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Summary:

The article presents the application of a linear regression model to the problem of space-time disaggregation of the GDP of the Polish economy. In the approach described, the structural parameters of linear regression are subject to estimation, where the annual GDP of voivodships (regions of Poland at NUTS 2 level) or the rate of its changes represent the explained variable, while the annual domestic GDP or the rate of its changes is the explanatory variable. The authors propose to estimate the quarterly GDP and its changes in individual regions as functions of regression parameters.

The study presents the results of GDP estimates and its changes in individual regions. Alternative approaches were subject to evaluation in terms of the level of statistical uncertainty associated with the estimation and in terms of the level of spatial diversity of the estimated values.

Key words: Disaggregation of observed macroeconomic categories, classical linear regression model, uncertainty of estimation.

JEL: C13, C43, E01

1. Introduction

The problem of space-time disaggregation of the observed time series of the basic macroeconomic categories has been the subject of intensive studies for over half a century. Originally, this issue was analysed as a particular example of the problem of completing data, where it was taken into account as one of potential empirical fields of application. The search for methods of precise space-time estimation of the values of macroeconomic categories subsequently emerged as a separate part of econometrics and statistical methods, due to its fundamental role in statistical reporting and scientific analysis of national accounts.

One of the first studies attempting to obtain quarterly approximations derived from annual data was the paper published by Lisman and Sandee (1964). The authors obtained a time series with a quarterly frequency, determining the absolute changes in the observed macroeconomic variable between consecutive years, the amplitude of seasonal variations occurring in a given year and the transition matrix constituting the background to the applied linear interpolation. The application of this method to acquire quarterly data requires access to dataset with annual frequency. A similar approach was proposed by Boot, Feibes, Lisman (1967), who obtained quarterly observations by minimising the sum of squares of differences between adjacent quarters, with the simultaneous limitation of the sum of quarterly values and the observed annual value. The study by Denton (1971) represents the same trend, with other auxiliary variables of the same frequency as the search variable additionally used. The afore-mentioned methods developed towards the dynamic approach on the basis of the ARIMA models (Wei, Stram, 1986, 1990; Guerrero, Martínez, 1995) or factor models (Angelini et al. 2006, Marcellino, 2007). The approaches described above allow to obtain time series of observations with the frequency higher than the primary series. Their primary value lies in the simplicity of the approach which links in an automatic manner the information on annual growth with the estimated information on the pattern of quarterly fluctuations arising mainly on the back of seasonal variations or, otherwise, subject to numerical interpolation. Such methods are commonly used in cases when there is a need to perform additional estimation of missing observations.

The second group of methods includes procedures using additional explanatory variables with the same frequency as the search variable. This group of methods was launched in the study by Chow and Lin (1971), who determined the best linear unbiased estimator (BLUE) and its

co-variance matrix for the searched values of a monthly frequency. The analysis was based on a regression model, where the explained variable was observed with a quarterly frequency, whereas in the case of explanatory variables, observations with a higher (monthly) frequency were available. According to Chow and Lin (1971) the vector of explanatory variables can consist of advanced or delayed variables in relation to the search variable, and additionally of the trend or zero-one variables.

Chow and Lin's approach (1971) allowed to formulate the most popular data interpolation method, and many methods proposed later constitute its modification. For example, Rossi (1982) added the condition of the equality of the aggregate and the sum of data available with a higher frequency (i.e. the quarterly value and the sum of monthly values constituting this value). Fernandez (1981) and Littermann (1983) considered complex stochastic structure, postulating a temporal dependence of error terms. The Chow and Lin estimator led to the generalisation presented by DiFonzo (1990), in which the estimator was determined for regions with a higher frequency of time series in the case where lower frequency of the variable is available and the explanatory variables with higher frequency are available too. The DiFonzo estimator, as a special case of the Aitken estimator in the linear regression model, depends on the unknown covariance matrix of error terms. The modifications proposed in literature take into account, inter alia, the number of estimated series, the dynamic nature of the series and the spatial covariance of residuals or their auto-correlation; c.f. Pavia-Mirallesa, Carber-Borrasa (2007), Salazar et al. (1997), Silva et al. (2001).

One of the latest approaches in the disaggregation of series is the modification of the Chow and Lin method, proposed in the study by Polasek et al. (2010). This latter study applies data on the GDP of Spanish regions at the NUTS2 level, making estimates of GDP observations for NUTS3. In the regression, regional information of social, economic and spatial nature was used for the NUTS3 level. The principal value of the study by Polasek et al. (2010) is the application of the Bayesian approach in the estimation of spatial regression parameters. The Bayesian approach allowed to make a formal presentation of information about statistical uncertainty associated with the estimated GDP observations for regions. In addition, in this approach a researcher may contribute expertise on the parameters of the model through a priori distributions, which allows to obtain estimates compliant with the researcher's intuition for values of macroeconomic indicators within the regions. This approach was subject to further modifications in a study by Polasek (2013).

A comprehensive and exhaustive review of disaggregation methods can be found in the study by Pavía-Miralles (2010), whereas comparative analyses are presented, inter alia, by Chen (2007), who juxtaposed the results of applying five selected disaggregation methods to 60 series from national accounts. The comparison of methods based on numerical interpolation was presented by Marcellino (1998).

The literature of the subject indicates that the most popular disaggregation methods are based on the regression approach, with the use of additional explanatory variables, selected from the set of available variables. In addition, the basic goal of the studies is to determine the scoring for interesting values, while omitting in-depth analysis of statistical uncertainty associated with the estimation of observations with a higher uncertainty. Authors of some studies determine the average errors of the estimation. However, the analysis of uncertainty is not a criterion for quality assessment of interesting value estimations.

Studies on space-time disaggregation of indicators have only recently been pursued in Poland. The Central Statistical Office publishes official GDP values for regions with an almost two-year delay with respect to the publication of GDP ratios for the whole country. It means that obtaining relatively valid GDP estimates for regions requires forecasts of those values. In this respect, the Institute for Structural Research seems to be a leading institution. It presented the review of methods applicable in the forecast of annual GDP for regions in the IBS report (2010). The proposed approaches use estimates of the annual share of regions in the creation of the national GDP, the convergence equation obtained in accordance with the Barro, Sala-i-Martin methodology (1995), cluster analysis or classical methods of time series analysis. Attempts aimed at forecasting the GDP of the Zachodniopomorskie regions were undertaken in the study by Batóg (2011). The approaches applied in that study resort to the classical analysis of time series based on deterministic trends. The obtained results of three-annual forecasts of the annual GDP do not differ significantly depending on the selection of the trend function, and *ex post* errors in each of the variants are low. The high precision of forecasts discussed in the study by Batóg (2011) is mainly derived from the adoption of a short-term forecasting horizon. Obtaining the time series of approximated values of Poland's GDP with a monthly frequency was subject to analyses in the study by Kelm (2008). This latter study uses the co-integration approach in order to determine long-term relations between indicators observed with a monthly frequency and the quarterly variable subject to breakdown. The classical approaches, mentioned at the beginning of this section, proposed in the studies by Boot, Feibes, Lisman (1967), Lisman, Sandee (1964) and Denton (1971) were used for the

case of the Polish economy in the study by Woźniak (2011) in order to break down quarterly indices of the business cycle into monthly frequency figures.

The objective of this study is the application of the classical linear regression model to the problem of space-time disaggregation of the GDP of the Polish economy. In the approach described, the structural parameters of linear regression are subject to estimation, where the annual GDP of voivodships (regions of Poland) or the rate of its changes represent the explained variable whereas the annual domestic GDP or the rate of its changes fulfil the role of the explanatory variable. The proposed approach consists in estimating the quarterly GDP and its changes for individual regions as functions of the considered regression parameters. We present the results of GDP estimates and its changes in regions according to three variants of the applied empirical series with an annual frequency. In the first variant, the period of 1995–2012 is subject to analysis, and an identical GDP deflator is adopted for the whole country and for individual regions with respect to 1995-2003, while for 2004–2012 the deflators[1] vary. In the second variant, the analysis focuses on 1995–2012 as well, and for the entire duration of this period an identical GDP deflator for Poland and for individual regions was adopted. In the third variant diversified deflators were adopted, however, due to the availability of observations, we have focused the analysis on the period between 2003 and 2012.

The proposed alternative approaches were subject to evaluation in terms of the level of statistical uncertainty associated with the estimation and in terms of the level of spatial diversity of the estimated values. Consequently, besides the evaluation parameters, the analysis covers confidence intervals of estimated values obtained from the regression models. This paper includes a discussion of the differences in dispersion between those intervals in the proposed approaches. Particular attention has been paid to the diversification of GDP estimates and its changes in regions in the period of economic slowdown in Poland in 2001-2002.

[1] CSO has been publishing the real GDP index and deflators of prices for voivodships since 2004; c.f. *Gross domestic product – Regional accounts*, Regional Statistical Office, Katowice

2. Application of linear regression in the problem of GDP quarterly estimation for regions

Let us assume that GDP in year *t* for the *i-th* region (voivodship), designated as $GDP_{ti}^{A\_R}$, may be described by the Classical Linear Regression Model, where Poland's GDP in year *t* is adopted as the explanatory variable (designated as $GDP_{t}^{A\_N}$). The regression equation takes the following form:

$$GDP_{ti}^{A\_R} = \beta_{0i} + \beta_{1i} GDP_{t}^{A\_N} + \varepsilon_{ti}, t = 1,...,T, i = 1,...,m. \quad (1)$$

Let us assume that $y_{[Tx1]}^{(i)}$ denotes the vector of observations on the explained variable, whereas $X_{[Tx2]}$ means the matrix of observations on the explanatory variable with the first column of ones:

$$y_{[Tx1]}^{(i)} = \begin{bmatrix} GDP_{1i}^{A\_R} \\ \vdots \\ GDP_{Ti}^{A\_R} \end{bmatrix}, \quad X_{[Tx2]} = \begin{bmatrix} 1 & GDP_{1}^{A\_N} \\ \vdots & \vdots \\ 1 & GDP_{T}^{A\_N} \end{bmatrix}.$$

Assuming that $\varepsilon_{ti} \sim N(0, \sigma_{ii}^2)$, $\operatorname{cov}(\varepsilon_{ti}, \varepsilon_{tj}) = \sigma_{ij}^2 \in R$ and assuming that $\operatorname{cov}(\varepsilon_{ti}, \varepsilon_{sj}) = 0$, if $t \neq s$, the regression system parameters in (1) may be estimated using the OLS estimator for each equation separately. Let $\hat{\beta}_{OLS}^{(i)} = (X'X)^{-1} X' y^{(i)}$ designate this estimator of the vector of parameters $\beta^{(i)} = (\beta_{0i}, \beta_{1i})'$. The problem of GDP estimation in quarter *n* in the *i-th* region (denoted by $GDP_{ni}^{Q\_R}$) may be considered as the issue of estimation of linear functions of structural parameters $\beta^{(i)}$. This is due to the fact that we adopt the following function of parameters as the definition of $GDP_{ni}^{Q\_R}$:

$$GDP_{ni}^{Q\_R} = \frac{1}{4}\beta_{0i} + \beta_{1i} GDP_{n}^{Q\_N} = \left(\frac{1}{4}, GDP_{n}^{Q\_N}\right)\beta^{(i)} = C_n^{GDP}\beta^{(i)}, \quad (2)$$

where $GDP_{n}^{Q\_N}$ denotes Poland's GDP in quarter *n*. In accordance with (2) we assume that the linear relation, estimated on the basis of annual data in regression (1), is transferable to the case of data with quarterly frequency. $GDP_{n}^{Q\_R}$ is defined in this manner is subject to unbiased estimation:

$$EST\_GDP_{ni}^{Q\_R} = C_n^{GDP}\hat{\beta}_{OLS}^{(i)}. \quad (3)$$

The variance of estimator (3) is provided by the formula:

$$V(EST\_GDP_{ni}^{Q\_R}) = \sigma_{ii}^2 C_n^{GDP} (X'X)^{-1} C_n^{GDP\prime}.$$

The approach presented above also allows to estimate the rate of GDP changes in quarter *n* in the *i-th* region in relation to the corresponding quarter of the previous year, designated as $\Delta GDP_{ni}^{Q\_R}$:

$$\Delta GDP_{ni}^{Q\_R} = \left(\frac{GDP_{ni}^{Q\_R}}{GDP_{n-4i}^{Q\_R}} - 1 \right) 100 = \left(\frac{\frac{1}{4}\beta_{0i} + \beta_{1i} GDP_n^{Q\_N}}{\frac{1}{4}\beta_{0i} + \beta_{1i} GDP_{n-4}^{Q\_N}} - 1 \right) 100. \qquad (4)$$

According to formula (4), $\Delta GDP_{ni}^{Q\_R}$ is a non-linear, differentiable function of regression (1) parameters, which is subject to the compliant and asymptotically normal estimation according to the naturally defined estimator:

$$EST\_\Delta GDP_{ni}^{Q\_R} = \left(\frac{EST\_GDP_{ni}^{Q\_R}}{EST\_GDP_{n-4i}^{Q\_R}} - 1 \right) 100 = \left(\frac{\frac{1}{4}\hat{\beta}_{0i} + \hat{\beta}_{1i} GDP_n^{Q\_N}}{\frac{1}{4}\hat{\beta}_{0i} + \hat{\beta}_{1i} GDP_{n-4}^{Q\_N}} - 1 \right) 100. \qquad (5)$$

The asymptotic variance of estimator (5) is provided by the formula:

$$V(EST\_\Delta GDP_{ni}^{Q\_R}) = \sigma_{ii}^2 C_n^{\Delta GDP}(\beta^{(i)})(X'X)^{-1} C_n^{\Delta GDP}(\beta^{(i)})',$$

where

$$C_n^{\Delta GDP}(\beta^{(i)}) = \left(\frac{\partial \Delta GDP_{ni}^{Q\_R}}{\partial \beta_{0i}}, \frac{\partial \Delta GDP_{ni}^{Q\_R}}{\partial \beta_{1i}} \right).$$

For the estimation of $\Delta GDP_{ni}^{Q\_R}$ rate of changes a different approach may be adopted, applying the regression model similar to that defined in equation (1), in which, however, the linear relation between the rates of changes is examined directly. Let us thus consider alternatively the following regression model:

$$\Delta GDP_{ti}^{A\_R} = \alpha_{0i} + \alpha_{1i} \Delta GDP_t^{A\_N} + \xi_{ti}, i = 1,...,m, t = 1,...,T, \qquad (6)$$

where $\Delta GDP_{ti}^{A\_R}$ denotes the rate of GDP changes, expressed in percentage, in the *i-th* region from year *t*-1 to year *t*, whereas $\Delta GDP_t^{A\_N}$ denotes the rate of Poland's GDP changes from year *t*-1 to year *t*. Similarly to the case of ε_{ti} in (1) we assume that $\xi_{ti} \sim N(0, \omega_{ii}^2)$, $\mathrm{cov}(\xi_{ti}, \xi_{tj}) = \omega_{ij}^2$ and $\mathrm{cov}(\xi_{ti}, \xi_{sj}) = 0$, if $t \neq s$. Let $\hat{\alpha}_{OLS}^{(i)} = (Z'Z)^{-1} Z'u^{(i)}$ denote the OLS estimator of the vector of parameters $\alpha^{(i)} = (\alpha_{0i}, \alpha_{1i})'$, where:

$$u_{[Tx1]}^{(i)} = \begin{bmatrix} \Delta GDP_{1i}^{A\_R} \\ \vdots \\ \Delta GDP_{Ti}^{A\_R} \end{bmatrix}, \; Z_{[Tx2]} = \begin{bmatrix} 1 & \Delta GDP_{1}^{A\_N} \\ \vdots & \vdots \\ 1 & \Delta GDP_{T}^{A\_N} \end{bmatrix}.$$

Quarterly GDP changes in individual region may be treated as linear functions of regression (6) structural parameters. Thus, we assume that in quarter *n* for the *i-th* region, the rate of GDP changes in relation to the corresponding quarter of the previous year is expressed by the formula:

$$\Delta GDP_{ti}^{A\_R} = \alpha_{0i} + \alpha_{1i}\Delta GDP_{t}^{A\_N} = (1, \Delta GDP_{t}^{A\_N})\alpha^{(i)} = S_n\alpha^{(i)}. \qquad (7)$$

The function of parameters (7) is subject to unbiased estimation according to the formula:

$$EST\_\Delta GDP_{ni}^{Q\_R} = \hat{\alpha}_{0i} + \hat{\alpha}_{1i}\Delta GDP_{n}^{Q\_N} = S_n\hat{\alpha}^{(i)}. \qquad (8)$$

The variance of estimator (8) is provided by the formula:

$$V(EST\_\Delta GDP_{ni}^{A\_R}) = \omega_{ii}^2 S_n (Z'Z)^{-1} S_n'.$$

3. Discussion of obtained results

In this part, the results of quarterly GDP estimates for regions are presented. Data related to the annual GDP required in the applied approach were derived from the Polish Central Statistical Office (CSO) publication on *Gross domestic product – Regional accounts* (various issues from 2003–2014 were used), whereas data concerning quarterly GDP in Poland come from the Eurostat. The period we have studied is not homogeneous due to the occurrence of important structural changes. First of all, the new administrative division of Poland into 16 regions entered into force in 2000 and the GDP was first converted by CSO to new regions in 1995. Secondly, the real GDP growth broken down by regions has been available since 2004 and thirdly, in the period 1995–2002, an identical GDP deflator is adopted for estimates of real changes in individual regions and in Poland.

Accordingly, analyses were conducted pursuing three baseline variants of observations with annual frequency. In the first variant, the period of 1995-2012 is subject to analysis. In this variant it is assumed that in 1995–2003 the GDP deflator in Poland and in regions is identical. In turn, the corresponding deflators for 2004-2012 vary. The second variant also considers the period of 1995–2012 and throughout its entire duration an identical GDP deflator for Poland and for particular regions was adopted. Thus, in the second variant we assume that the growth of prices in individual regions and in Poland does not differ significantly. In the third variant differing deflators were adopted. We conducted the analysis for 2003–2012 due to the availability of statistical data.

Tables 1, 2 and 3 contain estimates of the quarterly value of the GDP in regions. The results refer to the application of regression model (1), where formula (3) of the quarterly GDP estimator for the selected regions plays a key role. In Tables 1, 2 and 3 graphs of score assessments, obtained according to (3), as well as the limits of small sample size 95% confidence intervals are presented. The confidence intervals have been built on the basis of the estimator variance formula (3) and the Classical Linear Regression Model assumptions, which means that the distribution of the standardised version of estimator (3), which provides the basis for inference about the linear function of regression (1) parameters, is a *t*-Student distribution. The tables contain results of analyses making use of empirical data in the first variant (Table 1), second variant (Table 2) and third variant (Table 3). The numerical values of quarterly GDP estimates in regions (in PLN million, expressed in 2012 prices) are

contained in Tables 8, 9 and 10, respectively, for each of the considered empirical data variants. The dispersion of confidence intervals of the estimated GDP for regions is relatively small compared to the range of values in those categories. In each regions confidence intervals are most dispersed at the beginning of the sample, i.e. in 1995-1996 in the case of variants 1 and 2 and in 2002-2003 in variant 3. In each of the variants, the GDP of the Łódzkie region is estimated with the highest precision. The lowest precision estimation is obtained for the GDP of the Opolskie region, where confidence intervals are relatively the most dispersed taking into account the scale used in graphs included in Tables 1, 2 and 3. Comparing the GDP estimates of regions obtained according to various data variants, it should be stated that no significant differences exist, both in terms of scoring assessments and the dispersion of confidence intervals. The first and the second variant provide practically the same GDP estimates for regions, whereas in the case of the third variant, due to the lower number of observations the dispersion of confidence intervals is slightly higher.

