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Abstract

We present a new measure of extreme credit risk in the time domain, namely the conditional expected time to default ($CETD$). This measure has a clear interpretation and can be applied in a straightforward way to the analyses of loan performance in time. In contrast to the probability of default, $CETD$ provides direct information on the timing of a potential loan default under some stress scenarios. We apply a novel method to compute $CETD$ using Markov probability transition matrices, a popular approach in survival analysis literature. We employ the new measure to the analysis of changing credit risk in a large portfolio of corporate loans.

JEL Classification code: G21, G32, C13, C18

Keywords: credit risk, time to default, value at risk, conditional ETD

1 Introduction

One standard approach to analyzing the quality of corporate loans is to calculate the probabilities of default at some time horizon. For example, a one-year default probability measured at time t ($PD_t(1Y)$) provides information on the risk of default in the period from t up to t plus one year. Comparing the probabilities of default for different time horizons or at different periods allows for an assessment of how credit risk develops over time and how it is expected to change over time.

Our aim is to construct a measure of credit risk that provides information on the expected time to default (ETD) for a loan under a stress scenario. In contrast to the probability of default, ETD provides direct information on the timing of a potential loan default. Such information may help organize lending policies in credit institutions in a way that could decrease potential losses incurred due to loan defaults. Estimates of time to default are also used to predict the profitability of risky loan portfolios or to simulate credit losses in stress conditions (e.g., Andreeva, Ansell, and Crook, 2007; Bellotti and Crook, 2013). Time to default can greatly vary depending on the type of business, size, the industry in which the company operates, or the general economic situation. Analyzing time of default, dependent on a industry or the size of a company, may help diversify credit portfolios and thus spread losses over time rather than experience multiple losses at the same time. This can be particularly useful for minimizing the consequences of economic crises and economic breakdowns. The ETD conditional on a crisis scenario whereby a loan portfolio performs poorly, i.e. loans default rapidly in this portfolio, is of special interest for us. Such a *conditional* ETD measure indicates the time when a credit default can be expected for a given group of loans under stress conditions.

Survival analysis and hazard models are the most popular tools to model time to default for loan contracts (Cox, 1972; Lambrecht, Perraudin, and Satchell, 1997; Lando, 1994). For example, Glennon and Nigro (2005) described the distribution of the time to default with the hazard function and used a discrete-time hazard approach to estimate their model (e.g., Kiefer, 1988; Shumway, 2001). Stepanova and Thomas (2002) constructed credit-scoring models using survival-analysis tools, and Beran and Djaïdja (2007) presented an approach using survival analyses designed for rare defaults to model the time to default for retail clients.

In this class of models, it became important not only if, but when a borrower would default (Banasik, Crook, and Thomas, 1999). Carling, Jacobson, Lindé, and Roszbach (2007) estimated a duration model to explain the survival time to default for borrowers in the business loan portfolio of a major Swedish bank. Tong, Mues, and Thomas (2012) compared the ability of proportional hazard models and mixed cure models to predict defaults and to estimate the time to default. Bharath and Shumway (2008) employed predictions from the Merton distance to default model, as well as a number of other predictor factors, to estimate the time to default in a Cox proportional hazard model. They found that well specified reduced-form models outperform the Merton model in predicting defaults (cf., Charitou, Dionysiou, Lambertides, and Trigeorgis, 2013). Leland (2004) built structural models of credit risk, including the KMV model, and estimated expected default frequencies for different default horizons. Sarlija, Bensic, and Zekic-Susac (2009) used six neural network models and a survival analysis model to investigate the time to default for loans in one Croatian bank.

Most economists have considered probabilities of default, frequencies of rating transitions, or expected losses as a function of a given time horizon (e.g., one year; Crouhy, Galai, and Mark, 2000). However, fewer studies have focused explicitly on the timing of expected defaults. Bystrom and Kwon (2007) proposed the use of the *ETD* as a superior measure of credit risk. They derived the *ETD* from a model where defaults were analyzed under a risk-neutral measure. The risk-neutral probabilities of default are usually higher than the actual probabilities of default (i.e., PDs calculated under the physical measure). Hence, the *ETD* under the risk neutral measure should be lower than the *ETD* under the physical measure. Therefore, the results of Bystrom and Kwon (2007) should be treated as the lower limit of the actual *ETD* for a given financial instrument. In contrast, our *ETD* conditional on a "bad luck" scenario, is computed under the physical measure and provides an assessment of the actual credit risk. Ebnöther and Vanini (2007) developed a measure called the time-conditional expected shortfall (*TES*) to quantify the risk of a credit portfolio over a multiperiod horizon. *TES* reflects the expected cumulative loss at a given time in the future, conditional on the event that the earlier loss exceeds the chosen quantile (VaR). For example, *TES* measures the conditional expected future trend of losses, given that the annual loss is not acceptable to the lending institution.

