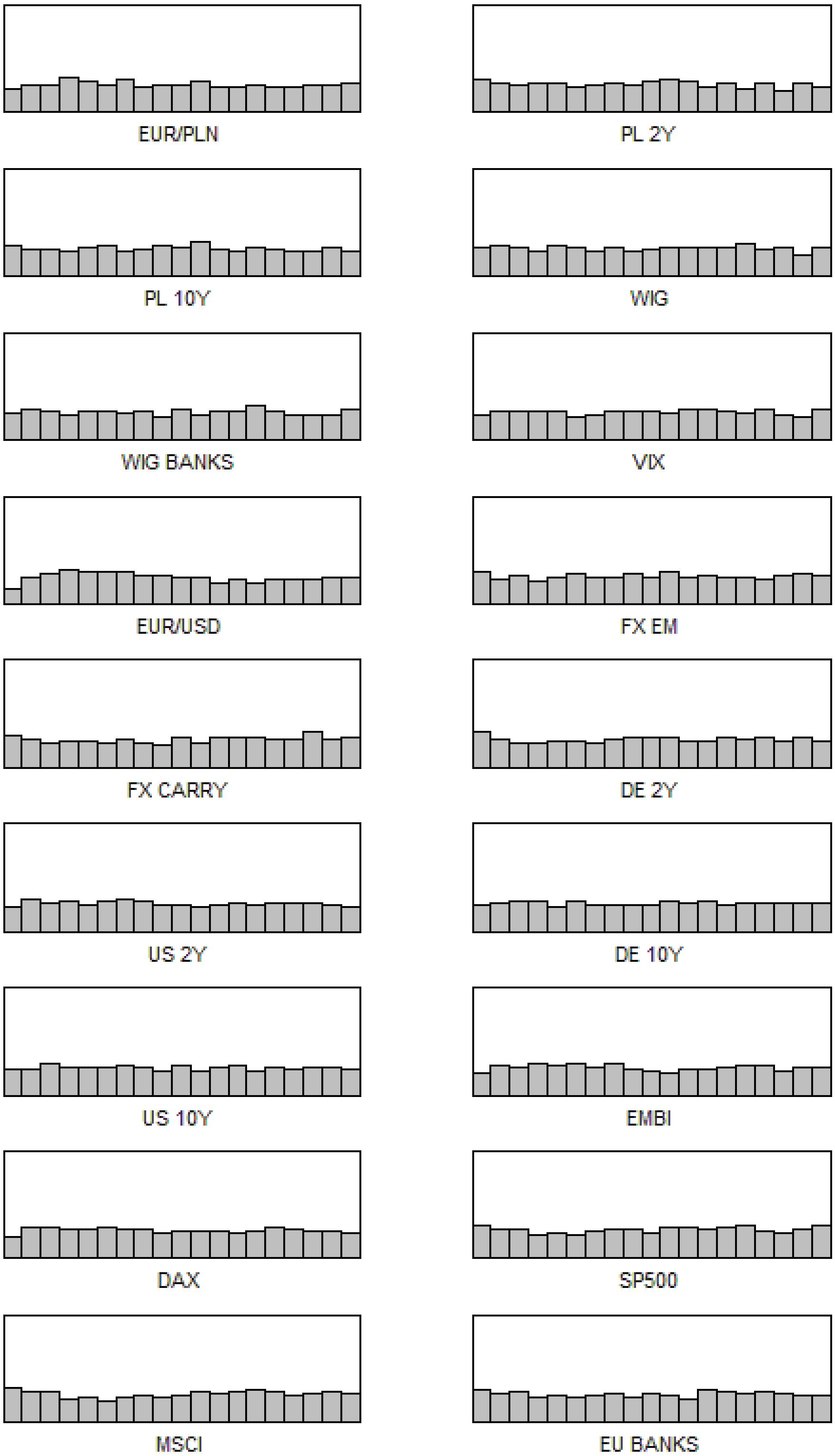

Figure 1. Histograms of the transformed variables.