The results of GDP change rate estimation for regions are included in Tables 4, 5, 6 and 7. Tables 4, 5 and 6 present the results of the application of regression model (1) and Table 7 – of regression model (7). In the case of regression equation (1) the GDP change rate is a non-linear function of parameters, which is subject to compliant and asymptotic estimation in accordance with estimator formula (5). According to the alternatively adopted regression equation (7), the GDP growth rate of a given region is a linear function of parameters, subject to unbiased estimation applying estimator (8), whose standardised form follows the small sample size *t*-Student distribution. Tables 4, 5, 6 and 7 present sample assessments and the limits of 95% confidence intervals for GDP rate of change in regions. Tables 4, 5 and 6 contain limits of estimated asymptotic confidence intervals, based on normal distribution, whereas Table 7 presents the course of the limits of small sample size confidence intervals built with the use of *t*-Student distribution. Results of the application of regression model (1) are analysed in the three variants of empirical data described above. Table 4 refers to the first variant, Table 5 – to the second variant and Table 6 – to the third variant. Results obtained in regression model (7) are presented only with the first data variant, since the application of the remaining variants would lead to very similar results.

The presented versions of GDP change rate estimates generate similar results in terms of quality. A major difference consists in the scale of statistical uncertainty, significantly different depending on the approach – either with the application of regression equation (1) or the alternative regression equation (7). In the case of regression (1) the dispersion of

confidence intervals presented in Tables 4, 5 and 6 is insignificant. In those cases a strong concentration of estimator (5) distribution is observed in periods characterised by slowdown of economic growth or recession (2001-2002 and 2008-2009). At the same time, the analysed confidence intervals are becoming relatively more dispersed in periods of expansion of the Polish economy (2004-2007). This effect confirming a strong dependence of estimator dispersion on the position of the central tendency of its distribution should be interpreted so that the approach using regression (1) will provide a more precise estimate of GDP change rate for regions in periods characterised by stagnation or activity slowdown. In turn, in the periods when economy will develop dynamically, the estimated values of GDP change rate for regions will be burdened with higher statistical uncertainty.

Like in the case of GDP level estimation, Tables 4,5 and 6 indicate that the GDP change rate of the Łódzkie region is estimated the most precisely, since in this case confidence intervals are most strongly concentrated around the central tendency. Similarly, the Opolskie region is characterised by the highest uncertainty of change rate estimation.

An explicitly dissimilar course and dispersion of confidence intervals was obtained on the basis of regression model (7) and estimator (8); c.f. Table 7. In this approach, the GDP change rate is estimated with a high uncertainty, as confirmed by strongly dispersed confidence intervals. The paths of scoring assessments of GDP change rate for regions basically run across areas of lower values than in the case of the approach presented in Tables 4, 5 and 6. Thus, regression (7) is used to obtain assessments of GDP change rate for regions. It turns out to be, on average, lower than when regression (1) is applied and the estimates are burdened with a significantly higher statistical uncertainty. This entails, in particular, a slightly different picture of the regional diversity of economic activity in Poland and its transformations over time. This picture emerges particularly clearly in the analysis of results for the economic slowdown period that took place in Poland in 2000-2002. In accordance with the value of estimated GDP rate of growth in regions, included in Tables 11, 12, 13 and 14, it can be said that both methods provide different pictures of economic growth slowdown in 2000-2002. The approach based on regression equation (1) (Table 11, 12 and 13), independently of the adopted empirical data variant, confines the negative values of GDP change rate for regions to 2001 Q4. In the remaining quarters of the examined period, irrespective of empirical data variants, estimations point to a positive rate of GDP change rate in regions. The decline in GDP for regions in 2001 Q4 estimated in regression model (1) is regionally very homogeneous and the values of GDP change rate range from -0.42% year-on-year in the case

of the Mazowieckie region to -0.22 in the case of the Zachodniopomorskie region. A different picture of this phase of the business cycle in the Polish economy is provided by estimates of GDP change rate in regions obtained with the use of regression (7) and estimator (8); c.f. Table 14. In this approach GDP change rates are strongly regionally diversified. 2001 Q4 does not post GDP decline in all regions. The GDP change rate in this quarter is estimated as positive in the Pomorskie (1.6% year-on-year), the Dolnośląskie (0.27% year-on-year) and the Kujawsko-Pomorskie (0.02% year-on-year) regions. In the remaining regions the GDP decline is strongly diversified regionally and its values range from -3.70% year-on-year in the case of the Opolskie region to -0.08% year-on-year in the case of the Łódzkie region . Results presented in Table 14 also indicate a strong regional diversity of the duration of the recession in 2001-2002 and of its depth. According to the estimates, the GDP decline in the Opolskie region lasted the longest, namely the seven quarters spanning from 2001 Q2 to 2002 Q4. In the case of the Lubelskie, the Łódzkie and the Zachodniopomorskie regions, the GDP decreased over two quarters, i.e. in 2001 Q4 and 2002 Q1. The Świętokrzyskie and the Zachodniopomorskie regions are characterised by strong volatility of economic growth in 2001 and 2002. In 2001 Q2, the GDP in these regions slightly increased, whereas in 2001 Q4 and 2002 Q1, estimates show a continued GDP decline.

The above-presented description of the period of slowdown in economic activity and recession in 2001 and 2002 is strongly regionally diversified and as such is burdened with large uncertainty reflected in the strong dispersion of confidence intervals as presented in figures in Table 7.

4. Summary

This study aimed at estimating the quarterly GDP value in particular regions on the basis of annual series of this variable on a regional scale and the GDP values for the entire Polish economy. The method proposed consists of two stages. In the first step we estimate parameters of the system of sixteen equations linking the GDP in Poland with the GDP in each of its regions. According to the proposed approach, in the second step, on the basis of the estimates from the first step, the estimated quarterly GDP for individual regions is treated as a function of regression parameters. Thus, wide-ranging inference is possible, including the determination of estimation errors and confidence intervals.

In the analysed period of 1995-2012, changes in the statistics of national accounts and in the administrative division took place, which encouraged us to consider three variants of empirical data. The results obtained in variants based on the same period of analysis (1995-2012) remain the same even with alternative assumptions concerning price developments. In turn, the variant for which data on GDP volume growth are available (2003-2012) provided results burdened with a higher statistical uncertainty.

The scoring assessments of GDP levels obtained in each of the three data variants basically do not differ from one another. Some differences may be perceived in the dispersion of confidence intervals. In the case of the shorter analysis period, estimation errors are higher, which leads to confidence intervals with greater dispersion. In the model-based approach using GDP growth rates, less precise results were obtained.

Results obtained on the basis of GDP levels generate precise estimates of the GDP change rate in regions, however, the regional diversification of the rate is insignificant. In accordance with this approach, regions with a higher share of added value creation in the GDP of the whole country demonstrate higher volatility of growth as measured by the GDP change rate. In the model-based approach where growth rates were used, relatively high regional diversification of growth rates was obtained, but the results are burdened with significant statistical uncertainty.

More developed regions, with a relatively high GDP per capita, high employment rates and a high share of the service sector, contribute more to the GDP of the country than regions of the so-called "Eastern Wall", with a high share of agriculture and relatively high unemployment

rates. Therefore, in the group of more developed regions the estimated growth rates are higher than those corresponding to the GDP change rate in the whole economy.

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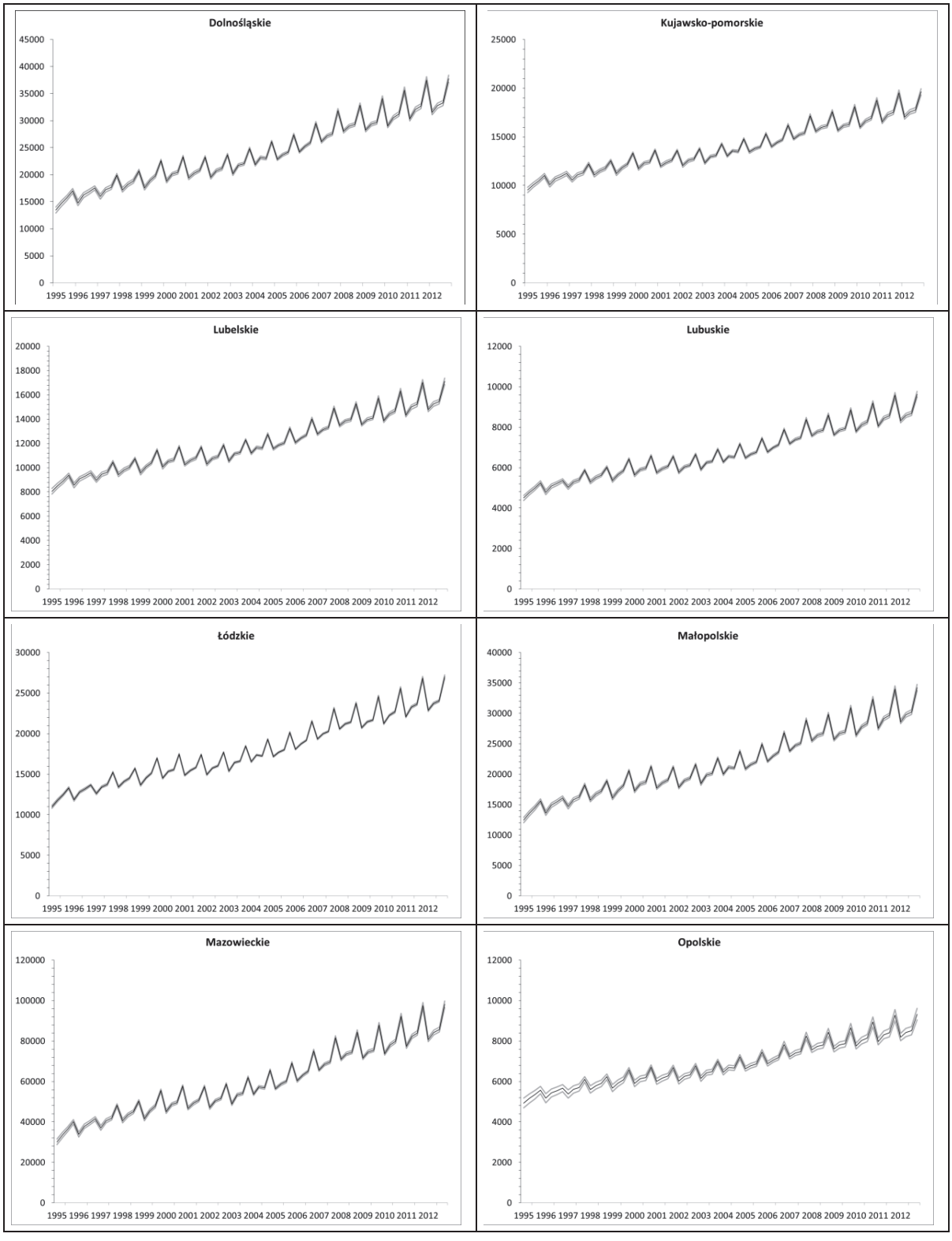
Table 1. Sample assessments and limits of small sample size (based on *t*-Student distribution) 95% confidence intervals, for quarterly region GDP, obtained on the basis of formula (3). Results obtained based on the first variant of empirical data.



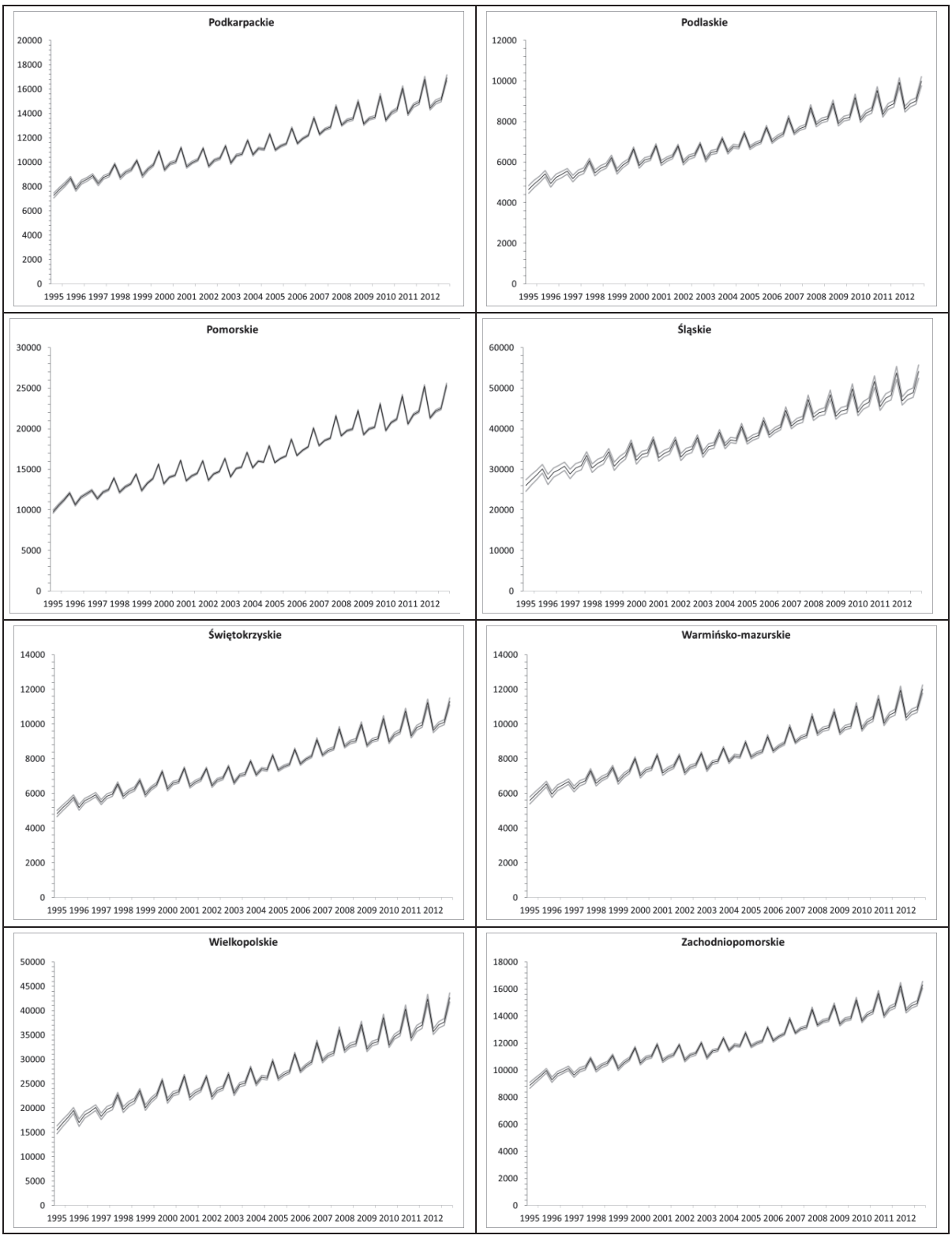

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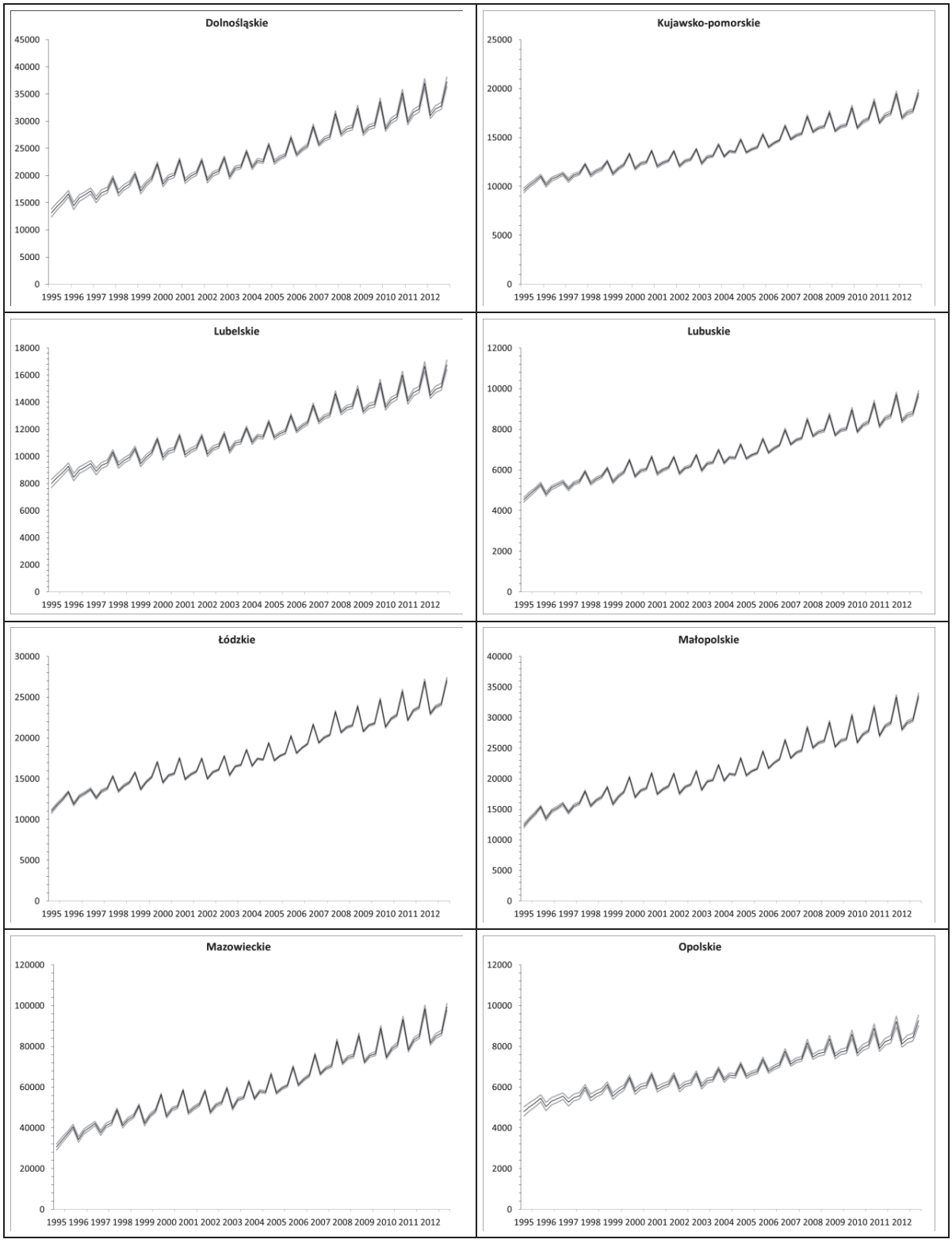
Table 2. Sample assessments and limits of small sample size (based on *t*-Student distribution) 95% confidence intervals for quarterly region GDP, obtained on the basis of formula (3). Results obtained based on the second variant of empirical data.