We extend the existing literature on credit risk analysis in four ways. First, we investigate a new measure of extreme credit risk focusing on a time domain of credit events - conditional expected time to default. Second, we employ a novel method to compute the distribution of time to default from rating transition matrices. Third, we employ a large and complete database of corporate loans containing all major exposures of nonfinancial firms towards banks in a country. Finally, this unique database allows us to examine differences in credit risk between different economic sectors and changes in risk over time.

To our best knowledge, our study is the first to analyze the "extreme" expected time to default, conditional on this time being shorter than the period that could be set by a lending institution. One can interpret this measure as a conditional Value-at-Risk for loan defaults in a time domain (*CVaR*, also known as an expected shortfall in financial investment literature). As proved in Pflug (2000), *CVaR* is a coherent measure of risk and it possesses several properties important for our context, including sub-additivity, monotonicity, translation equi-variability and positive homogeneity (cf., Artzner, Delbaen, Eber, and Heath, 1999). We propose the use of the expected time to default *under stress conditions* (*conditonal ETD*, *CETD*) as a measure of extreme credit risk (i.e., credit risk exceeding the threshold set by a lending institution).

Discriminating credit quality based on rating classes is a common business in corporate finance, and major agencies like Moody's, Standard & Poor's, and Fitch all publish assessments of clients' PDs depending on special rating scales (e.g., Moody's, 2011; Standard & Poor's, 2015; Fitch, 2015). Equally important, these rating scales enable economists to analyze *rating transitions* (or *migrations*) between credit quality grades. Such transitions are interpreted as changes in creditworthiness and in the credit risk of the counterparties that are considered (e.g., Bluhm, Overbeck, and Wagner, 2010, p. 51). Rating systems are widely used to assess the economic performance of large corporations that fund their activities in capital markets. For us, the most interesting are those rating systems that describe the current ability of large and small firms to pay back their loans to banks. In such systems, rating classes usually depend on delays in loan repayment and the loan losses already incurred by the banks.

In this study, we use a corporate loan rating system and rating transition matrices to analyze the expected times to default for corporate loan portfolios. We assume that developments in the credit risk of a loan portfolio are well described by the multi-state first-order Markov chain. Firms from different economic sectors show some heterogeneity with respect to their credit quality performance and how the economic fluctuations affect default frequencies over time (e.g., Virolainen, 2004; Fernandes, 2005). Thus, we analyze loan portfolios from 16 industries separately and allow the Markov transition matrices to change in time (e.g., (e.g., Jafry and Schuermann, 2004; Feng, Gouriéroux, and Jasiak, 2008; Frydman and Schuermann, 2008; Kadam and Lenk, 2008; Stefanescu, Tunaru, and Turnbull, 2009).

We employ a novel method to calculate the distribution of time to default from a Markov transition matrix, namely the graph reduction method of Górajski (2009). This method shrinks the computational burden associated with computing the distributions of time to default and enables us to compute these distributions for several loan portfolios and for different periods. The final *CETD* measure is a straightforward result from our calculations concerning the left tail of the computed distributions.

We identify five categories of credit quality for corporate loans in a bank portfolio. Because corporate loans experience changes in credit quality over the years, we assume that the Markov chain framework describes any credit migration between the five categories. Therefore, we interpret these categories as regimes in the Markov switching model. We estimate Markov chain transition matrices using unique data from the Polish banking sector to describe the quarterly frequencies of credit migration between the states. We further employ these estimated matrices, as well as the algorithm of Górajski (2009) to compute the probability distributions of the time to default for each credit category. This allows us to calculate and compare the *CETD* for Polish corporate loans from 16 separate economic sectors and for each quarter between Q1 2007 and Q1 2015.