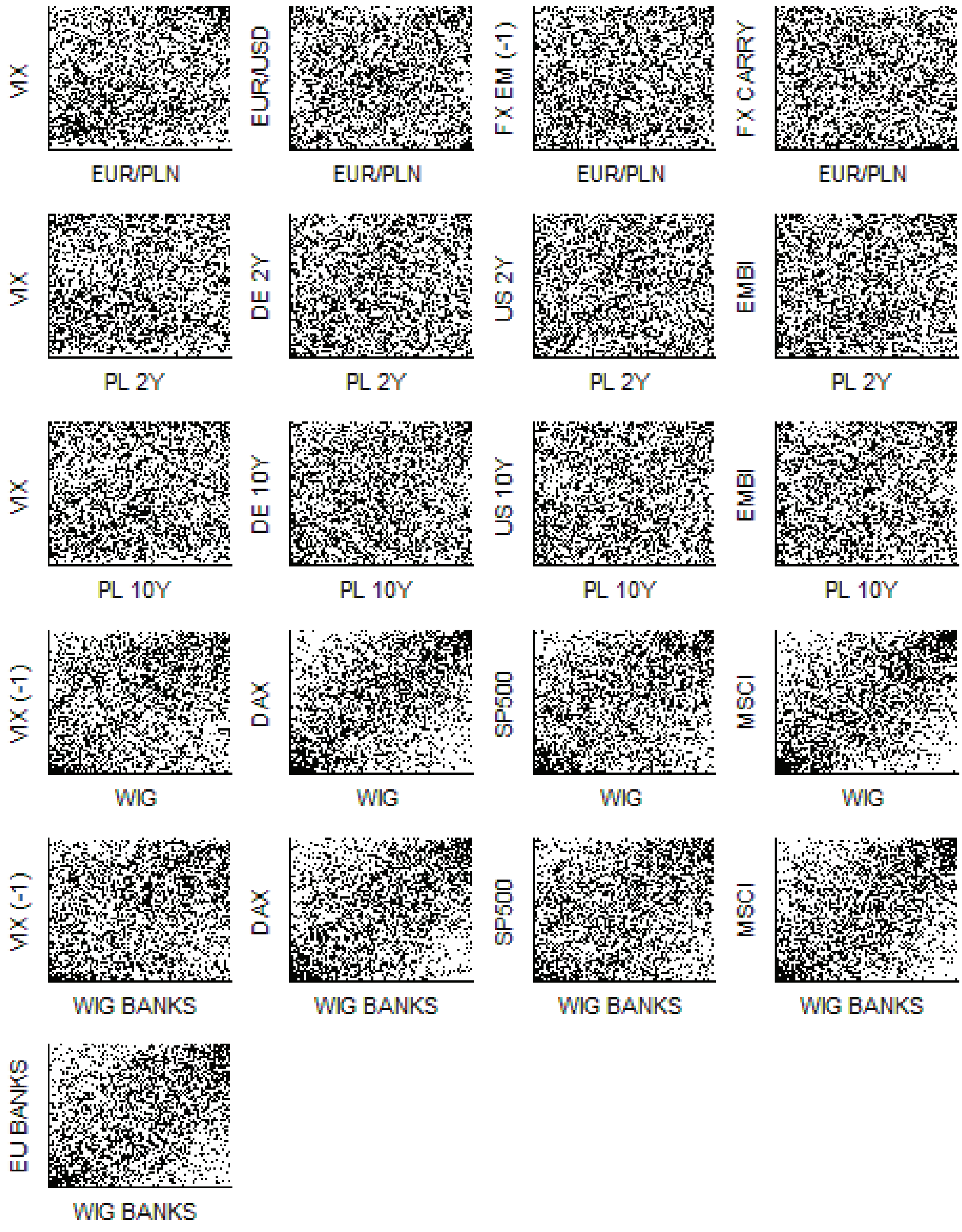

Figure 2. Bi-variate pseudo-samples of the transformed variables.

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