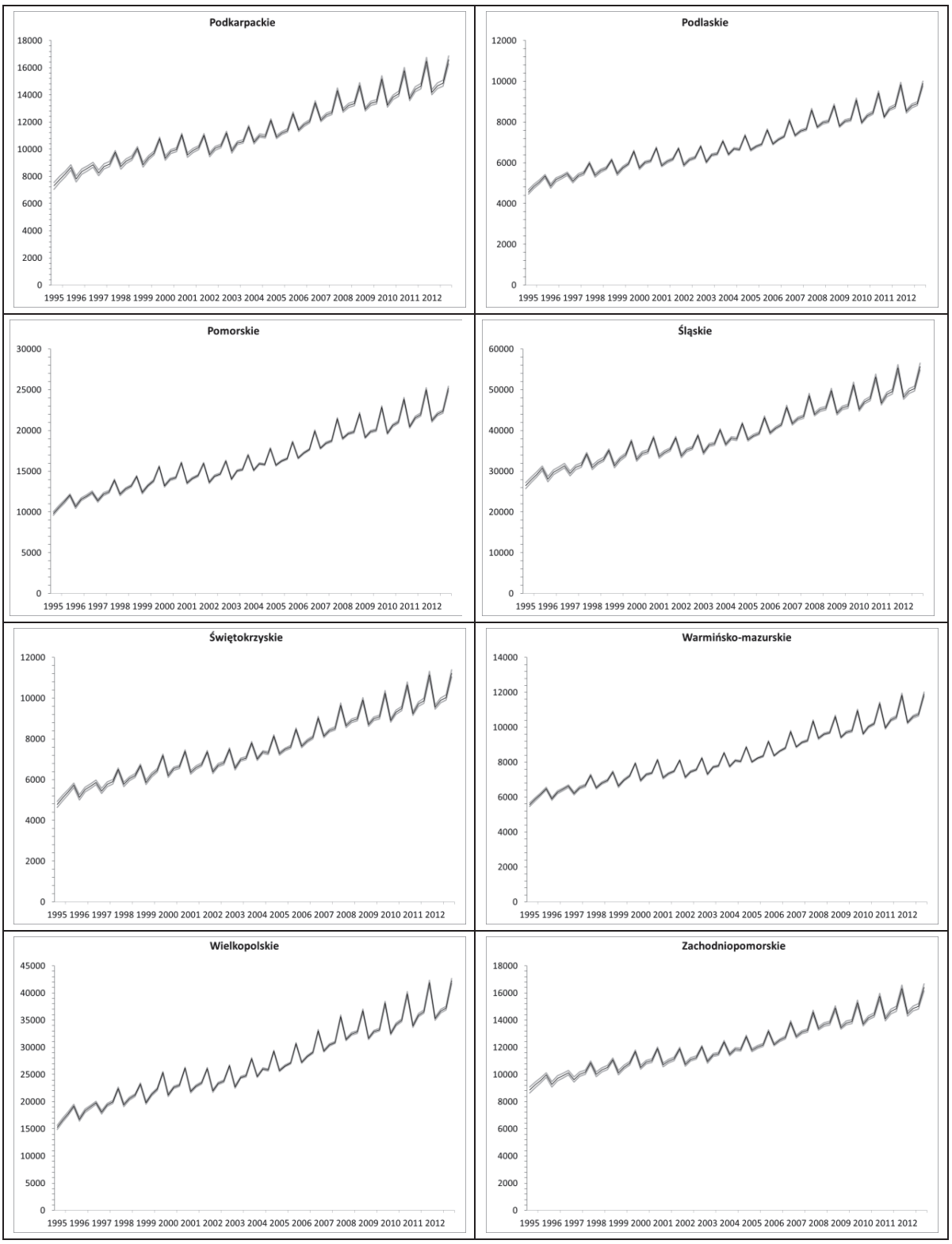

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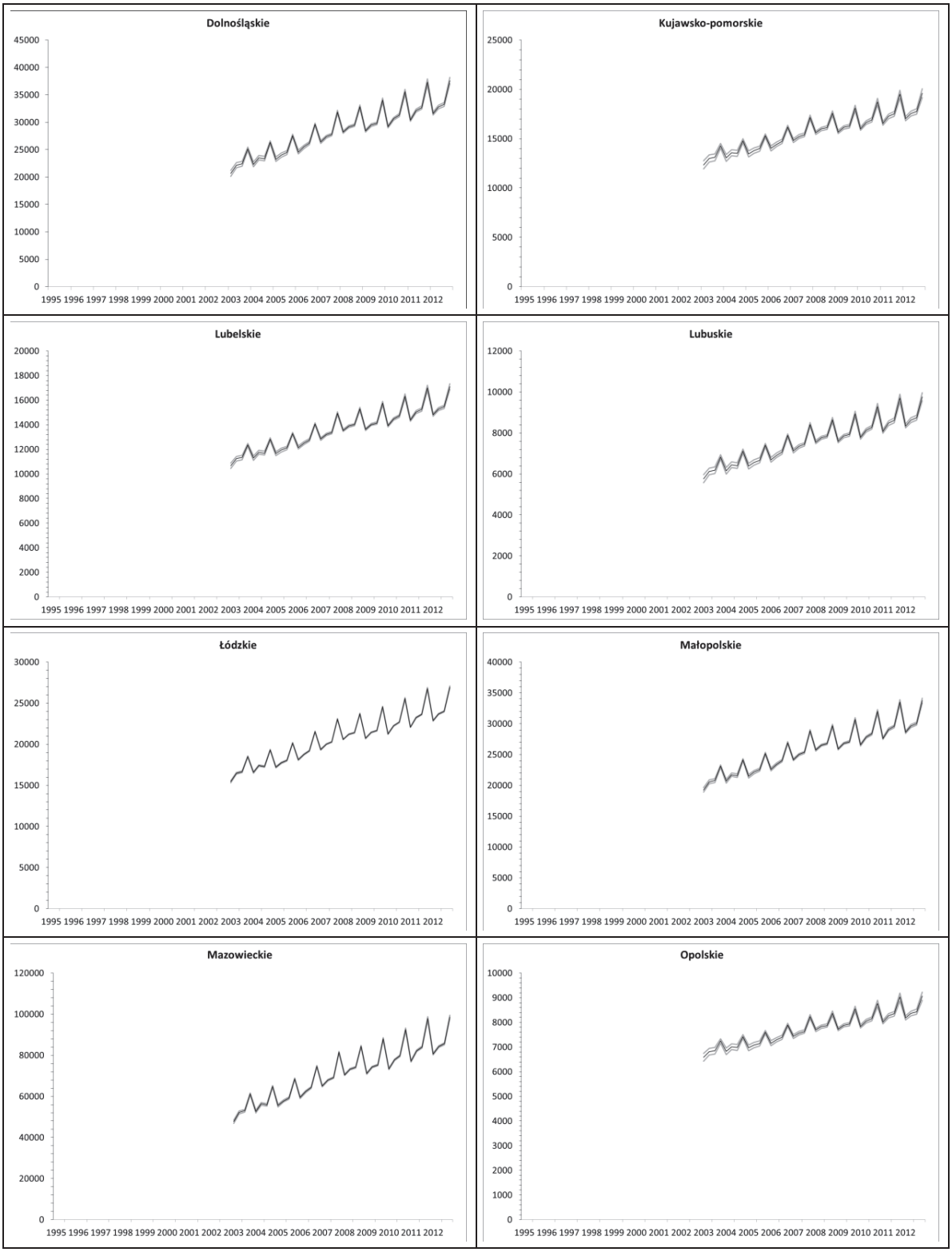
Table 3. Sample assessments and limits of small sample size (based on *t*-Student distribution) 95% confidence intervals for quarterly region GDP, obtained on the basis of formula (3). Results obtained based on the third variant of empirical data.



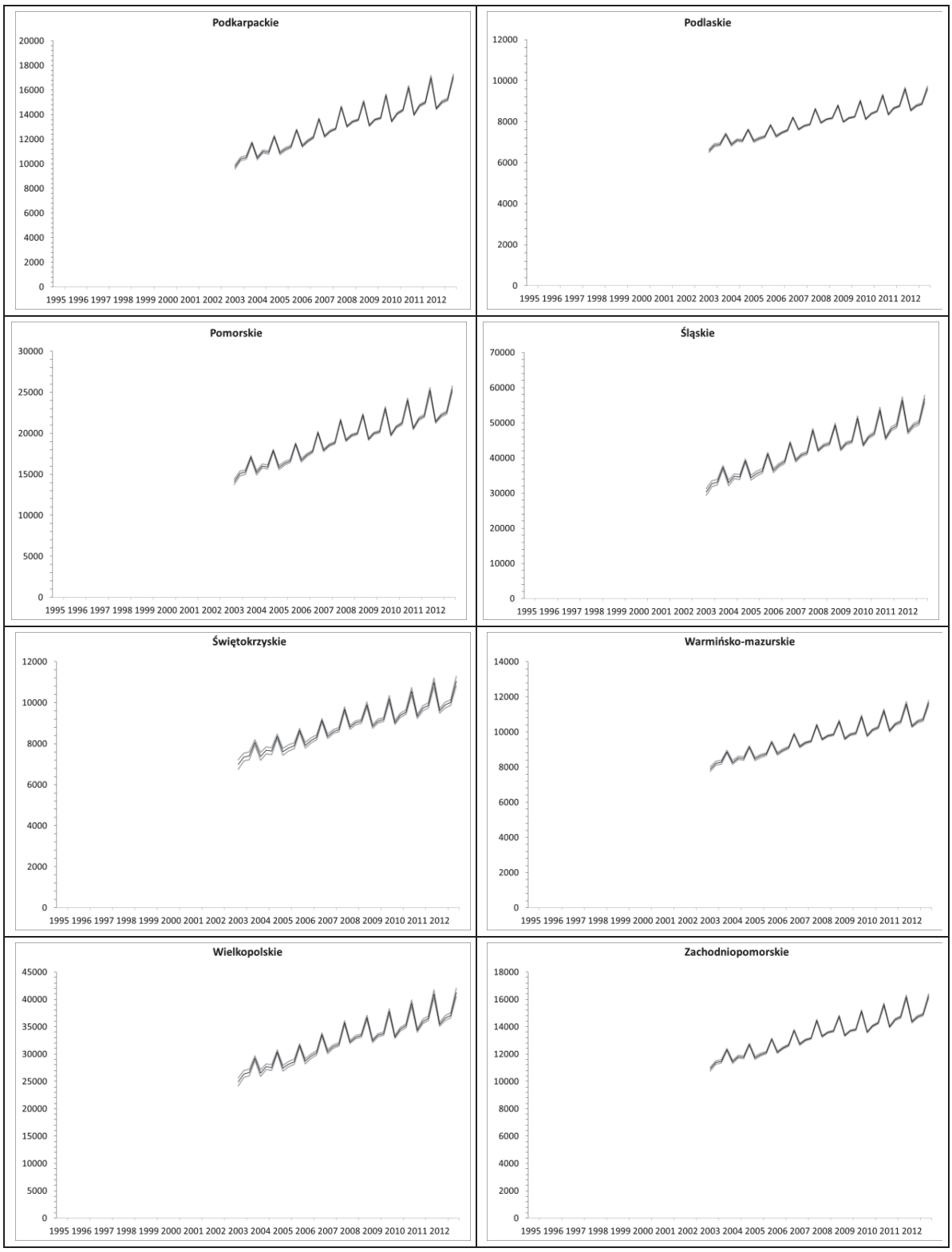

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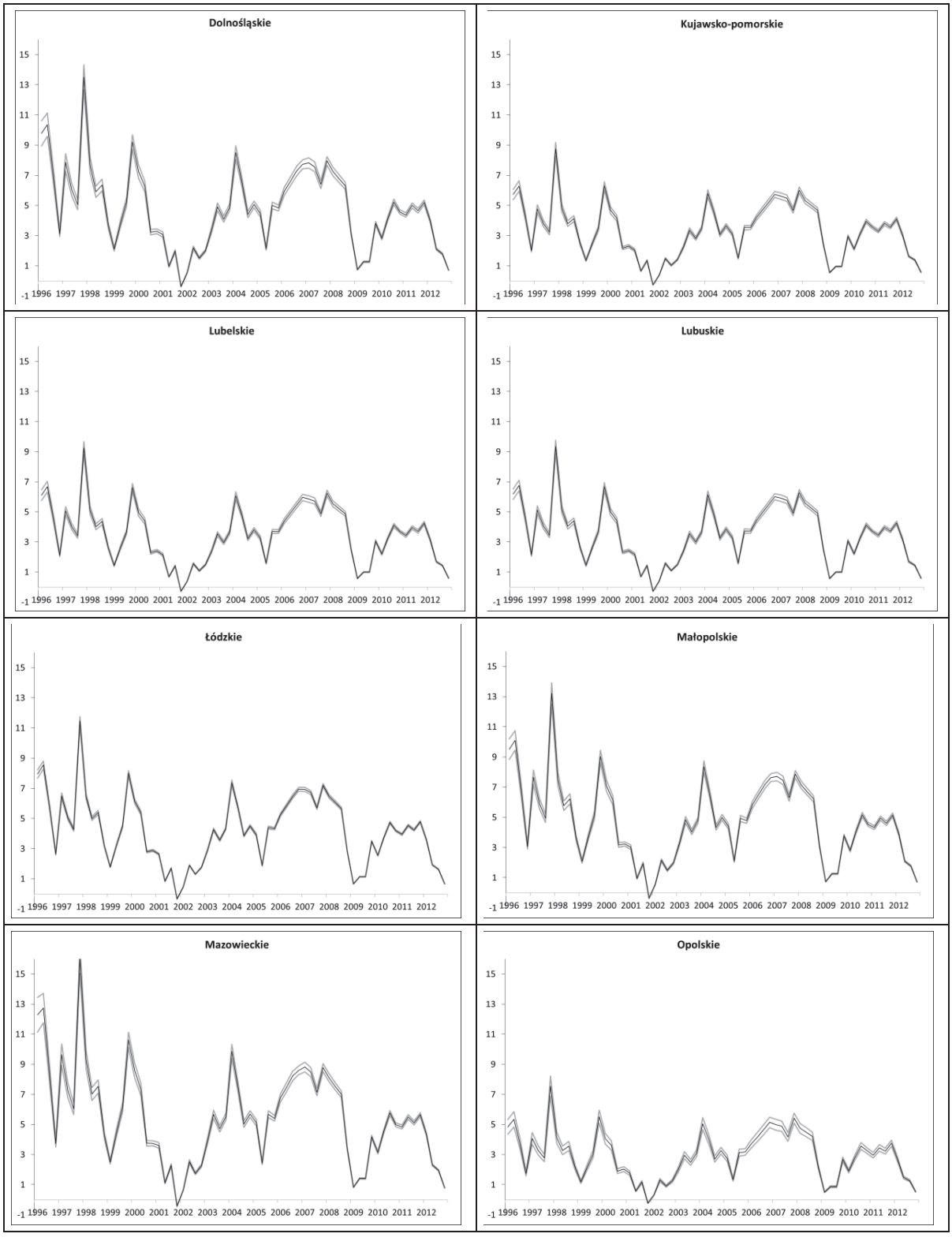
Table 4. Sample assessments and limits of asymptotic (based on normal distribution) 95% confidence intervals for quarterly year-on-year changes in region GDP, obtained on the basis of formula (5). Results obtained based on the first variant of empirical data.



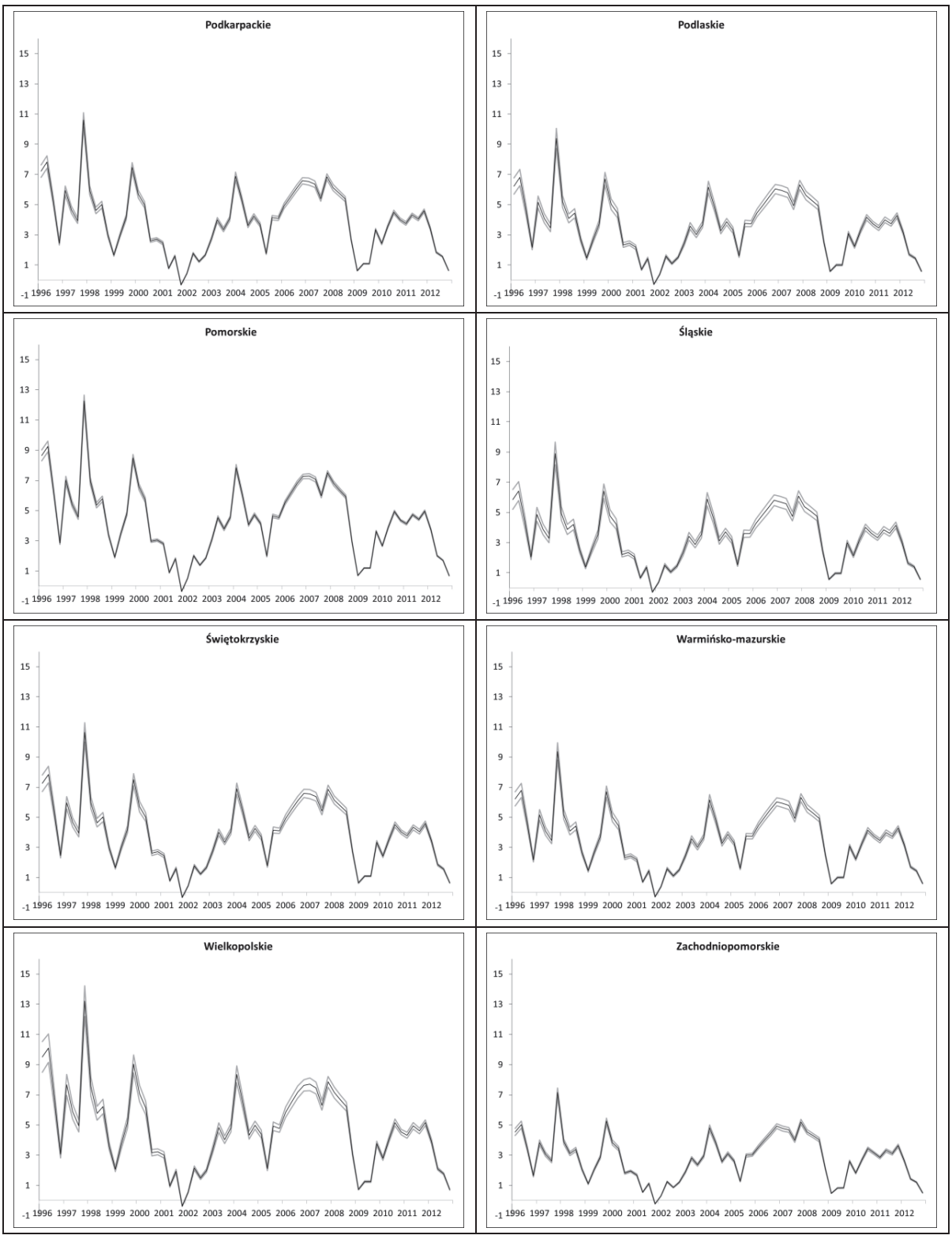

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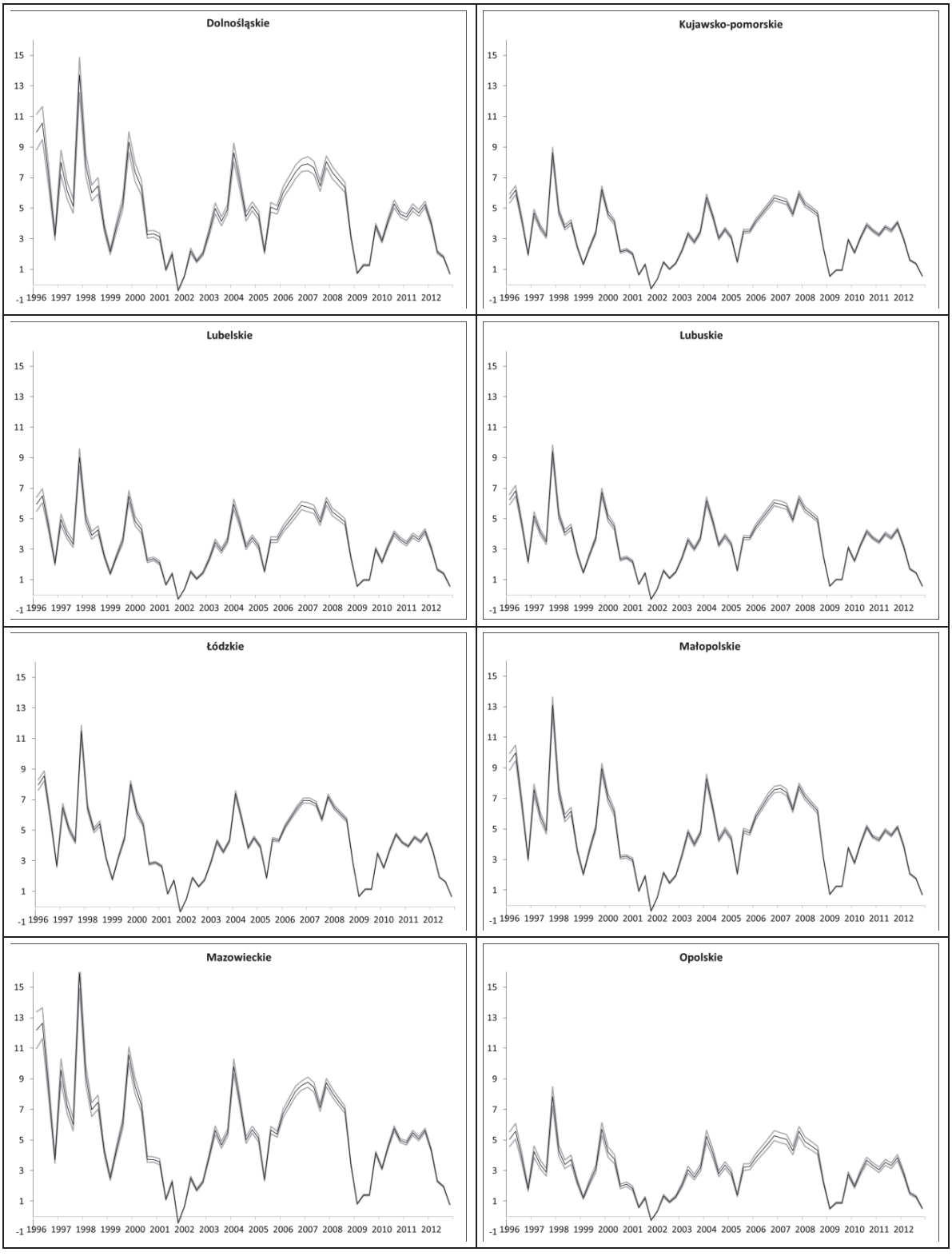
Table 5. Sample assessments and limits of asymptotic (based on normal distribution) 95% confidence intervals for quarterly year-on-year changes in region GDP, obtained on the basis of formula (5). Results obtained based on the second variant of empirical data.