Our database of corporate loans is of special interest because it enables us to identify quarterly credit ratings and an economic sector of each credit exposure. This database contains ca. 300,000 credit exposures for each quarter. The value of corporate loans provided by banks in Poland reached 285 billion zlotys (ca. EUR 70 billion) in 2015 and all larger loans are included in the database.

In the next section, we describe our simple theoretical model for rating transitions and define the *CETD* as our measure of credit risk. In the third section, we discuss our data and the identification method used to classify corporate loans into five risk categories. Empirical results present the changes of *CETD* in time and the differences across economic sectors. The final section presents our conclusions.

2 Conditional Distributions of Time to Default

This section presents a simple model used to estimate the transition probabilities between the five categories of corporate loans. We also present the novel algorithm used to efficiently calculate the distribution of time to default with the use of probability transition matrices. Next, we define *CETD* where the default happens extremely early, and we consider this a new measure of extreme credit risk.

We assume that the credit quality of a loan contract in a given economic sector $sec \in \{0, 1, 2, ...16\}$ may change over time. The time evolution of credit quality is well described by an absorbing Markov chain $X^{sec} = (X_t^{sec})_{t=1,2,...}$, where t denotes time in quarters. We further assume that a corporate loan X^{sec} can be assigned to one of the five credit risk categories (or states) $S^{sec} = \{1, 2, .., 5\}$ discussed in more detail in Section 3. The set S^{sec} consists of five quality categories of credit exposure. The state of default (*lost credit*), defined as 5 ($j = 5$), is assumed to be the absorbing state for X^{sec} while the other states (*normal* for $j = 1$, *under observation* for $j = 2$, *substandard* for $j = 3$, and *doubtful* for $j = 4$) are transient. Thus the set $\{X_t^{sec} = 5\}$ represents the default event at time t. Let $P^{sec} = [p_{ij}^{sec}]_{i,j \in S}$ be a matrix of transition probabilities for the Markov chain X^{sec}

$$p_{ij}^{sec} = Pr(X_t^{sec} = j | X_{t-1}^{sec} = i).$$

We estimate the transition matrices P^{sec} using the standard proportion method. The estimator of p_{ij}^{sec} given in formula (1) is a maximum-likelihood estimator that is consistent and asymptotically unbiased (cf., Anderson and Goodman, 1957).

$$\hat{p}_{ij}^{sec} = \frac{\sum_{t=1}^{T} n_{ij}^{sec}(t)}{\sum_{t=1}^{T} n_i^{sec}(t-1)} \tag{1}$$

where $T = 12$ quarters is the estimation window, and $n_{ij}^{sec}(t)$ and $n_i^{sec}(t-1)$ are a proportion of loans (credit exposures) that started in state i at the beginning of period $t-1$ and ended in state j in the period t, and a proportion of all individuals in period $t-1$ that started in state i, respectively.

2.1 Markov Chain Reduction Method

We use the graph-reduction method of Górajski (2009) to build an algorithm to determine the conditional distribution, $\mathcal{T}_{j_0}^{sec}$, of the time of absorption at the default state $j = 5$ under the condition that the initial state is $j_0 \in S \setminus \{5\}$. In particular, we compute the expected time of absorption $\mathbb{E}\mathcal{T}_{j_0}^{sec}$ as well as the variance of the absorption time $\mathbb{E}(\mathcal{T}_{j_0}^{sec})^2 - (\mathbb{E}\mathcal{T}_{j_0}^{sec})^2$.

Let us fix the initial state of a loan as $j_0 \in S^{sec} \setminus \{5\}$. Let $\tau^{sec} = [\tau_{ij}^{sec}]_{i,j \in S}$ be a matrix with random variables, where τ_{ij}^{sec} is equal to the random transition time between the states i and j. The probability distributions of τ_{ij}^{sec} for all $i,j \in S^{sec}$ are degenerate to one point: zero or one depending on whether there is a positive probability of transition between states or not,

$$\begin{aligned} Pr(\tau_{ij}^{sec} = 1) &= 1, \text{ if } p_{ij}^{sec} > 0, \\ Pr(\tau_{ij}^{sec} = 0) &= 1, \text{ if } p_{ij}^{sec} = 0. \end{aligned}$$