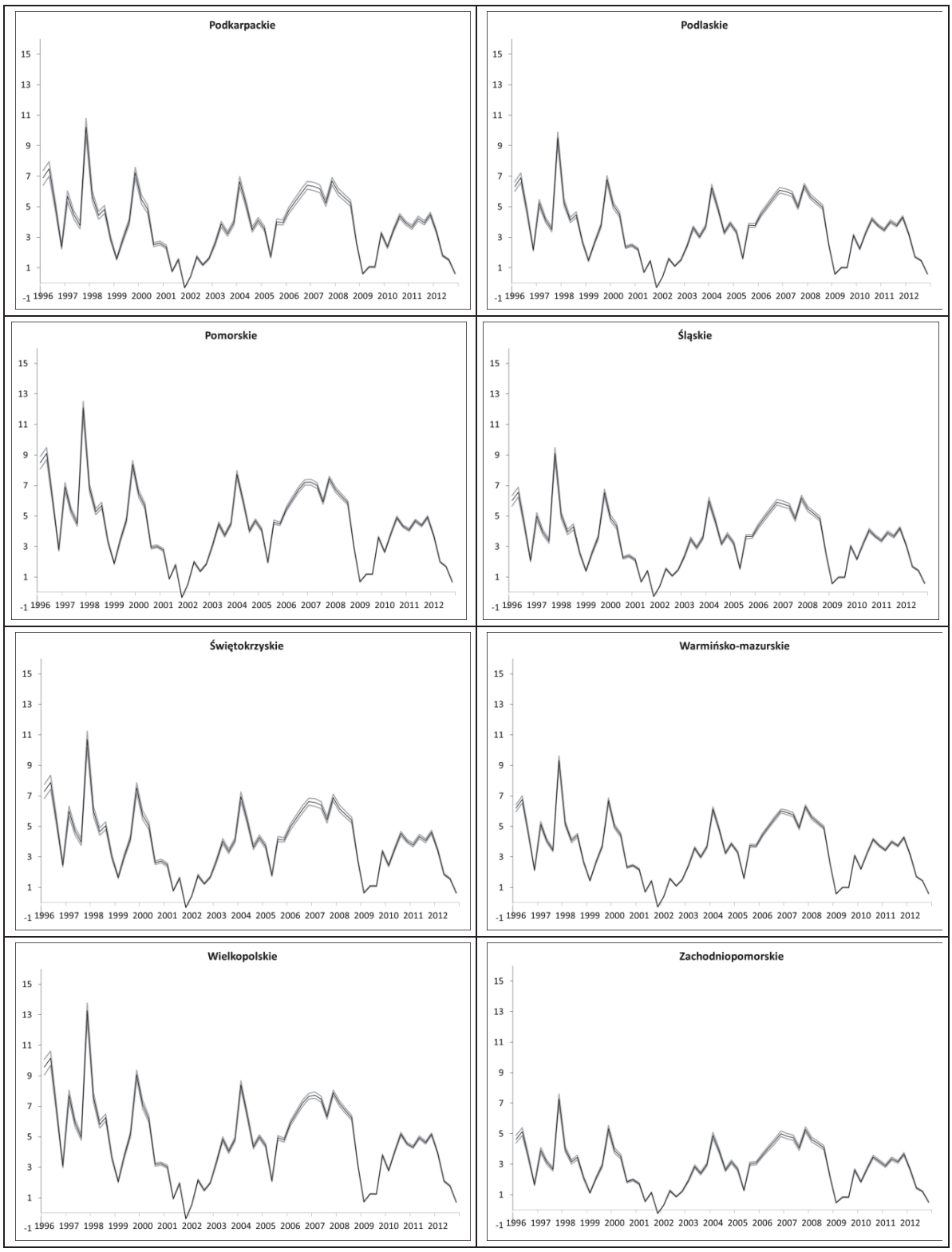

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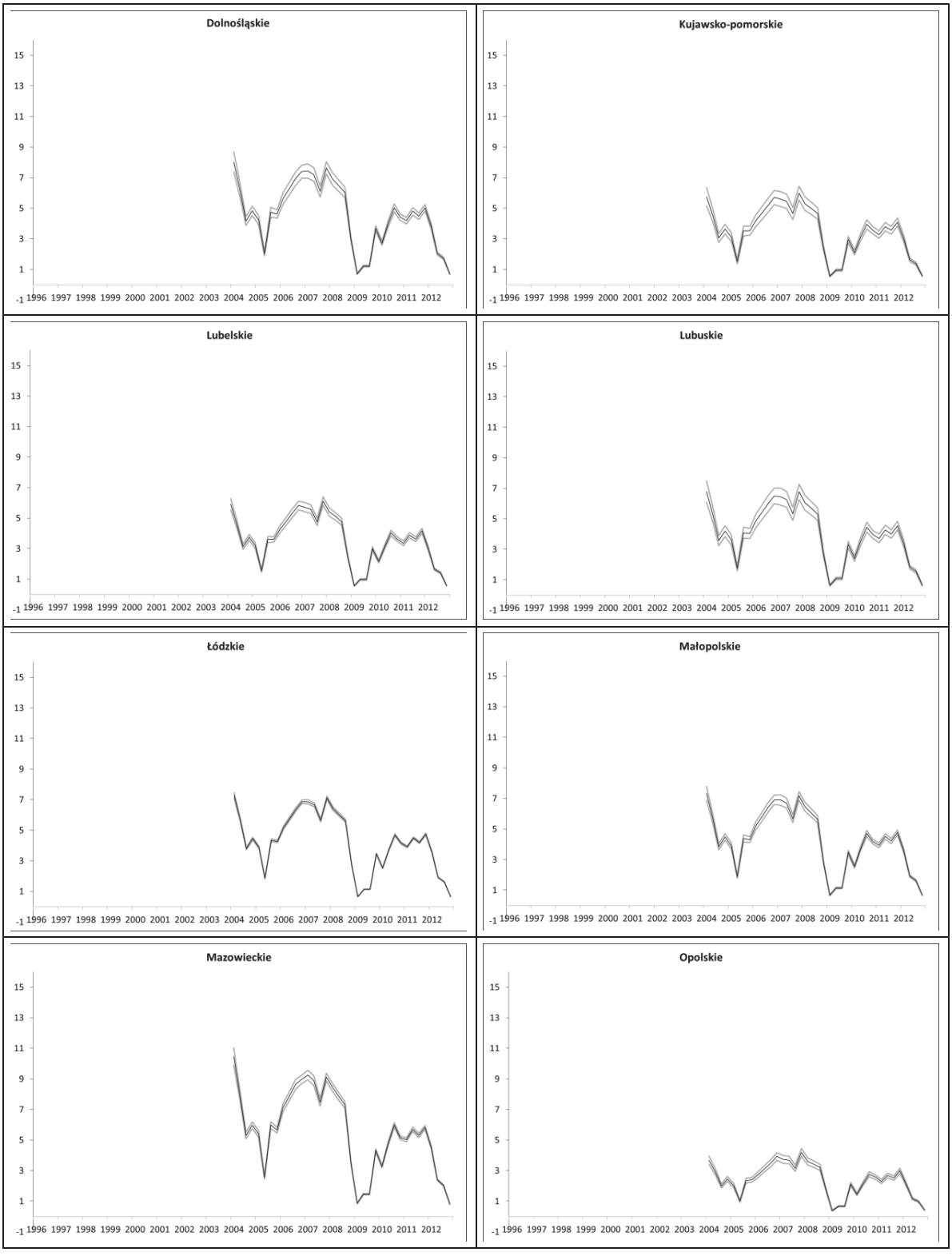
Table 6. Sample assessments and limits of asymptotic (based on normal distribution) 95% confidence intervals for quarterly year-on-year changes in region GDP, obtained on the basis of formula (5). Results obtained based on the third variant of empirical data.



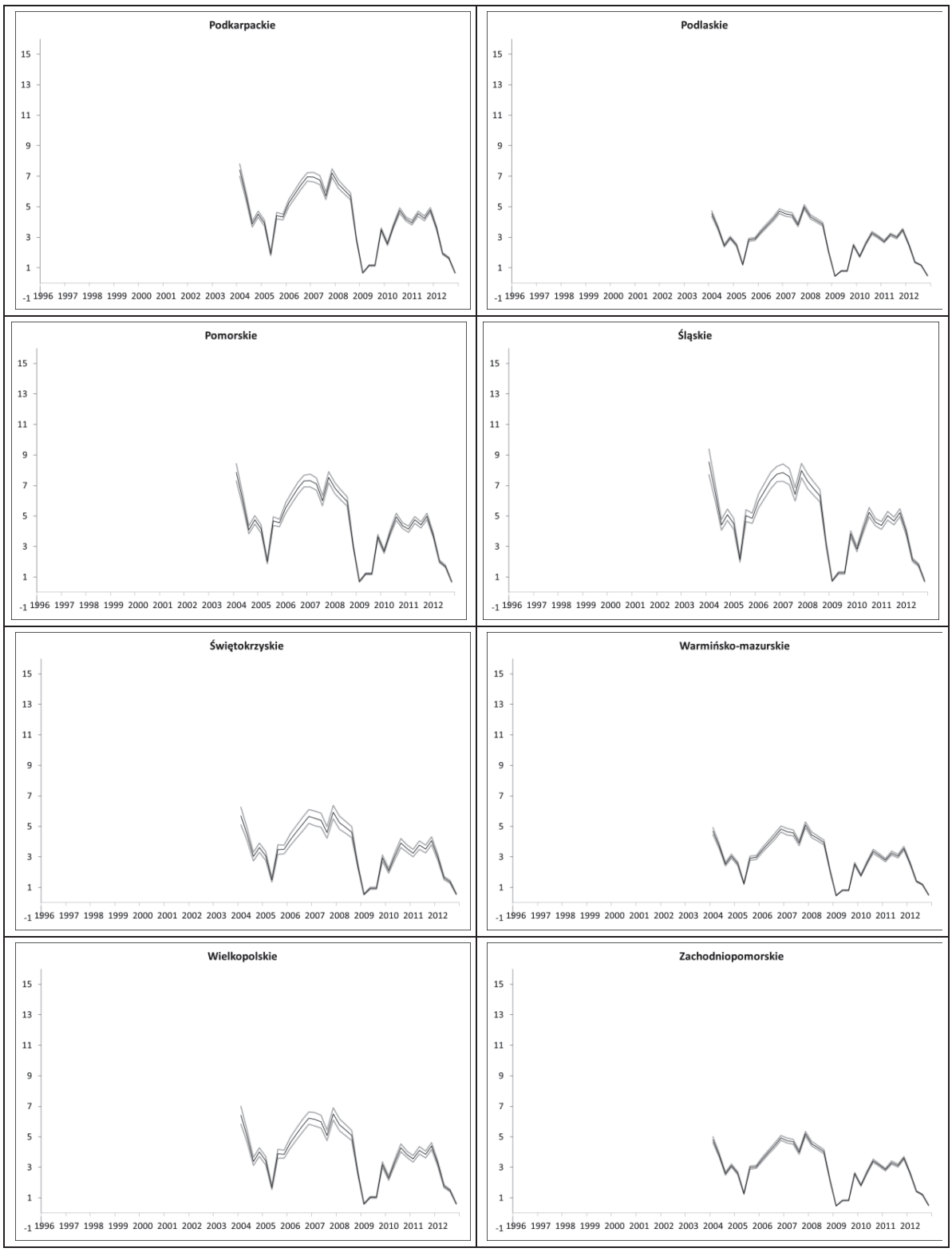

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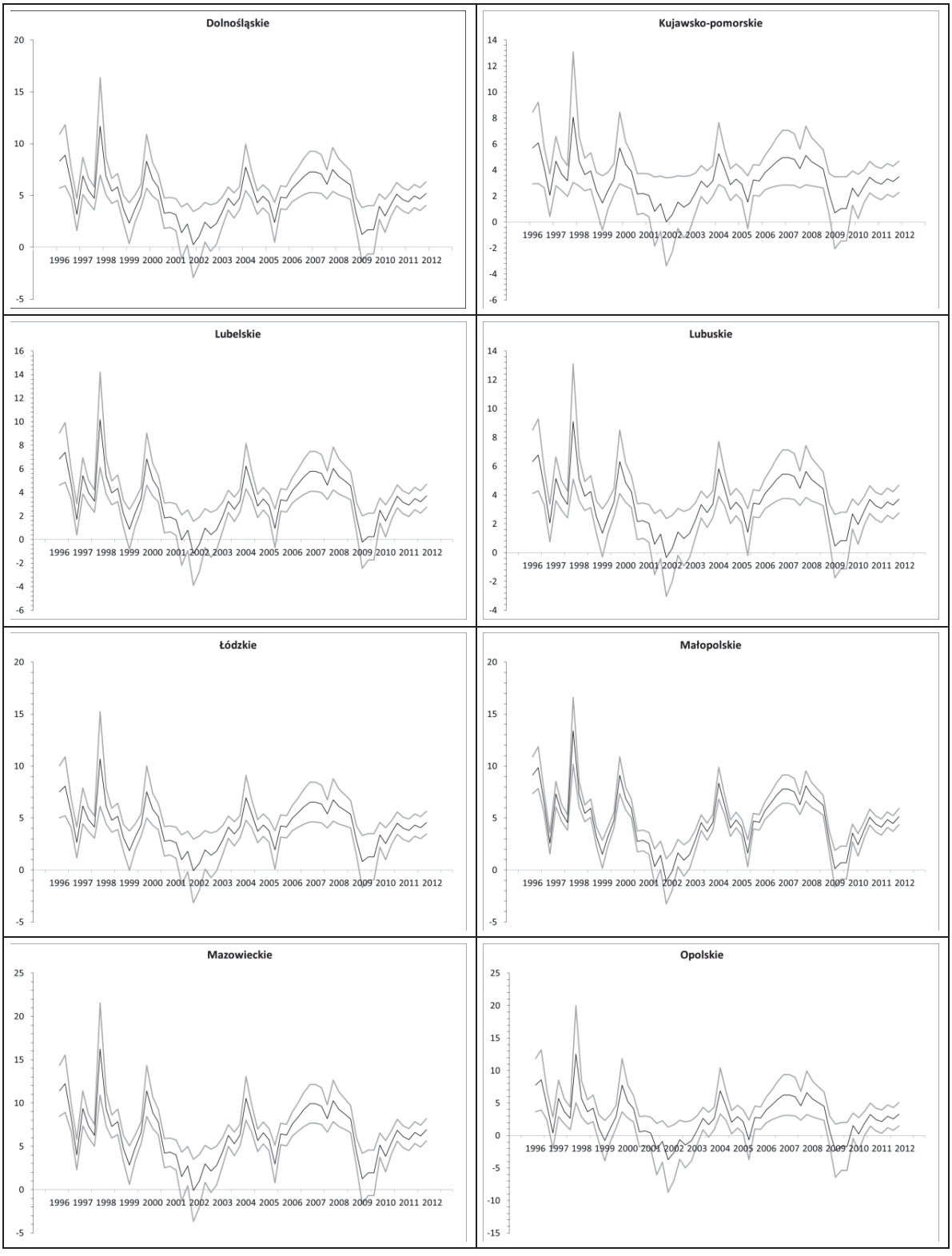
Table 7. Sample assessments and limits of small sample size (based on *t*-Student distribution) 95% confidence intervals for quarterly year-on-year changes in region GDP, obtained on the basis of formula (8). Results obtained based on the first variant of empirical data.



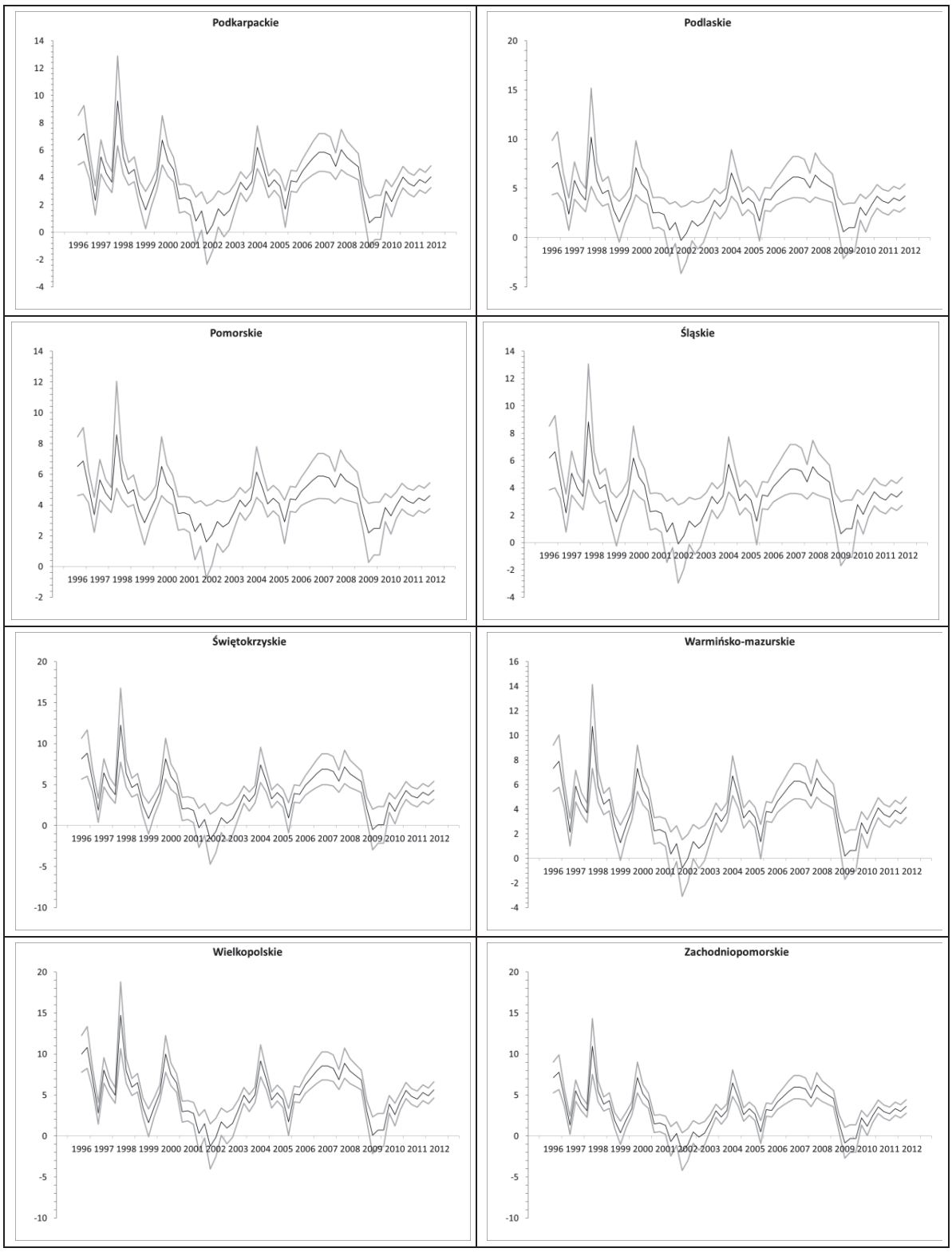

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Table 8 Estimated values of quarterly region GDP, obtained on the basis of formula (3). Results obtained based on the first variant of empirical data.