Moreover, let $M_k^{sec} = [m_{ij}^{sec}(k)]_{i,j\in S^{sec}}$ for $k = 1,2$ denote matrices collecting the first two moments of τ_{ij}^{sec}. Therefore, the time evolution of the credit quality in the economic sector $sec \in \{0,1,2\ldots,16\}$ is given by the following sextuple

$$\left(X^{sec}, S^{sec}, P^{sec}, \tau^{sec}, M_1^{sec}, M_2^{sec}\right).$$

The Markov chain reduction method tends to decrease the number of states by the specific reductions described in Górajski (2009, Section 3). We apply two types of reductions, namely the loop reduction and the state reduction. Assume that we have a Markov chain $\left(X^{sec,l}, S^{sec,l}, P^{sec,l}, \tau^{sec,l}, M_1^{sec,l}, M_2^{sec,l}\right)$ after l steps of the algorithm. At the beginning of stage $l+1$ we reduce all loops in states $S^{sec,l} \setminus \{5\}$.[1] The exit time from a state $i \in S^{sec,l} \setminus \{5\}$ with a loop to a state $j \in S^{sec,l}$, is a sum of random times $\tau_{ii,1}^{sec,l} + \tau_{ii,2}^{sec,l} + \cdots + \tau_{ii,k}^{sec,l}$ and $\tau_{ij}^{sec,l}$, where $k = 0,1,\ldots$ is the number of loops in the state i, and $\tau_{ii,1}^{sec,l}, \tau_{ii,2}^{sec,l}, \ldots, \tau_{ii,k}^{sec,l}$ are independently identically distributed times spent in a loop. In the loop reduction step, we remove the possibility of transition from i to i and add the time spent in loops to the time $\tau_{ij}^{sec,l}$ of the transition from i to j. Hence, the set $S^{sec,l+1} = S^{sec,l}$ and we have the transition probabilities

$$p_{ii}^{sec,l+1} = 0, \quad p_{ij}^{sec,l+1} = \frac{p_{ij}^{sec,l}}{1 - p_{ii}^{sec,l}},$$

the transition times from state i to state j

$$\tau_{ij}^{sec,l+1}(\omega) = \begin{cases} \tau_{ij}^{sec,l}(\omega) & \text{for } \omega \in \{X_t^{sec,l} = i, X_{t+1}^{sec,l} = j\} \\ (\tau_{1,ii}^{sec,l} + \tau_{ij}^{sec,l})(\omega) & \text{for } \omega \in \{X_t^{sec,l} = i, X_{t+1}^{sec,l} = i, X_{t+2}^{sec,l} = j\} \\ (\tau_{1,ii}^{sec,l} + \tau_{ii,2}^{sec,l} + \tau_{ij}^{sec,l})(\omega) & \text{for } \omega \in \{X_t^{sec,l} = i, X_{t+1}^{sec,l} = i, X_{t+2}^{sec,l} = i, X_{t+3}^{sec,l} = j\}, \\ \ldots & \ldots \end{cases}$$

and the first two moments of transition time from i to j,

$$\begin{aligned} m_{ij}^{l+1}(1) = \mathbb{E}\tau_{ij}^{sec,l+1} &= \frac{1}{p_{ij}^{l+1}}\left(\frac{qp_{ij}^l}{(1-q)^2} m_{ii}^l(1) + \frac{p_{ij}^l}{1-q} m_{ij}^l(1)\right) \\ m_{ij}^{l+1}(2) = \mathbb{E}(\tau_{ij}^{sec,l+1})^2 &= \frac{1}{p_{ij}^{l+1}}\left(\frac{p_{ij}^l(q^2+q)}{(1-q)^3} m_{ii}^l(2) + 2\frac{p_{ij}^l q}{(1-q)^2} m_{ii}^l(1) m_{ij}^l(1) + \frac{p_{ij}^l}{1-q} m_{ij}^l(2)\right) \end{aligned}$$

[1] A Markov chain X with transition probabilities $P = [p_{ij}]$ has a loop at state $i \in S$ if $p_{ii} > 0$.

for all $j \in S^{sec,l+1}$, where $q = p^l_{ii}$. Next, we can reduce a state $k \in S^{sec,l} \setminus \{j_0, 5\}$ in the Markov chain $X^{sec,l+1}$ without loops by deleting all paths to the state k and summing the time of transitions through the state k. To implement the reduction of k, we use the following formulae

$$p^{sec,l+2}_{ik} = 0, \quad p^{sec,l+2}_{ij} = p^{sec,l+1}_{ij} + p^{sec,l+1}_{ik} p^{sec,l+1}_{kj},$$

$$\tau^{sec,l+2}_{ij}(\omega) = \begin{cases} \tau^{sec,l+1}_{ij}(\omega) & \text{for } \omega \in \{X^{sec,l+1}_t = i, X^{sec,l+1}_{t+1} = j\} \\ (\tau^{sec,l+1}_{ik} + \tau^{sec,l+1}_{kj})(\omega) & \text{for } \omega \in \{X^{sec,l+1}_t = i, X^{sec,l+1}_{t+1} = i, X^{sec,l+1}_{t+2} = j\}, \end{cases} \tag{2}$$

$$m^{l+2}_{ij}(1) = \mathbb{E}\tau^{sec,l+2}_{ij} = \frac{1}{p^{l+2}_{ij}} \left(p^{l+1}_{ij} m^{l+1}_{ij}(1) + p^{l+1}_{ik} p^{l+1}_{kj} \left(m^{l+1}_{ik}(1) + m^{l+1}_{kj}(1) \right) \right),$$

$$m^{l+2}_{ij}(2) = \mathbb{E}(\tau^{sec,l+2}_{ij})^2 = \frac{1}{p^{l+2}_{ij}} \left(p^{l+1}_{ij} m^{l+1}_{ij}(2) + p^{l+1}_{ik} p^{l+1}_{jk} \left(m^{l+1}_{ik}(2) + 2m^{l+1}_{ik}(1) m^{l+1}_{kj}(1) + m^{l+1}_{kj}(2) \right) \right),$$

for all $i, j \in S^{sec,l}$. In (2) we consider two disjoint paths between states i and j. The first path links the states i and j directly, whereas the second starts at i and goes to j through the state k. Finally, we obtain the new Markov process $X^{sec,l+2}$ with a smaller number of states $S^{sec,l+2} = S^{sec,l+1} \setminus \{k\}$. The reduction method follows according to the scheme

$$(X^{sec}, S^{sec}, P^{sec}, \tau^{sec}, M^{sec}_1, M^{sec}_2) \underrightarrow{Reduct} \left(X^{sec,1}, S^{sec,1}, P^{sec,1}, \tau^{sec,1}, M^{sec,1}_1, M^{sec,1}_2 \right) \underrightarrow{Reduct} \ldots$$
$$\ldots \underrightarrow{Reduct} (X^{sec,n}, S^{sec,n}, P^{sec,n}, \tau^{sec,n}, M^{sec,n}_1, M^{sec,n}_2).$$

After a finite number of steps n, we obtain the reduced Markov chain $X^{sec,n}$ with just two states, $S^{sec,n} = \{j_0, 5\}$, and the random time transition between the initial state, j_0, and the default, $j = 5$ given by $\tau^{sec,n}_{j_0 5}$. Moreover, we have $m^{sec,n}_{j_0 5}(1) = \mathbb{E}\tau^{sec,n}_{j_0 5}$, $m^{sec,n}_{j_0 5}(2) = \mathbb{E}(\tau^{sec,n}_{j_0 5})^2$. Because the reduction method retains the distribution of transition times between the states, we have

$$\begin{aligned} \mathcal{T}^{sec}_{j_0} &= \tau^{sec}_{j_0 5}, \\ \mathbb{E}\mathcal{T}^{sec}_{j_0} &= m^{sec,n}_{j_0 5}(1), \\ \mathbb{E}(\mathcal{T}^{sec}_{j_0})^2 &= m^{sec,n}_{j_0 5}(2). \end{aligned}$$

2.2 Conditional Expected Time to Default

Using the probability distribution $\mathcal{T}^{sec}_{j_0}$, we introduce the *CETD* as a measure of extreme credit risk. Let $1 - \alpha \in (0, 1)$ be our chosen confidence level. The following condition defines the threshold level $VaR_\alpha(\mathcal{T}_{j_0}^{sec})$ for the probability distribution $\mathcal{T}_{j_0}^{sec}$

$$VaR_\alpha(\mathcal{T}_{j_0}^{sec}) = \inf\{T \geq 0 : Pr(\mathcal{T}_{j_0}^{sec} \leq T) \geq \alpha\}. \tag{3}$$

$VaR_\alpha(\mathcal{T}_{j_0}^{sec})$ can be interpreted as the value at risk of the time to default $\mathcal{T}_{j_0}^{sec}$ with tolerance level α, or equivalently T_α^{sec}, the longest time to default in the set of all $(100 \times \alpha)\,\%$ shortest times to default.