	Dolnośląskie	Kujawsko-Pomorskie	Lubelskie	Lubuskie	Łódzkie	Małopolskie	Mazowieckie	Opolskie	Podkarpackie	Podlaskie	Pomorskie	Śląskie	Świętokrzyskie	Warmińsko-mazurskie	Wielkopolskie	Zachodniopomorskie
1995Q1	13481.51	9548.96	8053.99	4506.67	10967.72	12446.28	30121.94	4935.80	7264.33	4652.98	9789.46	26027.95	4841.79	5605.08	15523.58	8886.09
1995Q2	14711.46	10059.59	8512.88	4766.55	11780.49	13550.47	33572.98	5158.22	7753.61	4923.34	10579.29	27449.46	5169.03	5930.15	16900.19	9262.14
1995Q3	15756.27	10493.36	8902.70	4987.31	12470.92	14488.45	36504.54	5347.15	8169.24	5153.00	11250.23	28656.99	5447.01	6206.29	18069.58	9581.59
1995Q4	17029.90	11022.12	9377.89	5256.42	13312.56	15631.84	40078.12	5577.47	8675.89	5432.95	12068.11	30128.98	5785.87	6542.90	19495.06	9971.00
1996Q1	14800.19	10096.43	8545.98	4785.30	11839.12	13630.12	33821.92	5174.26	7788.90	4942.84	10636.27	27552.00	5192.63	5953.60	16999.49	9289.27
1996Q2	16235.37	10692.26	9081.45	5088.54	12787.52	14918.55	37848.81	5433.79	8359.82	5258.31	11557.90	29210.71	5574.48	6332.91	18605.80	9728.08
1996Q3	16841.37	10943.85	9307.55	5216.59	13187.98	15462.59	39549.14	5543.37	8600.89	5391.51	11947.05	29911.09	5735.71	6493.07	19284.05	9913.36
1996Q4	17564.48	11244.06	9577.34	5369.38	13665.82	16111.75	41578.06	5674.13	8888.54	5550.46	12411.40	30746.81	5928.09	6684.19	20093.38	10134.45
1997Q1	15963.42	10579.36	8979.99	5031.08	12607.81	14674.41	37085.76	5384.61	8251.64	5198.53	11383.26	28896.40	5502.12	6261.04	18301.42	9644.93
1997Q2	17214.01	11098.56	9446.58	5295.32	13434.22	15797.12	40594.70	5610.76	8749.12	5473.42	12186.34	30341.76	5834.85	6591.56	19701.12	10027.29
1997Q3	17691.99	11297.00	9624.91	5396.32	13750.08	16226.23	41935.85	5697.19	8939.27	5578.48	12493.29	30894.19	5962.02	6717.89	20236.10	10173.44
1997Q4	19935.34	12228.36	10461.91	5870.33	15232.53	18240.19	48230.30	6102.86	9831.67	6071.59	13933.89	33486.92	6558.88	7310.79	22746.93	10859.34
1998Q1	17196.21	11091.17	9439.94	5291.56	13422.46	15781.15	40544.77	5607.54	8742.05	5469.51	12174.92	30321.19	5830.11	6586.86	19681.21	10021.85
1998Q2	18230.92	11520.74	9825.99	5510.19	14106.22	16710.05	43448.00	5794.65	9153.66	5696.95	12839.37	31517.06	6105.41	6860.32	20839.29	10338.21
1998Q3	18817.05	11764.08	10044.67	5634.04	14493.54	17236.24	45092.56	5900.64	9386.82	5825.78	13215.76	32194.46	6261.35	7015.23	21495.30	10517.42
1998Q4	20667.69	12532.40	10735.14	6025.07	15716.48	18897.65	50285.16	6235.29	10123.01	6232.57	14404.19	34333.34	6753.73	7504.35	23566.60	11083.25
1999Q1	17559.51	11242.00	9575.49	5368.33	13662.54	16107.30	41564.13	5673.24	8886.57	5549.36	12408.22	30741.08	5926.77	6682.87	20087.82	10132.93
1999Q2	18917.04	11805.59	10081.98	5655.17	14559.62	17326.01	45373.12	5918.72	9426.59	5847.76	13279.97	32310.03	6287.95	7041.66	21607.21	10547.99
1999Q3	19803.42	12173.59	10412.69	5842.45	15145.36	18121.76	47860.17	6079.01	9779.20	6042.60	13849.18	33334.46	6523.78	7275.92	22599.28	10819.00
1999Q4	22571.60	13322.83	11445.49	6427.35	16974.62	20606.88	55627.20	6579.58	10880.38	6651.07	15626.81	36533.77	7260.27	8007.54	25697.52	11665.37
2000Q1	18831.10	11769.91	10049.91	5637.01	14502.82	17248.86	45131.98	5903.18	9392.40	5828.87	13224.78	32210.70	6265.09	7018.94	21511.02	10521.72
2000Q2	20101.55	12297.36	10523.92	5905.45	15342.37	18389.40	48696.67	6132.92	9897.79	6108.13	14040.63	33679.02	6603.10	7354.72	22932.96	10910.16
2000Q3	20443.61	12439.37	10651.54	5977.72	15568.40	18696.48	49656.41	6194.77	10033.86	6183.32	14260.29	34074.35	6694.11	7445.12	23315.80	11014.74
2000Q4	23313.67	13630.92	11722.36	6584.15	17464.99	21273.07	57709.33	6713.77	11175.58	6814.18	16103.35	37391.41	7457.71	8203.66	26528.07	11892.25
2001Q1	19411.31	12010.80	10266.39	5759.60	14886.24	17769.74	46759.96	6008.10	9623.21	5956.41	13597.38	32881.28	6419.46	7172.29	22160.42	10699.11
2001Q2	20295.84	12378.02	10596.41	5946.50	15470.76	18563.83	49241.82	6168.05	9975.08	6150.84	14165.40	33903.58	6654.79	7406.07	23150.42	10969.56
2001Q3	20847.79	12607.17	10802.34	6063.12	15835.50	19059.34	50790.50	6267.86	10194.65	6272.16	14519.84	34541.49	6801.64	7551.95	23768.18	11138.32
2001Q4	23226.33	13594.65	11689.77	6565.70	17407.28	21194.66	57464.27	6697.98	11140.84	6794.98	16047.26	37290.47	7434.47	8180.58	26430.32	11865.55
2002Q1	19518.57	12055.33	10306.41	5782.27	14957.12	17866.03	47060.92	6027.50	9665.88	5979.98	13666.26	33005.25	6447.99	7200.64	22280.47	10731.91

2002Q2	20740.70	12562.71	10762.39	6040.50	15764.73	18963.20	50490.02	6248.50	10152.05	6248.62	14451.07	34417.72	6773.15	7523.64	23648.32	11105.57
2002Q3	21162.67	12737.90	10919.82	6129.66	16043.58	19342.02	51674.00	6324.80	10319.91	6341.37	14722.05	34905.41	6885.42	7635.17	24120.60	11234.59
2002Q4	23692.34	13788.12	11863.64	6664.16	17715.22	21613.02	58771.81	6782.25	11326.21	6897.42	16346.51	37829.05	7558.45	8303.74	26951.89	12008.03
2003Q1	20181.88	12330.71	10553.89	5922.42	15395.45	18461.52	48922.06	6147.44	9929.75	6125.79	14092.21	33771.86	6624.47	7375.95	23022.87	10934.72
2003Q2	21760.87	12986.25	11143.01	6256.05	16438.88	19879.05	53352.44	6432.98	10557.87	6472.86	15106.19	35596.78	7044.57	7793.27	24790.13	11417.49
2003Q3	22029.97	13097.97	11243.41	6312.91	16616.70	20120.63	54107.47	6481.64	10664.92	6532.01	15278.99	35907.78	7116.17	7864.39	25091.31	11499.76
2003Q4	24849.42	14268.50	12295.34	6908.65	18479.85	22651.78	62018.38	6991.48	11786.50	7151.75	17089.55	39166.34	7866.30	8609.55	28246.93	12361.81
2004Q1	21901.41	13044.60	11195.44	6285.75	16531.75	20005.22	53746.76	6458.39	10613.78	6503.75	15196.44	35759.20	7081.97	7830.41	24947.42	11460.46
2004Q2	23180.79	13575.75	11672.78	6556.07	17377.18	21153.78	57336.49	6689.74	11122.72	6784.97	16018.01	37237.84	7422.35	8168.54	26379.35	11851.63
2004Q3	22999.61	13500.53	11605.18	6517.79	17257.46	20991.12	56828.13	6656.98	11050.65	6745.15	15901.67	37028.44	7374.15	8120.66	26176.57	11796.23
2004Q4	26109.51	14791.65	12765.48	7174.90	19312.54	23783.03	65554.00	7219.35	12287.77	7428.73	17898.74	40622.69	8201.56	8942.58	29657.27	12747.08
2005Q1	22882.62	13451.96	11561.53	6493.07	17180.15	20886.09	56499.86	6635.82	11004.11	6719.43	15826.54	36893.22	7343.02	8089.74	26045.62	11760.46
2005Q2	23675.74	13781.23	11857.44	6660.65	17704.26	21598.12	58725.24	6779.25	11319.61	6893.77	16335.86	37809.87	7554.04	8299.35	26933.31	12002.96
2005Q3	24150.54	13978.35	12034.59	6760.98	18018.01	22024.37	60057.45	6865.11	11508.49	6998.13	16640.76	38358.62	7680.36	8424.84	27464.73	12148.13
2005Q4	27372.85	15316.14	13236.83	7441.83	20147.38	24917.19	69098.72	7447.80	12790.33	7706.43	18710.02	42082.79	8537.68	9276.48	31071.25	13133.34
2006Q1	24245.06	14017.60	12069.86	6780.95	18080.48	22109.22	60322.66	6882.20	11546.09	7018.91	16701.46	38467.86	7705.51	8449.82	27570.52	12177.03
2006Q2	25242.11	14431.53	12441.85	6991.62	18739.34	23004.32	63120.20	7062.50	11942.71	7238.07	17341.72	39620.19	7970.78	8713.33	28686.44	12481.87
2006Q3	25912.77	14709.96	12692.08	7133.33	19182.53	23606.40	65001.96	7183.77	12209.50	7385.49	17772.40	40395.30	8149.22	8890.58	29437.07	12686.92
2006Q4	29489.62	16194.94	14026.60	7889.10	21546.18	26817.51	75038.01	7830.58	13632.38	8171.71	20069.34	44529.23	9100.86	9835.92	33440.41	13780.54
2007Q1	26141.68	14805.00	12777.49	7181.70	19333.80	23811.91	65644.27	7225.17	12300.57	7435.81	17919.40	40659.88	8210.12	8951.09	29693.28	12756.92
2007Q2	27151.22	15224.12	13154.14	7395.00	20000.92	24718.21	68476.84	7407.72	12702.16	7657.71	18567.69	41826.64	8478.71	9217.90	30823.19	13065.58
2007Q3	27569.16	15397.64	13310.08	7483.31	20277.10	25093.42	69649.53	7483.30	12868.42	7749.58	18836.08	42309.68	8589.91	9328.36	31290.96	13193.36
2007Q4	31837.41	17169.66	14902.55	8385.17	23097.64	28925.22	81625.50	8255.14	14566.33	8687.78	21577.01	47242.67	9725.51	10456.43	36068.13	14498.37
2008Q1	28038.06	15592.31	13485.02	7582.39	20586.96	25514.38	70965.19	7568.09	13054.95	7852.65	19137.20	42851.61	8714.67	9452.29	31815.78	13336.73
2008Q2	28991.01	15987.94	13840.57	7783.74	21216.69	26369.88	73638.99	7740.42	13434.03	8062.11	19749.15	43952.97	8968.20	9704.14	32882.34	13628.09
2008Q3	29307.94	16119.52	13958.81	7850.71	21426.12	26654.40	74528.24	7797.73	13560.11	8131.78	19952.67	44319.26	9052.53	9787.91	33237.06	13724.99
2008Q4	32829.04	17581.35	15272.53	8594.70	23752.93	29815.46	84407.86	8434.46	14960.81	8905.75	22213.80	48388.75	9989.34	10718.51	37178.00	14801.56
2009Q1	28245.53	15678.44	13562.43	7626.23	20724.06	25700.63	71547.29	7605.61	13137.48	7898.25	19270.42	43091.38	8769.86	9507.12	32047.97	13400.16
2009Q2	29359.64	16140.98	13978.10	7861.63	21460.29	26700.82	74673.31	7807.08	13580.67	8143.14	19985.87	44379.01	9066.28	9801.57	33294.93	13740.80
2009Q3	29680.26	16274.09	14097.72	7929.38	21672.16	26988.65	75572.90	7865.06	13708.22	8213.62	20191.76	44749.56	9151.58	9886.31	33653.77	13838.83
2009Q4	34084.25	18102.47	15740.85	8859.92	24582.40	30942.32	87929.78	8661.44	15460.13	9181.66	23019.86	49839.46	10323.30	11050.26	38582.88	15185.34
2010Q1	29044.53	16010.16	13860.53	7795.05	21252.05	26417.93	73789.15	7750.10	13455.32	8073.88	19783.51	44014.82	8982.44	9718.29	32942.24	13644.45
2010Q2	30568.36	16642.80	14429.08	8117.03	22259.03	27785.94	78064.78	8025.65	14061.51	8408.83	20762.07	45775.99	9387.87	10121.03	34647.77	14110.36
2010Q3	31233.77	16919.05	14677.34	8257.63	22698.75	28383.31	79931.81	8145.98	14326.21	8555.10	21189.37	46545.03	9564.91	10296.89	35392.52	14313.81
2010Q4	35645.43	18750.61	16323.32	9189.79	25614.06	32343.86	92310.18	8943.75	16081.17	9524.82	24022.40	51643.78	10738.66	11462.87	40330.20	15662.67
2011Q1	30315.13	16537.67	14334.59	8063.52	22091.69	27558.60	77354.25	7979.86	13960.77	8353.17	20599.45	45483.31	9320.50	10054.10	34364.34	14032.94

2011Q2	32098.06	17277.87	14999.80	8440.24	23269.88	29159.22	82356.85	8302.27	14670.02	8745.07	21744.39	47543.92	9794.86	10525.32	36359.86	14578.07
2011Q3	32689.66	17523.48	15220.53	8565.25	23660.83	29690.33	84016.78	8409.25	14905.36	8875.11	22124.30	48227.66	9952.26	10681.67	37022.00	14758.95
2011Q4	37502.93	19521.78	17016.36	9582.27	26841.53	34011.43	97522.03	9279.65	16820.09	9933.12	25215.22	53790.58	11232.86	11953.79	42409.18	16230.60
2012Q1	31512.39	17034.72	14781.29	8316.50	22882.86	28633.44	80713.56	8196.36	14437.04	8616.34	21368.29	46867.04	9639.04	10370.53	35704.36	14399.00
2012Q2	32778.32	17560.29	15253.61	8583.98	23719.42	29769.93	84265.56	8425.29	14940.63	8894.60	22181.24	48330.14	9975.85	10705.11	37121.24	14786.06
2012Q3	33275.05	17766.52	15438.94	8688.94	24047.66	30215.86	85659.29	8515.11	15138.23	9003.79	22500.22	48904.22	10108.00	10836.39	37677.19	14937.93
2012Q4	37772.19	19633.56	17116.82	9639.16	27019.46	34253.15	98277.51	9328.34	16927.20	9992.30	25388.13	54101.77	11304.50	12024.95	42710.54	16312.92

Table 9 Estimated values of quarterly region GDP, obtained on the basis of formula (3). Results obtained based on the second variant of empirical data.

	Dolnośląskie	Kujawsko-Pomorskie	Lubelskie	Lubuskie	Łódzkie	Małopolskie	Mazowieckie	Opolskie	Podkarpackie	Podlaskie	Pomorskie	Śląskie	Świętokrzyskie	Warmińsko-mazurskie	Wielkopolskie	Zachodniopomorskie
1995Q1	13126.55	9614.636	7998.447	4536.342	11007.07	12302.82	30610.28	4803.223	7304.349	4568.856	9818.331	26466.06	4787.58	5563.808	15284.85	8860.843
1995Q2	14349.71	10119.7	8441.831	4800.945	11824.32	13380.5	34088.88	5029.622	7774.286	4838.364	10595.44	27945.72	5113.184	5884.972	16648.76	9243.524
1995Q3	15388.75	10548.75	8818.475	5025.718	12518.55	14295.96	37043.85	5221.941	8173.484	5067.304	11255.57	29202.65	5389.776	6157.793	17807.37	9568.601
1995Q4	16655.34	11071.75	9277.604	5299.716	13364.81	15411.91	40645.96	5456.379	8660.108	5346.381	12060.28	30734.85	5726.942	6490.362	19219.72	9964.87
1996Q1	14437.94	10156.14	8473.816	4820.032	11883.27	13458.24	34339.81	5045.953	7808.186	4857.806	10651.5	28052.46	5136.672	5908.14	16747.15	9271.129
1996Q2	15865.2	10745.48	8991.185	5128.787	12836.89	14715.75	38398.85	5310.129	8356.537	5172.284	11558.28	29779.02	5516.607	6282.895	18338.65	9717.665
1996Q3	16467.85	10994.33	9209.641	5259.157	13239.54	15246.72	40112.76	5421.676	8588.075	5305.071	11941.16	30508.05	5677.032	6441.133	19010.66	9906.212
1996Q4	17186.96	11291.27	9470.314	5414.721	13720.01	15880.31	42157.87	5554.779	8864.358	5463.518	12398.04	31377.96	5868.46	6629.95	19812.52	10131.2
1997Q1	15594.75	10633.81	8893.149	5070.281	12656.19	14477.47	37629.71	5260.071	8252.63	5112.694	11386.45	29451.85	5444.614	6211.883	18037.08	9633.052
1997Q2	16838.43	11147.35	9343.973	5339.323	13487.14	15573.23	41166.66	5490.268	8730.451	5386.723	12176.6	30956.34	5775.68	6538.436	19423.88	10022.15
1997Q3	17313.77	11343.63	9516.282	5442.154	13804.74	15992.04	42518.52	5578.252	8913.079	5491.46	12478.6	31531.36	5902.217	6663.247	19953.93	10170.87
1997Q4	19544.73	12264.84	10324.98	5924.77	15295.34	17957.65	48863.23	5991.187	9770.21	5983.023	13896	34230.16	6496.096	7249.029	22441.62	10868.85
1998Q1	16820.73	11140.04	9337.559	5335.495	13475.32	15557.64	41116.34	5486.993	8723.653	5382.824	12165.36	30934.93	5770.97	6533.79	19404.15	10016.62
1998Q2	17849.73	11564.94	9710.561	5558.095	14162.84	16464.25	44042.74	5677.453	9118.992	5609.551	12819.11	32179.71	6044.888	6803.973	20551.56	10338.55
1998Q3	18432.61	11805.62	9921.852	5684.189	14552.29	16977.81	45700.44	5785.342	9342.936	5737.982	13189.44	32884.83	6200.052	6957.021	21201.52	10520.91
1998Q4	20273.04	12565.57	10588.99	6082.323	15781.95	18599.34	50934.5	6125.992	10050.03	6143.497	14358.71	35111.19	6689.97	7440.26	23253.73	11096.71
1999Q1	17182.03	11289.23	9468.524	5413.653	13716.72	15875.96	42143.84	5553.866	8862.461	5462.431	12394.9	31371.99	5867.146	6628.654	19807.02	10129.65
1999Q2	18532.05	11846.68	9957.898	5705.701	14618.73	17065.42	45983.24	5803.747	9381.141	5759.893	13252.61	33005.12	6226.522	6983.131	21312.4	10552.03
1999Q3	19413.54	12210.67	10277.43	5896.391	15207.69	17842.07	48490.14	5966.905	9719.808	5954.118	13812.65	34071.46	6461.174	7214.583	22295.33	10827.81
1999Q4	22166.43	13347.39	11275.33	6491.915	17047.01	20267.53	56319.19	6476.446	10777.46	6560.682	15561.64	37401.63	7193.99	7937.407	25365.01	11689.09
2000Q1	18446.59	11811.39	9926.917	5687.211	14561.62	16990.12	45740.17	5787.928	9348.304	5741.061	13198.31	32901.73	6203.77	6960.689	21217.1	10525.29
2000Q2	19710.02	12333.09	10384.9	5960.528	15405.78	18103.29	49333.32	6021.782	9833.716	6019.444	14001.01	34430.12	6540.097	7292.43	22625.93	10920.57
2000Q3	20050.19	12473.55	10508.21	6034.115	15633.06	18402.99	50300.73	6084.744	9964.408	6094.395	14217.13	34841.61	6630.649	7381.747	23005.24	11026.99
2000Q4	22904.4	13652.12	11542.84	6651.558	17540.08	20917.74	58417.95	6613.04	11060.99	6723.285	16030.5	38294.36	7390.438	8131.177	26187.91	11919.97
2001Q1	19023.59	12049.65	10136.08	5812.034	14947.15	17498.5	47381.15	5894.728	9569.99	5868.197	13564.9	33599.74	6357.37	7112.194	21860.51	10705.81
2001Q2	19903.24	12412.88	10454.94	6002.326	15534.88	18273.53	49882.83	6057.546	9907.951	6062.017	14123.77	34663.85	6591.532	7343.164	22841.39	10981.02
2001Q3	20452.14	12639.53	10653.92	6121.069	15901.62	18757.14	51443.87	6159.144	10118.84	6182.961	14472.51	35327.86	6737.649	7487.289	23453.45	11152.75
2001Q4	22817.54	13616.25	11511.35	6632.769	17482.05	20841.21	58170.93	6596.964	11027.62	6704.147	15975.31	38189.29	7367.316	8108.371	26091.06	11892.8
2002Q1	19130.26	12093.7	10174.74	5835.109	15018.42	17592.48	47684.51	5914.472	9610.972	5891.7	13632.67	33728.78	6385.765	7140.202	21979.45	10739.18