Given the fixed level of tolerance α or the corresponding threshold level $VaR_\alpha(\mathcal{T}_{j_0}^{sec})$ for a credit exposure in state j_0, we define *the conditional expected time to default*, $CETD_\alpha(\mathcal{T}_{j_0}^{sec})$, by

$$CETD_\alpha(\mathcal{T}_{j_0}^{sec}) = \mathbb{E}(\mathcal{T}_{j_0}^{sec} | \mathcal{T}_{j_0}^{sec} \leq VaR_\alpha(\mathcal{T}_{j_0}^{sec})). \tag{4}$$

$CETD_{\alpha,j_0}^{sec}$ is coherent because it is a conditional value at risk for the time to default $\mathcal{T}_{j_0}^{sec}$ (Artzner, Delbaen, Eber, and Heath, 1999). Hence, it assumes sub-additivity, positive homogeneity, and translation-equi-variability,

$$CETD_\alpha(\mathcal{T}_{j_0}^{sec} + \mathcal{T}_{\tilde{j}_0}^{sec}) \leq CETD_\alpha(\mathcal{T}_{j_0}^{sec}) + CETD_\alpha(\mathcal{T}_{\tilde{j}_0}^{sec}), \tag{5}$$

$$CETD_\alpha(w\mathcal{T}_{j_0}^{sec}) = wCETD_\alpha(\mathcal{T}_{j_0}^{sec}), \tag{6}$$

$$CETD_\alpha(\mathcal{T}_{j_0}^{sec} + t) = t + CETD_\alpha(\mathcal{T}_{j_0}^{sec}), \tag{7}$$

for all $j_0, \tilde{j}_0 \in S^{sec} \setminus \{5\}$ and $t, w > 0$. Interestingly, the most popular risk measure in the time domain, namely the probability to default (PD), does not possess the above properties. Properties (5) -(7) are important in the analyses of credit risk for portfolio expositions. The first two properties are essential in finding the upper boundary for the risk of the portfolio, whereas the third property enables time shifting in risk analysis.

Using the value at risk in the domain of losses, $VaR(L)$, where L is a loss, enables us to measure the value of extreme losses that occur on a fixed time horizon. However, this measure does not take into account the importance of changing default times. We believe that our $CETD$ is a desirable measure of the time distance to default and a useful measure of credit risk, complementary to the traditional $VaR(L)$.

3 Data and Identification of Credit Quality Ratings

In our empirical research, we consider the claims of commercial banks to non-financial enterprises in Poland. The database contains data from all banks in the Polish banking system. Claims include the following balance sheet items: loans, debt and equity instruments, and remaining receivables. The data are based on the so-called large exposure reporting, i.e., the database contains all exposures toward enterprises in excess of 500,000 zlotys (ca. EUR 120,000) per firm from banks that are either joint-stock companies, state-run banks, or non-associated cooperative banks. The database also includes exposures toward enterprises in excess of 100,000 zlotys per firm from (smaller) associated cooperative banks. The data are available at the bank-enterprise level, which means that each record of the database corresponds to a particular firm exposure in a particular bank. All claims to a firm in a given bank are aggregated and are treated as a single exposure.

We use quarterly observations, and our sample starts in Q1 2004 and ends in Q1 2015. Up to Q2 2013, all banks were reporting quality-of-loan exposures classified into one of the five categories, which is keeping with the Polish Accounting Standards (PAS). These five categories are as follows:

1. Normal - when any delay in the repayment of the principal or interest is less than a month, and the economic and financial situation of debtors does not raise any concerns.

2. Under observation - when the delay in the repayment of the principal or interest is more than a month and no more than three months, and the economic and financial situation of debtors does not raise concerns, or when an exposure requires special attention due to the risk associated with the region, state, industry, customer group, or product group.

3. Substandard - when the delay in repayment of the principal or interest is more than three months but no more than six months, or the economic and financial situation of the obligors may constitute a threat to the timely repayment of the exposure.

4. Doubtful - when the delay in the repayment of the principal or interest is more than six months but no more than twelve months, or the economic and financial situation of the obligors is significantly deteriorated, especially when incurred losses significantly breach their equity (net assets).

5. Lost - when the delay in the repayment of the principal or interest is more than twelve months, or the economic and financial situation of the obligors is significantly deteriorated, and the obligors are unable to repay debt (e.g., the company's bankruptcy was announced).

We code all the risk categories of loans with numbers from 1 (normal credit exposure) to 5 (lost credit exposure, or default).

Unfortunately, most banks have stopped providing full information on the quality of exposures since Q3 2013 due to a change in reporting standards. From Q3 2013, our database includes information about just two subcategories of loans from banks reporting according to International Accounting Standard (IAS). Under the IAS No. 39, outstanding loans can only be divided into exposures at risk of losing value and exposures not at risk of losing value, without isolating additional subcategories. The banks reporting under the IAS hold a majority of loans in the Polish banking sector.

With respect to the data from Q3 2013 until the end of the sample, we matched each exposure to one of the five categories using the following information about provisioning. Under Polish law, all banks set provisions for the risk associated with their activities (also known as specific provisions for credit exposures). Each exposure is classified as either normal, under observation, or a risk group (i.e., substandard, doubtful, and lost). For corporations, rules of provisioning indicate that the amount overdue and the economic and financial situation of a debtor are two independent factors that influence its classification to a particular category.

The size of a specific provision is associated with the risk group that is assigned (i.e., 1.5% of the provisioning basis is assumed for normal duties in the group of loans and receivables under observation, 20% for loans below standard, 50% for doubtful receivables, and 100% for lost claims under the national regulations). The basis for provisioning against the credit risk for claims classified as under observation exposures or risk group exposures can be further reduced by the value of collateral, guarantees, or any rights secured by the law. Because the provision can be reduced and we do not have sufficient information about the collateral or guarantees, we assume that this reduction is dependent on the class of economic activity. We calculate the ranges of provision coverage for each category of loan quality in seperate sections of the economy (Table 1).

At this step, we employ data on exposures and provisions from Q2 2013.[2] Using mathematical notation, the set S^{sec} corresponds to the five quality categories of credit exposure where each state $j \in S^{sec}$ is identified by the value of reserve requirement R_t^{sec} in period t. More precisely, the credit exposure X_t^{sec} is in the state j in period t if the value of provision requirement R_t^{sec} belongs to the fixed interval $[r_j^{sec}, r_{j+1}^{sec})$, i.e.,

$$\{X_t^{sec} = j\} = \{R_t^{sec} \in [r_j^{sec}, r_{j+1}^{sec})\},$$

for all $j = 1, 2, \ldots, 5$, where $r_1^{sec} < r_2^{sec} ... < r_6^{sec}$.

We are aware that in practice the loan quality depends on several factors that are

[2]These are the most recent data available. However the results did not change much from quarter to quarter.

Table 1: Industrial classification of economic activities

Classification symbol	Description	Assigned Number (sec)
-	All nonfinancial corporate sectors of the Polish economy	0
B	Mining	1
C	Manufacturing	2
D	Electricity, gas and steam supply	3
E	Water supply, sewerage, waste management	4
F	Construction	5
G	Retail trade and repairs	6
H	Transportation and storage	7
I	Hotels and restaurants	8
J	Information and communication	9
L	Real estate activities	10
M	Professional, scientific and technical activities	11
N	Administrative activities	12
P	Education	13
Q	Health care	14
R	Arts, entertainment and recreation	15
S	Other services	16

Source: own preparation

related to the terms of the loan contract, lending policy of the bank and the financial situation of the borrowers, for example, and they correlate with each other. In this research, we assume that the state in a previous period aptly identifies the probability of that state changing in the next period.

It must be noted that any exposure can be removed from the database or reclassified for reasons not necessarily connected with the changing of credit quality. For example, the exposure will be erased from the register when its value falls below the threshold of 500,000 zlotys (or 100,000 zlotys in the case of associated cooperative banks). Banks may also remove defaulted loans from the book.[3] Moreover, banks can merge and harmonize their risk policies, which may cause some loans to be reclassified. We decided to account for loans erased from the books of banks by removing the appropriate observations for loans leaving the book from our estimated sample.

[3] In the Polish banking sector, any sales of corporate loan tranches were of insignificant size.

4 Empirical Results

This section includes calculations for the credit-risk measures of the aggregate portfolios of corporate loans from 16 individual economic sectors in Poland as well as for the portfolio of loans from the entire nonfinancial corporate sector of the Polish economy.