2002Q2	20345.65	12595.56	10615.31	6098.03	15830.47	18663.31	51141	6139.431	10077.92	6159.495	14404.84	35199.03	6709.299	7459.325	23334.7	11119.43
2002Q3	20765.29	12768.83	10767.43	6188.81	16110.85	19033.04	52334.43	6217.104	10239.15	6251.958	14671.45	35706.67	6821.007	7569.51	23802.63	11250.72
2002Q4	23280.98	13807.62	11679.34	6733.022	17791.69	21249.52	59488.91	6682.742	11205.67	6806.259	16269.75	38749.91	7490.682	8230.054	26607.82	12037.79
2003Q1	19789.91	12366.08	10413.86	5977.809	15459.16	18173.67	49560.51	6036.568	9864.408	6037.046	14051.77	34526.75	6561.363	7313.406	22715.01	10945.56
2003Q2	21360.18	13014.48	10983.07	6317.502	16508.32	19557.18	54026.27	6327.215	10467.71	6383.035	15049.41	36426.31	6979.367	7725.711	24465.98	11436.84
2003Q3	21627.79	13124.98	11080.08	6375.392	16687.12	19792.96	54787.33	6376.747	10570.52	6441.999	15219.43	36750.04	7050.604	7795.977	24764.39	11520.57
2003Q4	24431.66	14282.76	12096.46	6981.947	18560.52	22263.35	62761.4	6895.727	11647.77	7059.799	17000.81	40141.9	7796.994	8532.19	27890.93	12397.8
2004Q1	21499.94	13072.19	11033.73	6347.735	16601.7	19680.32	54423.74	6353.084	10521.4	6413.829	15138.2	36595.38	7016.571	7762.408	24621.83	11480.57
2004Q2	22772.25	13597.55	11494.94	6622.971	17451.79	20801.31	58042.13	6588.581	11010.22	6694.168	15946.54	38134.5	7355.26	8096.479	26040.55	11878.63
2004Q3	22592.07	13523.15	11429.62	6583.994	17331.41	20642.56	57529.71	6555.231	10941	6654.468	15832.07	37916.54	7307.297	8049.17	25839.64	11822.26
2004Q4	25684.8	14800.21	12550.71	7253.035	19397.79	23367.45	66325.25	7127.674	12129.22	7335.911	17796.97	41657.82	8130.578	8861.226	29288.27	12789.85
2005Q1	22475.73	13475.11	11387.45	6558.825	17253.67	20540.05	57198.82	6533.696	10896.3	6628.832	15758.15	37775.79	7276.325	8018.62	25709.9	11785.85
2005Q2	23264.47	13800.8	11673.36	6729.451	17780.66	21234.98	59441.97	6679.687	11199.33	6802.622	16259.26	38729.94	7486.288	8225.721	26589.41	12032.62
2005Q3	23736.65	13995.77	11844.52	6831.596	18096.15	21651	60784.82	6767.084	11380.74	6906.661	16559.25	39301.14	7611.982	8349.7	27115.93	12180.35
2005Q4	26941.16	15318.98	13006.13	7524.82	20237.22	24474.38	69898.28	7360.219	12611.91	7612.736	18595.18	43177.65	8465.021	9191.109	30689.21	13182.92
2006Q1	23830.65	14034.59	11878.59	6851.931	18158.95	21733.82	61052.14	6784.483	11416.86	6927.372	16618.97	39414.85	7637.004	8374.382	27220.75	12209.76
2006Q2	24822.18	14444.01	12238.02	7066.427	18821.44	22607.43	63872.02	6968.01	11797.8	7145.845	17248.92	40614.31	7900.95	8634.729	28326.39	12519.97
2006Q3	25489.14	14719.41	12479.78	7210.708	19267.06	23195.06	65768.8	7091.459	12054.05	7292.8	17672.66	41421.13	8078.494	8809.852	29070.09	12728.64
2006Q4	29046.24	16188.21	13769.2	7980.205	21643.71	26329.09	75884.99	7749.855	13420.68	8076.563	19932.59	45724.16	9025.39	9743.838	33036.53	13841.52
2007Q1	25716.79	14813.42	12562.31	7259.956	19419.17	23395.64	66416.24	7133.596	12141.51	7342.961	17817.3	41696.52	8139.095	8869.627	29323.95	12799.86
2007Q2	26720.75	15227.97	12926.23	7477.139	20089.95	24280.18	69271.43	7319.422	12527.23	7564.17	18455.14	42911.01	8406.347	9133.235	30443.43	13113.97
2007Q3	27136.39	15399.6	13076.9	7567.052	20367.66	24646.39	70453.48	7396.354	12686.92	7655.751	18719.21	43413.81	8516.989	9242.369	30906.9	13244
2007Q4	31381.05	17152.31	14615.56	8485.29	23203.71	28386.21	82525.08	8182.014	14317.72	8591.011	21415.97	48548.6	9646.917	10356.89	35640.03	14572
2008Q1	27602.7	15592.15	13245.93	7667.928	20679.22	25057.24	71779.65	7482.665	12866.08	7758.497	19015.47	43977.91	8641.121	9364.808	31426.87	13389.89
2008Q2	28550.38	15983.46	13589.46	7872.937	21312.41	25892.21	74474.8	7658.075	13230.17	7967.306	19617.56	45124.32	8893.393	9613.64	32483.61	13686.39
2008Q3	28865.56	16113.61	13703.71	7941.12	21522.99	26169.9	75371.15	7716.412	13351.27	8036.752	19817.8	45505.59	8977.294	9696.397	32835.06	13785
2008Q4	32367.21	17559.51	14973.03	8698.622	23862.6	29255.08	85329.65	8364.545	14696.6	8808.298	22042.51	49741.55	9909.431	10615.83	36739.68	14880.53
2009Q1	27809.01	15677.34	13320.72	7712.56	20817.07	25239.02	72366.4	7520.853	12945.34	7803.956	19146.55	44227.49	8696.043	9418.981	31656.93	13454.44
2009Q2	28916.97	16134.84	13722.35	7952.242	21557.35	26215.2	75517.37	7725.929	13371.02	8048.081	19850.47	45567.79	8990.981	9709.897	32892.39	13801.08
2009Q3	29235.82	16266.5	13837.93	8021.217	21770.38	26496.12	76424.15	7784.945	13493.52	8118.335	20053.04	45953.5	9075.857	9793.617	33247.93	13900.84
2009Q4	33615.49	18074.96	15425.52	8968.659	24696.63	30354.89	88879.69	8595.594	15176.19	9083.341	22835.58	51251.6	10241.72	10943.59	38131.6	15271.07
2010Q1	28603.6	16005.44	13608.75	7884.451	21347.97	25939.1	74626.16	7667.926	13250.62	7979.033	19651.37	45188.7	8907.561	9627.615	32542.96	13703.04
2010Q2	30119.02	16631.19	14158.08	8212.278	22360.49	27274.28	78935.93	7948.42	13832.85	8312.937	20614.17	47021.91	9310.965	10025.52	34232.77	14177.16
2010Q3	30780.75	16904.43	14397.95	8355.429	22802.62	27857.31	80817.86	8070.903	14087.08	8458.742	21034.58	47822.41	9487.118	10199.27	34970.65	14384.19
2010Q4	35168.04	18716.04	15988.31	9304.519	25733.96	31722.79	93295.07	8882.962	15772.68	9425.427	23821.96	53129.73	10655.01	11351.24	39862.82	15756.81
2011Q1	29867.19	16527.2	14066.79	8157.799	22192.22	27052.4	78219.72	7901.807	13736.09	8257.448	20454.17	46717.27	9243.926	9959.394	33951.95	14098.37

2011Q2	31640.27	17259.34	14709.52	8541.364	23376.9	28614.59	83262.26	8229.993	14417.31	8648.125	21580.66	48862.17	9715.919	10424.95	35929.08	14653.1
2011Q3	32228.6	17502.28	14922.78	8668.637	23769.99	29132.95	84935.45	8338.89	14643.34	8777.757	21954.44	49573.88	9872.533	10579.43	36585.11	14837.17
2011Q4	37015.29	19478.81	16657.92	9704.128	26968.18	33350.33	98548.53	9224.875	16482.39	9832.444	24995.57	55364.35	11146.75	11836.27	41922.64	16334.75
2012Q1	31057.83	17018.85	14498.39	8415.368	22987.75	28101.43	81605.86	8122.188	14193.54	8519.793	21210.62	48157.59	9560.876	10272.02	35279.62	14470.88
2012Q2	32316.77	17538.69	14954.75	8687.712	23828.9	29210.64	85186.22	8355.21	14677.22	8797.185	22010.46	49680.54	9896.005	10602.58	36683.44	14864.75
2012Q3	32810.76	17742.66	15133.81	8794.574	24158.95	29645.87	86591.07	8446.643	14867.01	8906.028	22324.31	50278.11	10027.5	10732.29	37234.26	15019.3
2012Q4	37283.06	19589.37	16754.98	9762.054	27147.09	33586.25	99310.04	9274.437	16585.26	9891.444	25165.69	55688.27	11218.03	11906.58	42221.22	16418.52

Table 10 Estimated values of quarterly region GDP, obtained on the basis of formula (3). Results obtained based on the third variant of empirical data.

	Dolnośląskie	Kujawsko-Pomorskie	Lubelskie	Lubuskie	Łódzkie	Małopolskie	Mazowieckie	Opolskie	Podkarpackie	Podlaskie	Pomorskie	Śląskie	Świętokrzyskie	Warmińsko-mazurskie	Wielkopolskie	Zachodniopomorskie
2004Q1	22305.12	13057.44	11293.10	6158.05	16573.01	20686.61	52666.17	6829.17	10464.25	6878.17	15185.83	32899.25	7380.62	8251.92	26506.25	11427.54
2004Q2	23538.31	13586.55	11761.83	6448.90	17410.25	21739.14	56378.95	7009.58	11001.10	7101.31	16009.79	34827.38	7675.89	8526.76	27694.79	11817.19
2004Q3	23363.67	13511.62	11695.45	6407.71	17291.68	21590.08	55853.17	6984.03	10925.08	7069.71	15893.10	34554.33	7634.08	8487.84	27526.47	11762.01
2004Q4	26361.28	14797.78	12834.84	7114.70	19326.83	24148.56	64878.17	7422.59	12230.05	7612.12	17895.98	39241.21	8351.84	9155.91	30415.57	12709.16
2005Q1	23250.90	13463.24	11652.59	6381.11	17215.12	21493.83	55513.65	6967.54	10875.98	7049.31	15817.76	34378.01	7607.08	8462.71	27417.79	11726.38
2005Q2	24015.39	13791.25	11943.17	6561.42	17734.15	22146.33	57815.31	7079.38	11208.79	7187.64	16328.55	35573.32	7790.13	8633.09	28154.60	11967.93
2005Q3	24473.05	13987.61	12117.12	6669.36	18044.86	22536.94	59193.19	7146.34	11408.03	7270.45	16634.34	36288.88	7899.71	8735.08	28595.69	12112.54
2005Q4	27579.01	15320.26	13297.69	7401.90	20153.57	25187.90	68544.41	7600.74	12760.17	7832.46	18709.61	41145.18	8643.42	9427.30	31589.21	13093.92
2006Q1	24564.15	14026.70	12151.75	6690.84	18106.72	22614.70	59467.50	7159.67	11447.69	7286.93	16695.22	36431.33	7921.53	8755.39	28683.50	12141.32
2006Q2	25525.20	14439.05	12517.04	6917.51	18759.19	23434.96	62360.94	7300.27	11866.07	7460.83	17337.34	37933.96	8151.64	8969.57	29609.75	12444.98
2006Q3	26171.64	14716.41	12762.76	7069.97	19198.08	23986.70	64307.21	7394.84	12147.49	7577.80	17769.27	38944.70	8306.43	9113.65	30232.79	12649.24
2006Q4	29619.35	16195.68	14073.22	7883.11	21538.81	26929.33	74687.31	7899.25	13648.41	8201.66	20072.88	44335.32	9131.96	9882.03	33555.68	13738.61
2007Q1	26392.30	14811.08	12846.63	7122.01	19347.88	24175.03	64971.53	7427.13	12243.55	7617.73	17916.70	39289.70	8359.27	9162.82	30445.46	12718.96
2007Q2	27365.38	15228.59	13216.49	7351.51	20008.53	25005.56	67901.21	7569.49	12667.17	7793.81	18566.87	40811.15	8592.26	9379.69	31383.31	13026.42
2007Q3	27768.23	15401.44	13369.62	7446.53	20282.04	25349.40	69114.10	7628.43	12842.55	7866.70	18836.04	41441.03	8688.73	9469.47	31771.58	13153.71
2007Q4	31882.36	17166.65	14933.39	8416.85	23075.22	28860.83	81500.63	8230.33	14633.58	8611.14	21584.92	47873.63	9673.83	10386.38	35736.77	14453.65
2008Q1	28220.20	15595.37	13541.41	7553.12	20588.89	25735.16	70474.86	7694.55	13039.31	7948.48	19138.03	42147.70	8796.95	9570.20	32207.19	13296.52
2008Q2	29138.74	15989.47	13890.54	7769.76	21212.51	26519.13	73240.33	7828.94	13439.18	8114.69	19751.75	43583.87	9016.89	9774.92	33092.47	13586.75
2008Q3	29444.22	16120.55	14006.66	7841.81	21419.91	26779.87	74160.07	7873.63	13572.17	8169.97	19955.87	44061.51	9090.03	9843.00	33386.90	13683.27
2008Q4	32838.19	17576.76	15296.70	8642.28	23724.16	29676.63	84378.37	8370.17	15049.69	8784.10	22223.57	49368.10	9902.70	10599.40	36657.99	14755.66
2009Q1	28420.17	15681.17	13617.42	7600.29	20724.66	25905.83	71076.92	7723.81	13126.36	7984.67	19271.64	42460.37	8844.83	9614.77	32399.92	13359.71
2009Q2	29494.06	16141.93	14025.60	7853.56	21453.75	26822.40	74310.10	7880.92	13593.87	8178.99	19989.16	44139.43	9101.96	9854.11	33434.93	13699.02
2009Q3	29803.10	16274.52	14143.07	7926.45	21663.56	27086.17	75240.54	7926.13	13728.40	8234.91	20195.65	44622.62	9175.96	9922.98	33732.78	13796.67
2009Q4	34048.08	18095.88	15756.58	8927.63	24545.58	30709.28	88021.03	8547.18	15576.40	9003.02	23031.96	51259.81	10192.40	10869.05	37824.08	15137.95
2010Q1	29190.32	16011.61	13910.15	7781.93	21247.53	26563.16	73395.64	7836.48	13461.64	8124.03	19786.22	43664.53	9029.24	9786.41	33142.19	13603.05
2010Q2	30659.14	16641.82	14468.45	8128.35	22244.75	27816.80	77817.84	8051.37	14101.07	8389.80	20767.62	45961.08	9380.94	10113.76	34557.83	14067.15
2010Q3	31300.52	16917.01	14712.24	8279.62	22680.20	28364.23	79748.87	8145.21	14380.29	8505.86	21196.16	46963.90	9534.51	10256.71	35175.99	14269.81
2010Q4	35552.89	18741.53	16328.55	9282.54	25567.23	31993.64	92551.60	8767.34	16231.50	9275.31	24037.41	53612.64	10552.72	11204.42	39274.41	15613.42
2011Q1	30415.05	16537.09	14375.67	8070.78	22079.03	27608.47	77082.95	8015.66	13994.81	8345.64	20604.53	45579.43	9322.49	10059.36	34322.58	13990.03

2011Q2	32133.60	17274.45	15028.89	8476.10	23245.79	29075.26	82257.05	8267.09	14742.96	8656.60	21752.79	48266.45	9733.99	10442.37	35978.91	14533.03
2011Q3	32703.84	17519.12	15245.63	8610.59	23632.94	29561.97	83973.89	8350.52	14991.21	8759.79	22133.80	49158.05	9870.53	10569.46	36528.51	14713.21
2011Q4	37343.32	19509.74	17009.09	9704.82	26782.80	33521.79	97942.11	9029.28	17010.95	9599.29	25233.70	56412.05	10981.43	11603.45	41000.03	16179.15
2012Q1	31569.08	17032.24	14814.31	8342.96	22862.53	28593.44	80557.43	8184.50	14497.20	8554.45	21375.60	47383.80	9598.82	10316.56	35434.83	14354.66
2012Q2	32789.30	17555.79	15278.12	8630.75	23690.97	29634.91	84231.20	8363.02	15028.41	8775.25	22190.90	49291.67	9890.99	10588.51	36610.88	14740.22
2012Q3	33268.09	17761.22	15460.11	8743.67	24016.03	30043.56	85672.70	8433.07	15236.85	8861.89	22510.81	50040.28	10005.64	10695.22	37072.34	14891.50
2012Q4	37602.86	19621.09	17107.74	9766.03	26959.00	33743.30	98723.49	9067.25	17123.93	9646.25	25407.11	56817.84	11043.57	11661.29	41250.17	16261.15

Table 11 Estimated values of year-on-year rate of changes in quarterly region GDP, obtained on the basis of formula (5). Results obtained based on the first variant of empirical data.

	Dolnośląskie	Kujawsko-Pomorskie	Lubelskie	Lubuskie	Łódzkie	Małopolskie	Mazowieckie	Opolskie	Podkarpackie	Podlaskie	Pomorskie	Śląskie	Świętokrzyskie	Warmińsko-mazurskie	Wielkopolskie	Zachodniopomorskie
1996Q1	9.78	5.73	6.11	6.18	7.95	9.51	12.28	4.83	7.22	6.23	8.65	5.86	7.25	6.22	9.51	4.54
1996Q2	10.36	6.29	6.68	6.76	8.55	10.10	12.74	5.34	7.82	6.80	9.25	6.42	7.84	6.79	10.09	5.03
1996Q3	6.89	4.29	4.55	4.60	5.75	6.72	8.34	3.67	5.28	4.63	6.19	4.38	5.30	4.62	6.72	3.46
1996Q4	3.14	2.01	2.13	2.15	2.65	3.07	3.74	1.73	2.45	2.16	2.84	2.05	2.46	2.16	3.07	1.64
1997Q1	7.86	4.78	5.08	5.14	6.49	7.66	9.65	4.07	5.94	5.17	7.02	4.88	5.96	5.16	7.66	3.83
1997Q2	6.03	3.80	4.02	4.06	5.06	5.89	7.25	3.26	4.66	4.09	5.44	3.87	4.67	4.08	5.89	3.08
1997Q3	5.05	3.23	3.41	3.45	4.26	4.94	6.03	2.77	3.93	3.47	4.57	3.29	3.95	3.46	4.94	2.62
1997Q4	13.50	8.75	9.24	9.33	11.46	13.21	16.00	7.56	10.61	9.39	12.27	8.91	10.64	9.37	13.21	7.15
1998Q1	7.72	4.84	5.12	5.18	6.46	7.54	9.33	4.14	5.94	5.21	6.95	4.93	5.96	5.20	7.54	3.91
1998Q2	5.91	3.80	4.02	4.06	5.00	5.78	7.03	3.28	4.62	4.08	5.36	3.87	4.64	4.08	5.78	3.10
1998Q3	6.36	4.13	4.36	4.41	5.41	6.22	7.53	3.57	5.01	4.43	5.78	4.21	5.02	4.43	6.22	3.38
1998Q4	3.67	2.49	2.61	2.64	3.18	3.60	4.26	2.17	2.96	2.65	3.38	2.53	2.97	2.65	3.60	2.06
1999Q1	2.11	1.36	1.44	1.45	1.79	2.07	2.51	1.17	1.65	1.46	1.92	1.38	1.66	1.46	2.07	1.11
1999Q2	3.76	2.47	2.61	2.63	3.21	3.69	4.43	2.14	2.98	2.65	3.43	2.52	2.99	2.64	3.68	2.03
1999Q3	5.24	3.48	3.66	3.70	4.50	5.14	6.14	3.02	4.18	3.72	4.79	3.54	4.19	3.72	5.14	2.87
1999Q4	9.21	6.31	6.62	6.68	8.01	9.04	10.62	5.52	7.48	6.71	8.49	6.41	7.50	6.71	9.04	5.25
2000Q1	7.24	4.70	4.95	5.00	6.15	7.09	8.58	4.05	5.69	5.04	6.58	4.78	5.71	5.03	7.08	3.84
2000Q2	6.26	4.17	4.38	4.43	5.38	6.14	7.32	3.62	5.00	4.45	5.73	4.24	5.01	4.45	6.14	3.43
2000Q3	3.23	2.18	2.29	2.32	2.79	3.17	3.75	1.90	2.60	2.33	2.97	2.22	2.61	2.33	3.17	1.81
2000Q4	3.29	2.31	2.42	2.44	2.89	3.23	3.74	2.04	2.71	2.45	3.05	2.35	2.72	2.45	3.23	1.94
2001Q1	3.08	2.05	2.15	2.17	2.64	3.02	3.61	1.78	2.46	2.19	2.82	2.08	2.46	2.18	3.02	1.69
2001Q2	0.97	0.66	0.69	0.70	0.84	0.95	1.12	0.57	0.78	0.70	0.89	0.67	0.78	0.70	0.95	0.54
2001Q3	1.98	1.35	1.42	1.43	1.72	1.94	2.28	1.18	1.60	1.44	1.82	1.37	1.61	1.43	1.94	1.12
2001Q4	-0.37	-0.27	-0.28	-0.28	-0.33	-0.37	-0.42	-0.24	-0.31	-0.28	-0.35	-0.27	-0.31	-0.28	-0.37	-0.22
2002Q1	0.55	0.37	0.39	0.39	0.48	0.54	0.64	0.32	0.44	0.40	0.51	0.38	0.44	0.40	0.54	0.31
2002Q2	2.19	1.49	1.57	1.58	1.90	2.15	2.53	1.30	1.77	1.59	2.02	1.52	1.78	1.59	2.15	1.24
2002Q3	1.51	1.04	1.09	1.10	1.31	1.48	1.74	0.91	1.23	1.10	1.39	1.05	1.23	1.10	1.48	0.86

2002Q4	2.01	1.42	1.49	1.50	1.77	1.97	2.28	1.26	1.66	1.51	1.86	1.44	1.67	1.51	1.97	1.20
2003Q1	3.40	2.28	2.40	2.42	2.93	3.33	3.95	1.99	2.73	2.44	3.12	2.32	2.74	2.43	3.33	1.89
2003Q2	4.92	3.37	3.54	3.57	4.28	4.83	5.67	2.95	4.00	3.59	4.53	3.43	4.01	3.58	4.83	2.81
2003Q3	4.10	2.83	2.96	2.99	3.57	4.03	4.71	2.48	3.34	3.01	3.78	2.87	3.35	3.00	4.02	2.36
2003Q4	4.88	3.48	3.64	3.67	4.32	4.81	5.52	3.09	4.06	3.69	4.55	3.54	4.07	3.68	4.81	2.95
2004Q1	8.52	5.79	6.08	6.13	7.38	8.36	9.86	5.06	6.89	6.17	7.84	5.88	6.91	6.16	8.36	4.81
2004Q2	6.53	4.54	4.75	4.80	5.71	6.41	7.47	3.99	5.35	4.82	6.04	4.61	5.36	4.82	6.41	3.80
2004Q3	4.40	3.07	3.22	3.25	3.86	4.33	5.03	2.71	3.62	3.26	4.08	3.12	3.63	3.26	4.33	2.58
2004Q4	5.07	3.67	3.82	3.85	4.51	4.99	5.70	3.26	4.25	3.87	4.74	3.72	4.26	3.87	4.99	3.12
2005Q1	4.48	3.12	3.27	3.30	3.92	4.40	5.12	2.75	3.68	3.32	4.15	3.17	3.69	3.31	4.40	2.62
2005Q2	2.14	1.51	1.58	1.60	1.88	2.10	2.42	1.34	1.77	1.60	1.98	1.54	1.77	1.60	2.10	1.28
2005Q3	5.00	3.54	3.70	3.73	4.41	4.92	5.68	3.13	4.14	3.75	4.65	3.59	4.15	3.75	4.92	2.98
2005Q4	4.84	3.55	3.69	3.72	4.32	4.77	5.41	3.16	4.09	3.74	4.53	3.59	4.10	3.73	4.77	3.03
2006Q1	5.95	4.20	4.40	4.43	5.24	5.86	6.77	3.71	4.93	4.46	5.53	4.27	4.94	4.45	5.85	3.54
2006Q2	6.62	4.72	4.93	4.97	5.85	6.51	7.48	4.18	5.50	4.99	6.16	4.79	5.52	4.99	6.51	3.99
2006Q3	7.30	5.23	5.46	5.51	6.46	7.18	8.23	4.64	6.09	5.54	6.80	5.31	6.10	5.53	7.18	4.44
2006Q4	7.73	5.74	5.97	6.01	6.94	7.63	8.60	5.14	6.58	6.04	7.27	5.81	6.60	6.03	7.62	4.93
2007Q1	7.82	5.62	5.86	5.91	6.93	7.70	8.82	4.98	6.53	5.94	7.29	5.70	6.55	5.93	7.70	4.76
2007Q2	7.56	5.49	5.72	5.77	6.73	7.45	8.49	4.89	6.36	5.80	7.07	5.57	6.37	5.79	7.45	4.68
2007Q3	6.39	4.67	4.87	4.91	5.71	6.30	7.15	4.17	5.40	4.93	5.99	4.74	5.41	4.92	6.30	3.99
2007Q4	7.96	6.02	6.24	6.29	7.20	7.86	8.78	5.42	6.85	6.32	7.51	6.09	6.86	6.31	7.86	5.21
2008Q1	7.25	5.32	5.54	5.58	6.48	7.15	8.11	4.75	6.13	5.61	6.80	5.39	6.15	5.60	7.15	4.55
2008Q2	6.78	5.02	5.22	5.26	6.08	6.68	7.54	4.49	5.76	5.28	6.36	5.08	5.77	5.28	6.68	4.31
2008Q3	6.31	4.69	4.87	4.91	5.67	6.22	7.00	4.20	5.38	4.93	5.93	4.75	5.39	4.93	6.22	4.03
2008Q4	3.11	2.40	2.48	2.50	2.84	3.08	3.41	2.17	2.71	2.51	2.95	2.43	2.71	2.51	3.08	2.09
2009Q1	0.74	0.55	0.57	0.58	0.67	0.73	0.82	0.50	0.63	0.58	0.70	0.56	0.63	0.58	0.73	0.48
2009Q2	1.27	0.96	0.99	1.00	1.15	1.25	1.40	0.86	1.09	1.01	1.20	0.97	1.09	1.00	1.25	0.83
2009Q3	1.27	0.96	1.00	1.00	1.15	1.25	1.40	0.86	1.09	1.01	1.20	0.97	1.09	1.01	1.25	0.83
2009Q4	3.82	2.96	3.07	3.09	3.49	3.78	4.17	2.69	3.34	3.10	3.63	3.00	3.34	3.10	3.78	2.59
2010Q1	2.83	2.12	2.20	2.21	2.55	2.79	3.13	1.90	2.42	2.22	2.66	2.14	2.42	2.22	2.79	1.82
2010Q2	4.12	3.11	3.23	3.25	3.72	4.06	4.54	2.80	3.54	3.26	3.88	3.15	3.55	3.26	4.06	2.69
2010Q3	5.23	3.96	4.11	4.14	4.74	5.17	5.77	3.57	4.51	4.16	4.94	4.01	4.52	4.15	5.17	3.43
2010Q4	4.58	3.58	3.70	3.72	4.20	4.53	4.98	3.26	4.02	3.74	4.36	3.62	4.02	3.73	4.53	3.14
2011Q1	4.37	3.29	3.42	3.44	3.95	4.32	4.83	2.96	3.76	3.46	4.12	3.34	3.76	3.46	4.32	2.85

2011Q2	5.00	3.82	3.96	3.98	4.54	4.94	5.50	3.45	4.33	4.00	4.73	3.86	4.34	3.99	4.94	3.31
2011Q3	4.66	3.57	3.70	3.73	4.24	4.60	5.11	3.23	4.04	3.74	4.41	3.62	4.05	3.74	4.60	3.11
2011Q4	5.21	4.11	4.25	4.27	4.79	5.16	5.65	3.76	4.59	4.29	4.97	4.16	4.60	4.28	5.15	3.63
2012Q1	3.95	3.01	3.12	3.14	3.58	3.90	4.34	2.71	3.41	3.15	3.73	3.04	3.42	3.15	3.90	2.61
2012Q2	2.12	1.63	1.69	1.70	1.93	2.09	2.32	1.48	1.84	1.71	2.01	1.65	1.85	1.71	2.09	1.43
2012Q3	1.79	1.39	1.43	1.44	1.63	1.77	1.95	1.26	1.56	1.45	1.70	1.40	1.56	1.45	1.77	1.21
2012Q4	0.72	0.57	0.59	0.59	0.66	0.71	0.77	0.52	0.64	0.60	0.69	0.58	0.64	0.60	0.71	0.51

Table 12 Estimated values of year-on-year rate of changes in quarterly region GDP, obtained on the basis of formula (5). Results obtained based on the second variant of empirical data.

	Dolnośląskie	Kujawsko-Pomorskie	Lubelskie	Lubuskie	Łódzkie	Małopolskie	Mazowieckie	Opolskie	Podkarpackie	Podlaskie	Pomorskie	Śląskie	Świętokrzyskie	Warmińsko-mazurskie	Wielkopolskie	Zachodniopomorskie
1996Q1	9.99	5.63	5.94	6.25	7.96	9.39	12.18	5.05	6.90	6.32	8.49	5.99	7.29	6.19	9.57	4.63
1996Q2	10.56	6.18	6.51	6.83	8.56	9.98	12.64	5.58	7.49	6.90	9.09	6.56	7.89	6.76	10.15	5.13
1996Q3	7.01	4.22	4.44	4.64	5.76	6.65	8.28	3.82	5.07	4.69	6.09	4.47	5.33	4.60	6.76	3.53
1996Q4	3.19	1.98	2.08	2.17	2.66	3.04	3.72	1.80	2.36	2.19	2.80	2.09	2.47	2.15	3.08	1.67
1997Q1	8.01	4.70	4.95	5.19	6.50	7.57	9.58	4.24	5.69	5.25	6.90	4.99	5.99	5.14	7.70	3.90
1997Q2	6.13	3.74	3.92	4.10	5.07	5.83	7.21	3.39	4.47	4.15	5.35	3.95	4.70	4.07	5.92	3.13
1997Q3	5.14	3.18	3.33	3.48	4.27	4.89	6.00	2.89	3.78	3.51	4.50	3.35	3.97	3.45	4.96	2.67
1997Q4	13.72	8.62	9.02	9.42	11.48	13.08	15.91	7.86	10.22	9.51	12.08	9.09	10.70	9.34	13.27	7.28
1998Q1	7.86	4.76	5.00	5.23	6.47	7.46	9.27	4.31	5.71	5.28	6.84	5.04	5.99	5.18	7.58	3.98
1998Q2	6.01	3.75	3.92	4.10	5.01	5.72	6.99	3.41	4.45	4.14	5.28	3.95	4.66	4.06	5.81	3.16
1998Q3	6.46	4.07	4.26	4.45	5.42	6.16	7.48	3.71	4.82	4.49	5.70	4.29	5.05	4.41	6.25	3.44
1998Q4	3.73	2.45	2.56	2.66	3.18	3.57	4.24	2.25	2.86	2.68	3.33	2.57	2.98	2.64	3.62	2.10
1999Q1	2.15	1.34	1.40	1.46	1.79	2.05	2.50	1.22	1.59	1.48	1.89	1.41	1.67	1.45	2.08	1.13
1999Q2	3.82	2.44	2.55	2.66	3.22	3.65	4.41	2.22	2.87	2.68	3.38	2.57	3.00	2.63	3.70	2.06
1999Q3	5.32	3.43	3.58	3.73	4.50	5.09	6.10	3.14	4.03	3.77	4.73	3.61	4.21	3.70	5.16	2.92
1999Q4	9.34	6.22	6.48	6.73	8.02	8.97	10.57	5.72	7.24	6.79	8.38	6.52	7.53	6.68	9.08	5.34
2000Q1	7.36	4.63	4.84	5.05	6.16	7.02	8.53	4.21	5.48	5.10	6.48	4.88	5.74	5.01	7.12	3.91
2000Q2	6.36	4.11	4.29	4.47	5.38	6.08	7.29	3.76	4.82	4.51	5.65	4.32	5.04	4.43	6.16	3.49
2000Q3	3.28	2.15	2.25	2.34	2.80	3.14	3.73	1.97	2.52	2.36	2.93	2.26	2.62	2.32	3.18	1.84
2000Q4	3.33	2.28	2.37	2.46	2.89	3.21	3.73	2.11	2.63	2.48	3.01	2.39	2.73	2.44	3.24	1.98
2001Q1	3.13	2.02	2.11	2.19	2.65	2.99	3.59	1.85	2.37	2.21	2.78	2.12	2.48	2.18	3.03	1.72
2001Q2	0.98	0.65	0.67	0.70	0.84	0.94	1.11	0.59	0.75	0.71	0.88	0.68	0.79	0.70	0.95	0.55
2001Q3	2.00	1.33	1.39	1.44	1.72	1.92	2.27	1.22	1.55	1.45	1.80	1.40	1.61	1.43	1.95	1.14
2001Q4	-0.38	-0.26	-0.27	-0.28	-0.33	-0.37	-0.42	-0.24	-0.30	-0.28	-0.34	-0.27	-0.31	-0.28	-0.37	-0.23
2002Q1	0.56	0.37	0.38	0.40	0.48	0.54	0.64	0.33	0.43	0.40	0.50	0.38	0.45	0.39	0.54	0.31
2002Q2	2.22	1.47	1.53	1.59	1.90	2.13	2.52	1.35	1.72	1.61	1.99	1.54	1.79	1.58	2.16	1.26
2002Q3	1.53	1.02	1.07	1.11	1.32	1.47	1.73	0.94	1.19	1.12	1.37	1.07	1.24	1.10	1.49	0.88

2002Q4	2.03	1.41	1.46	1.51	1.77	1.96	2.27	1.30	1.61	1.52	1.84	1.47	1.67	1.50	1.98	1.22
2003Q1	3.45	2.25	2.35	2.45	2.93	3.30	3.93	2.06	2.64	2.47	3.07	2.37	2.75	2.43	3.35	1.92
2003Q2	4.99	3.33	3.46	3.60	4.28	4.79	5.64	3.06	3.87	3.63	4.47	3.49	4.03	3.57	4.85	2.85
2003Q3	4.15	2.79	2.90	3.01	3.58	3.99	4.69	2.57	3.24	3.04	3.73	2.92	3.37	2.99	4.04	2.40
2003Q4	4.94	3.44	3.57	3.70	4.32	4.77	5.50	3.19	3.95	3.73	4.49	3.59	4.09	3.67	4.82	2.99
2004Q1	8.64	5.71	5.95	6.19	7.39	8.29	9.81	5.24	6.66	6.24	7.73	5.99	6.94	6.14	8.39	4.89
2004Q2	6.61	4.48	4.66	4.84	5.72	6.36	7.43	4.13	5.18	4.87	5.96	4.69	5.39	4.80	6.44	3.86
2004Q3	4.46	3.03	3.15	3.27	3.86	4.29	5.01	2.80	3.50	3.30	4.03	3.17	3.64	3.25	4.34	2.62
2004Q4	5.13	3.62	3.76	3.88	4.51	4.96	5.68	3.36	4.13	3.91	4.68	3.78	4.28	3.86	5.01	3.16
2005Q1	4.54	3.08	3.21	3.33	3.93	4.37	5.10	2.84	3.56	3.35	4.10	3.23	3.70	3.30	4.42	2.66
2005Q2	2.16	1.49	1.55	1.61	1.88	2.08	2.41	1.38	1.72	1.62	1.96	1.56	1.78	1.60	2.11	1.30
2005Q3	5.07	3.49	3.63	3.76	4.41	4.89	5.66	3.23	4.02	3.79	4.59	3.65	4.17	3.73	4.94	3.03
2005Q4	4.89	3.51	3.63	3.75	4.33	4.74	5.39	3.26	3.98	3.77	4.49	3.65	4.11	3.72	4.78	3.07
2006Q1	6.03	4.15	4.31	4.47	5.25	5.81	6.74	3.84	4.78	4.50	5.46	4.34	4.96	4.44	5.88	3.60
2006Q2	6.70	4.66	4.84	5.01	5.85	6.46	7.45	4.32	5.34	5.05	6.09	4.87	5.54	4.97	6.53	4.05
2006Q3	7.38	5.17	5.36	5.55	6.47	7.13	8.20	4.79	5.92	5.59	6.72	5.39	6.13	5.51	7.21	4.50
2006Q4	7.81	5.67	5.87	6.05	6.95	7.58	8.56	5.29	6.41	6.09	7.19	5.90	6.62	6.01	7.65	5.00
2007Q1	7.91	5.55	5.76	5.95	6.94	7.65	8.79	5.15	6.35	6.00	7.21	5.79	6.57	5.91	7.73	4.83
2007Q2	7.65	5.43	5.62	5.81	6.74	7.40	8.45	5.04	6.18	5.85	6.99	5.65	6.40	5.77	7.47	4.74
2007Q3	6.46	4.62	4.78	4.94	5.71	6.26	7.12	4.30	5.25	4.98	5.92	4.81	5.43	4.91	6.32	4.05
2007Q4	8.04	5.96	6.15	6.33	7.21	7.81	8.75	5.58	6.68	6.37	7.44	6.18	6.89	6.29	7.88	5.28
2008Q1	7.33	5.26	5.44	5.62	6.49	7.10	8.08	4.89	5.97	5.66	6.72	5.47	6.17	5.58	7.17	4.61
2008Q2	6.85	4.96	5.13	5.29	6.08	6.64	7.51	4.63	5.61	5.33	6.30	5.16	5.79	5.26	6.70	4.36
2008Q3	6.37	4.64	4.79	4.94	5.67	6.18	6.98	4.33	5.24	4.98	5.87	4.82	5.40	4.91	6.24	4.08
2008Q4	3.14	2.37	2.45	2.51	2.84	3.06	3.40	2.23	2.65	2.53	2.93	2.46	2.72	2.50	3.09	2.12
2009Q1	0.75	0.55	0.56	0.58	0.67	0.73	0.82	0.51	0.62	0.59	0.69	0.57	0.64	0.58	0.73	0.48
2009Q2	1.28	0.95	0.98	1.01	1.15	1.25	1.40	0.89	1.06	1.01	1.19	0.98	1.10	1.00	1.26	0.84
2009Q3	1.28	0.95	0.98	1.01	1.15	1.25	1.40	0.89	1.07	1.02	1.19	0.98	1.10	1.00	1.26	0.84
2009Q4	3.86	2.94	3.02	3.10	3.50	3.76	4.16	2.76	3.26	3.12	3.60	3.04	3.35	3.09	3.79	2.62
2010Q1	2.86	2.09	2.16	2.23	2.55	2.77	3.12	1.96	2.36	2.24	2.64	2.17	2.43	2.22	2.80	1.85
2010Q2	4.16	3.08	3.18	3.27	3.73	4.04	4.53	2.88	3.45	3.29	3.85	3.19	3.56	3.25	4.08	2.72
2010Q3	5.28	3.92	4.05	4.17	4.74	5.14	5.75	3.67	4.40	4.19	4.89	4.07	4.53	4.14	5.18	3.48
2010Q4	4.62	3.55	3.65	3.74	4.20	4.51	4.97	3.34	3.93	3.77	4.32	3.66	4.04	3.73	4.54	3.18
2011Q1	4.42	3.26	3.37	3.47	3.95	4.29	4.82	3.05	3.66	3.49	4.09	3.38	3.78	3.45	4.33	2.88

2011Q2	5.05	3.78	3.89	4.01	4.55	4.91	5.48	3.54	4.23	4.03	4.69	3.91	4.35	3.98	4.96	3.36
2011Q3	4.70	3.54	3.65	3.75	4.24	4.58	5.09	3.32	3.95	3.77	4.37	3.66	4.06	3.73	4.62	3.15
2011Q4	5.25	4.08	4.19	4.29	4.80	5.13	5.63	3.85	4.50	4.32	4.93	4.21	4.62	4.27	5.17	3.67
2012Q1	3.99	2.97	3.07	3.16	3.58	3.88	4.33	2.79	3.33	3.18	3.70	3.08	3.43	3.14	3.91	2.64
2012Q2	2.14	1.62	1.67	1.71	1.93	2.08	2.31	1.52	1.80	1.72	1.99	1.67	1.85	1.70	2.10	1.44
2012Q3	1.81	1.37	1.41	1.45	1.64	1.76	1.95	1.29	1.53	1.46	1.68	1.42	1.57	1.44	1.77	1.23
2012Q4	0.72	0.57	0.58	0.60	0.66	0.71	0.77	0.54	0.62	0.60	0.68	0.59	0.64	0.59	0.71	0.51

Table 13 Estimated values of year-on-year rate of changes in quarterly region GDP, obtained on the basis of formula (5). Results obtained based on the third variant of empirical data.

	Dolnośląskie	Kujawsko-Pomorskie	Lubelskie	Lubuskie	Łódzkie	Małopolskie	Mazowieckie	Opolskie	Podkarpackie	Podlaskie	Pomorskie	Śląskie	Świętokrzyskie	Warmińsko-mazurskie	Wielkopolskie	Zachodniopomorskie
2005Q1	8.03	5.76	5.91	6.78	7.28	7.34	10.47	3.68	7.41	4.56	7.87	8.55	5.68	4.69	6.41	4.80
2005Q2	6.17	4.52	4.63	5.27	5.64	5.68	7.89	2.94	5.73	3.61	6.06	6.55	4.46	3.71	5.00	3.80
2005Q3	4.17	3.06	3.13	3.56	3.81	3.84	5.31	2.00	3.87	2.45	4.09	4.42	3.02	2.52	3.38	2.58
2005Q4	4.83	3.65	3.73	4.20	4.46	4.49	5.97	2.45	4.52	2.97	4.75	5.09	3.61	3.05	4.00	3.11
2006Q1	4.24	3.11	3.18	3.62	3.87	3.90	5.41	2.03	3.93	2.49	4.16	4.49	3.07	2.55	3.44	2.62
2006Q2	2.03	1.51	1.54	1.74	1.86	1.87	2.55	1.00	1.89	1.22	1.99	2.14	1.49	1.25	1.66	1.28
2006Q3	4.75	3.52	3.61	4.08	4.36	4.39	5.98	2.32	4.42	2.84	4.66	5.02	3.48	2.91	3.88	2.98
2006Q4	4.62	3.53	3.61	4.04	4.28	4.30	5.65	2.40	4.33	2.89	4.55	4.85	3.49	2.96	3.86	3.03
2007Q1	5.65	4.19	4.28	4.85	5.18	5.21	7.12	2.76	5.26	3.37	5.55	5.97	4.13	3.46	4.62	3.54
2007Q2	6.29	4.70	4.81	5.43	5.78	5.82	7.86	3.12	5.86	3.80	6.18	6.64	4.64	3.90	5.17	3.99
2007Q3	6.94	5.21	5.33	6.01	6.39	6.43	8.64	3.48	6.48	4.23	6.82	7.32	5.15	4.33	5.73	4.43
2007Q4	7.40	5.71	5.83	6.50	6.87	6.91	8.96	3.93	6.96	4.71	7.29	7.75	5.65	4.82	6.23	4.92
2008Q1	7.44	5.59	5.72	6.44	6.85	6.90	9.26	3.74	6.95	4.54	7.32	7.85	5.53	4.65	6.14	4.76
2008Q2	7.21	5.47	5.59	6.27	6.66	6.70	8.88	3.69	6.75	4.46	7.09	7.58	5.41	4.57	5.99	4.67
2008Q3	6.10	4.65	4.75	5.33	5.65	5.68	7.47	3.16	5.72	3.81	6.00	6.41	4.60	3.90	5.09	3.99
2008Q4	7.64	6.00	6.11	6.77	7.13	7.17	9.12	4.19	7.22	4.99	7.53	7.98	5.93	5.10	6.50	5.20
2009Q1	6.93	5.30	5.41	6.05	6.41	6.45	8.47	3.60	6.50	4.34	6.82	7.27	5.24	4.45	5.79	4.54
2009Q2	6.48	5.00	5.10	5.69	6.02	6.05	7.86	3.43	6.09	4.12	6.38	6.79	4.94	4.21	5.45	4.30
2009Q3	6.04	4.67	4.76	5.31	5.61	5.64	7.30	3.21	5.68	3.86	5.95	6.32	4.62	3.94	5.08	4.03
2009Q4	3.00	2.39	2.43	2.68	2.81	2.83	3.53	1.70	2.84	2.01	2.96	3.12	2.37	2.05	2.58	2.09
2010Q1	0.71	0.55	0.56	0.62	0.66	0.66	0.85	0.38	0.67	0.46	0.70	0.74	0.54	0.47	0.60	0.48
2010Q2	1.22	0.95	0.97	1.08	1.14	1.14	1.46	0.66	1.15	0.79	1.20	1.27	0.94	0.81	1.03	0.83
2010Q3	1.22	0.96	0.97	1.08	1.14	1.14	1.46	0.67	1.15	0.79	1.20	1.27	0.95	0.81	1.04	0.83
2010Q4	3.68	2.95	3.01	3.30	3.46	3.48	4.32	2.11	3.50	2.49	3.64	3.83	2.93	2.54	3.18	2.59
2011Q1	2.71	2.11	2.15	2.39	2.52	2.54	3.26	1.46	2.55	1.75	2.67	2.84	2.08	1.79	2.29	1.82
2011Q2	3.95	3.10	3.16	3.50	3.69	3.71	4.72	2.16	3.73	2.58	3.89	4.13	3.06	2.64	3.36	2.69
2011Q3	5.02	3.95	4.02	4.46	4.69	4.72	5.99	2.76	4.75	3.29	4.95	5.25	3.91	3.36	4.28	3.43
2011Q4	4.42	3.57	3.63	3.98	4.16	4.18	5.15	2.58	4.21	3.02	4.37	4.59	3.54	3.09	3.83	3.14
2012Q1	4.20	3.28	3.35	3.71	3.91	3.94	5.02	2.29	3.96	2.73	4.14	4.39	3.25	2.79	3.56	2.84

2012Q2	4.81	3.80	3.87	4.28	4.50	4.52	5.70	2.68	4.55	3.18	4.74	5.02	3.76	3.25	4.11	3.31
2012Q3	4.48	3.56	3.63	4.00	4.20	4.22	5.30	2.52	4.25	2.99	4.42	4.67	3.52	3.05	3.84	3.11
2012Q4	5.04	4.10	4.17	4.55	4.75	4.78	5.82	2.99	4.80	3.49	4.98	5.22	4.06	3.56	4.39	3.62

Table 14 Estimated values of year-on-year rate of changes in quarterly region GDP, obtained on the basis of formula (8). Results obtained based on the first variant of empirical data.

	Dolnośląskie	Kujawsko-Pomorskie	Lubelskie	Lubuskie	Łódzkie	Małopolskie	Mazowieckie	Opolskie	Podkarpackie	Podlaskie	Pomorskie	Śląskie	Świętokrzyskie	Warmińsko-mazurskie	Wielkopolskie	Zachodniopomorskie
1996Q1	8.33	5.70	6.83	6.32	7.52	9.12	11.42	7.75	6.73	7.12	6.52	6.20	8.16	7.34	10.02	7.15
1996Q2	8.90	6.10	7.40	6.79	8.06	9.85	12.24	8.56	7.22	7.65	6.87	6.65	8.86	7.91	10.82	7.79
1996Q3	6.17	4.18	4.69	4.54	5.49	6.39	8.34	4.69	4.90	5.15	5.20	4.52	5.54	5.16	7.01	4.73
1996Q4	3.16	2.06	1.70	2.05	2.65	2.58	4.04	0.41	2.33	2.39	3.37	2.16	1.88	2.13	2.79	1.34
1997Q1	6.90	4.69	5.42	5.14	6.17	7.32	9.38	5.72	5.52	5.81	5.65	5.09	6.43	5.90	8.03	5.54
1997Q2	5.50	3.70	4.02	3.98	4.85	5.54	7.37	3.73	4.32	4.53	4.79	3.99	4.72	4.48	6.06	3.97
1997Q3	4.72	3.16	3.25	3.34	4.12	4.55	6.27	2.63	3.66	3.82	4.32	3.38	3.77	3.70	4.98	3.10
1997Q4	11.70	8.07	10.19	9.10	10.70	13.39	16.23	12.54	9.61	10.21	8.58	8.83	12.26	10.73	14.74	10.94
1998Q1	6.86	4.66	5.38	5.10	6.14	7.27	9.32	5.67	5.49	5.78	5.63	5.05	6.38	5.86	7.97	5.50
1998Q2	5.44	3.66	3.96	3.93	4.79	5.46	7.29	3.64	4.27	4.47	4.76	3.94	4.65	4.42	5.98	3.90
1998Q3	5.83	3.94	4.35	4.25	5.16	5.96	7.85	4.20	4.60	4.83	4.99	4.25	5.12	4.82	6.52	4.34
1998Q4	3.66	2.41	2.20	2.46	3.12	3.21	4.75	1.12	2.76	2.84	3.67	2.55	2.48	2.63	3.49	1.90
1999Q1	2.32	1.47	0.87	1.36	1.86	1.51	2.84	-0.78	1.62	1.62	2.85	1.51	0.85	1.28	1.62	0.40
1999Q2	3.70	2.44	2.24	2.49	3.16	3.26	4.81	1.18	2.79	2.88	3.70	2.59	2.53	2.67	3.55	1.95
1999Q3	4.94	3.31	3.47	3.52	4.33	4.83	6.58	2.94	3.85	4.02	4.45	3.55	4.04	3.92	5.28	3.34
1999Q4	8.32	5.70	6.83	6.31	7.52	9.12	11.42	7.75	6.73	7.12	6.52	6.20	8.16	7.33	10.02	7.15
2000Q1	6.55	4.44	5.07	4.85	5.84	6.87	8.88	5.22	5.22	5.49	5.44	4.81	6.00	5.54	7.53	5.15
2000Q2	5.79	3.91	4.32	4.22	5.13	5.91	7.80	4.15	4.57	4.80	4.97	4.22	5.08	4.78	6.47	4.30
2000Q3	3.29	2.15	1.83	2.15	2.77	2.74	4.22	0.59	2.44	2.50	3.44	2.26	2.03	2.26	2.97	1.49
2000Q4	3.38	2.21	1.92	2.23	2.85	2.85	4.35	0.72	2.52	2.58	3.50	2.33	2.14	2.35	3.09	1.58
2001Q1	3.15	2.05	1.69	2.04	2.64	2.56	4.02	0.39	2.32	2.37	3.36	2.15	1.86	2.11	2.77	1.33
2001Q2	1.40	0.81	-0.05	0.59	0.99	0.34	1.52	-2.10	0.83	0.77	2.29	0.79	-0.27	0.35	0.32	-0.64
2001Q3	2.25	1.41	0.79	1.29	1.79	1.42	2.73	-0.89	1.55	1.55	2.81	1.45	0.76	1.21	1.51	0.31
2001Q4	0.27	0.02	-1.17	-0.34	-0.08	-1.09	-0.09	-3.70	-0.13	-0.26	1.60	-0.10	-1.64	-0.79	-1.25	-1.91
2002Q1	1.05	0.57	-0.40	0.30	0.66	-0.10	1.02	-2.59	0.53	0.45	2.08	0.51	-0.69	0.00	-0.16	-1.03
2002Q2	2.43	1.54	0.97	1.44	1.96	1.64	2.99	-0.64	1.71	1.71	2.92	1.59	0.98	1.39	1.76	0.51
2002Q3	1.86	1.14	0.41	0.97	1.42	0.93	2.18	-1.44	1.22	1.19	2.57	1.15	0.29	0.82	0.97	-0.12
2002Q4	2.30	1.45	0.84	1.33	1.83	1.48	2.80	-0.82	1.59	1.59	2.84	1.49	0.82	1.26	1.58	0.37
2003Q1	3.42	2.24	1.96	2.27	2.90	2.91	4.41	0.78	2.56	2.63	3.53	2.37	2.19	2.39	3.16	1.64

2003Q2	4.72	3.16	3.25	3.34	4.12	4.55	6.27	2.63	3.66	3.82	4.32	3.38	3.77	3.70	4.98	3.10
2003Q3	4.04	2.68	2.57	2.77	3.48	3.69	5.29	1.66	3.08	3.19	3.90	2.85	2.94	3.01	4.02	2.33
2003Q4	4.75	3.18	3.28	3.36	4.15	4.59	6.31	2.67	3.69	3.84	4.34	3.41	3.81	3.73	5.02	3.13
2004Q1	7.72	5.27	6.24	5.82	6.95	8.36	10.56	6.89	6.22	6.57	6.15	5.73	7.43	6.73	9.18	6.47
2004Q2	6.10	4.13	4.62	4.48	5.42	6.30	8.24	4.59	4.84	5.08	5.16	4.46	5.45	5.09	6.91	4.65
2004Q3	4.31	2.87	2.84	3.00	3.73	4.04	5.68	2.04	3.31	3.44	4.07	3.06	3.28	3.29	4.40	2.64
2004Q4	4.93	3.31	3.46	3.51	4.32	4.82	6.57	2.93	3.84	4.01	4.45	3.55	4.03	3.91	5.27	3.33
2005Q1	4.38	2.91	2.91	3.05	3.79	4.12	5.77	2.14	3.37	3.50	4.11	3.11	3.35	3.35	4.49	2.71
2005Q2	2.41	1.52	0.95	1.42	1.94	1.62	2.96	-0.67	1.69	1.69	2.90	1.57	0.95	1.37	1.73	0.49
2005Q3	4.84	3.24	3.37	3.44	4.23	4.71	6.44	2.80	3.76	3.93	4.39	3.48	3.92	3.82	5.14	3.23
2005Q4	4.75	3.18	3.28	3.36	4.15	4.59	6.31	2.67	3.69	3.84	4.34	3.41	3.81	3.73	5.02	3.13
2006Q1	5.65	3.81	4.17	4.10	4.99	5.73	7.59	3.94	4.45	4.66	4.88	4.10	4.90	4.63	6.27	4.13
2006Q2	6.23	4.22	4.75	4.58	5.54	6.46	8.42	4.77	4.95	5.20	5.24	4.56	5.61	5.22	7.08	4.79
2006Q3	6.82	4.64	5.34	5.07	6.10	7.21	9.27	5.61	5.45	5.74	5.60	5.02	6.33	5.82	7.91	5.46
2006Q4	7.27	4.95	5.79	5.44	6.52	7.79	9.91	6.25	5.84	6.15	5.88	5.38	6.88	6.27	8.55	5.96
2007Q1	7.27	4.95	5.79	5.44	6.52	7.79	9.91	6.25	5.84	6.15	5.88	5.38	6.88	6.27	8.55	5.96
2007Q2	7.08	4.82	5.59	5.28	6.34	7.54	9.63	5.97	5.67	5.97	5.76	5.22	6.64	6.08	8.27	5.74
2007Q3	6.09	4.12	4.61	4.46	5.41	6.28	8.22	4.56	4.82	5.07	5.15	4.45	5.43	5.08	6.89	4.63
2007Q4	7.51	5.13	6.03	5.64	6.75	8.09	10.26	6.59	6.04	6.38	6.03	5.57	7.17	6.52	8.89	6.24
2008Q1	6.83	4.64	5.35	5.08	6.11	7.23	9.28	5.63	5.46	5.75	5.61	5.03	6.34	5.83	7.93	5.47
2008Q2	6.44	4.37	4.96	4.76	5.74	6.73	8.72	5.07	5.13	5.39	5.37	4.73	5.87	5.44	7.38	5.03
2008Q3	6.04	4.09	4.57	4.43	5.37	6.23	8.16	4.50	4.79	5.03	5.13	4.42	5.38	5.03	6.83	4.58
2008Q4	3.32	2.17	1.85	2.18	2.79	2.77	4.26	0.63	2.46	2.53	3.46	2.28	2.06	2.28	3.01	1.51
2009Q1	1.23	0.70	-0.22	0.45	0.83	0.13	1.28	-2.33	0.69	0.62	2.19	0.65	-0.48	0.18	0.09	-0.83
2009Q2	1.69	1.02	0.24	0.84	1.27	0.72	1.94	-1.68	1.08	1.04	2.47	1.02	0.09	0.65	0.74	-0.31
2009Q3	1.69	1.02	0.24	0.84	1.27	0.72	1.94	-1.68	1.08	1.04	2.47	1.02	0.09	0.65	0.74	-0.31
2009Q4	3.94	2.61	2.48	2.69	3.39	3.57	5.16	1.52	3.00	3.10	3.84	2.77	2.83	2.92	3.89	2.22
2010Q1	3.04	1.97	1.58	1.95	2.54	2.42	3.86	0.24	2.23	2.28	3.29	2.07	1.73	2.01	2.62	1.21
2010Q2	4.17	2.77	2.70	2.88	3.60	3.85	5.48	1.84	3.19	3.31	3.98	2.95	3.10	3.14	4.20	2.47
2010Q3	5.14	3.45	3.67	3.69	4.52	5.09	6.87	3.23	4.02	4.20	4.58	3.71	4.29	4.13	5.57	3.57
2010Q4	4.62	3.08	3.15	3.25	4.02	4.42	6.12	2.48	3.57	3.72	4.26	3.30	3.65	3.60	4.83	2.98
2011Q1	4.39	2.92	2.92	3.06	3.81	4.13	5.79	2.15	3.38	3.51	4.12	3.12	3.37	3.37	4.51	2.72
2011Q2	4.95	3.32	3.48	3.53	4.34	4.85	6.60	2.96	3.86	4.03	4.46	3.57	4.06	3.94	5.30	3.36
2011Q3	4.66	3.12	3.19	3.29	4.06	4.48	6.18	2.54	3.61	3.76	4.28	3.34	3.70	3.64	4.89	3.03
2011Q4	5.19	3.49	3.72	3.72	4.56	5.15	6.93	3.29	4.06	4.24	4.60	3.75	4.34	4.17	5.63	3.62
2012Q1	3.95	3.01	3.12	3.14	3.58	3.90	4.34	2.71	3.41	3.15	3.73	3.04	3.42	3.15	3.90	2.61

2012Q2	2.12	1.63	1.69	1.70	1.93	2.09	2.32	1.48	1.84	1.71	2.01	1.65	1.85	1.71	2.09	1.43
2012Q3	1.79	1.39	1.43	1.44	1.63	1.77	1.95	1.26	1.56	1.45	1.70	1.40	1.56	1.45	1.77	1.21
2012Q4	0.72	0.57	0.59	0.59	0.66	0.71	0.77	0.52	0.64	0.60	0.69	0.58	0.64	0.60	0.71	0.51

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