Given the sample of quarterly data on loan exposures from Q1 2004 to Q1 2015, we estimate the transition matrices P_t^{sec} using twelve-quarter rolling windows of historical data for each estimated matrix. Based on the estimated transition probabilities, the Markov chain reduction method enables us to compute the probability distribution of time to default $\mathcal{T}_{j0}^{sec}(t)$ for all quarters $t \in [2007Q1, 2015Q1]$ and economic sectors $sec \in \{0, 1, 2, .., 16\}$ (cf., Section 2.1). Figures 1-4 present the cumulative distribution functions of $\mathcal{T}_{j0}^{sec}(t)$. Because the samples used to estimate transition matrices are three-year samples, the PDs as well as other further measures of credit risk observed at time t should be treated as representative of the whole three-year period of the samples ending at period t. One may consider shorter estimation samples or some estimation methods to give more weight to the most recent observations in order to construct more "up-to-date" measures of credit risk.

All figures reveal a limited risk of rapidly deteriorating loan quality within all economic sectors throughout the sample. The time to default of a loan is long even for small cumulative probabilities of default. For example, in 2007 a corporate good-quality loan representative of the economy of the time required more than 20 quarters to default with the probability of 0.05 (cf., Figure 1, the first row and column). In turn, analogous loans from the under-observation, substandard, and doubtful categories required, respectively, more than eight quarters, around two quarters, and only one quarter to default with a probability of 0.05 (cf., Figures 2, 3, and 4, the first row and column). These results suggest that upon entering the state of substandard and doubtful quality, the deterioration process intensifies significantly.

Interestingly, the changes of PDs in time are parallel for all of the loan categories investigated (cf., Figures 1-4). Independently from the initial state of a loan exposure, the least credit risk for most sectors is observed in the period between Q1 2007 and Q4 2008. The risk increases rapidly in the first quarters of 2009 and reveals a slow upward trend in the following years. In some sectors, short-term improvement in the risk conditions is observable in 2012 (sectors C, H, and N) and in 2013 (sectors E, L). Sector R (arts, entertainment, and recreation) has been less dependent on external economic conditions than other sectors, and therefore its associated risk was lowest in 2010, but then it also increased.

The results, when broken down by sector, confirm anecdotal evidence and do not contradict the indices of credit risk for particular economic activities presented in the Financial Stability Reports (FSR) published semi-annually by the Polish central bank (e.g., Narodowy Bank Polski, 2015). As the result of the global financial crisis, the quality of corporate loans deteriorated at the end of 2008 and beginning of 2009. In response to this, credit policies at commercial banks tightened. However, the process of quality deterioration was different depending on the section of the economy. In sectors dominated by huge national infrastructural suppliers such as those represented by section D (electricity, gas and steam supply) and Q (health care), the changes in (high quality) debt servicing were almost invisible (cf., Figures 1-4).

The number of insolvencies rapidly rose mid 2012, and this was a result of worsening conditions in the construction companies (sector F), which had over-invested in earlier years (also during the financial crisis). There were also new bankruptcies of large infrastructural companies, which had to implement their investment projects under unfavorable rules that had been agreed upon prior to the crisis. These problems affected the whole chain of suppliers in the economy, and other sectors as well. A worsening performance of the construction sector (i.e., rapidly falling time to default as well as small values of this measure in general) is confirmed by the increasing levels of non-performing loan ratios. A negative trend in world coal prices strongly affected the Polish mining industry (sector B) from the beginning of 2014. The time to default decreased dramatically especially in the group of well-servicing debtors. All calculated measures suggest that the risk in sector H (transportation and storage) was the most stable over time. Although the PDs were relatively high, the financial crisis did not affect this activity much.

Another credit risk measure, namely ETD, behaves in a very similar way to the estimated default probabilities over time. Figures 5-8 present changing mean and standard deviation of $\mathcal{T}_{j0}^{sec}(t)$. Importantly, the unconditional expected times to default are extremely long, reaching even thousands of quarters for some sectors. In practice, such a result complicates the analysis of risk in loan portfolios, because risk scenarios require more plausible time horizons for potential defaults. However, this fact does not prohibit treating ETD as an index of credit risk and observing its changes in time. An interesting finding is the parallel change in the values of ETD and standard deviation of $\mathcal{T}_{j0}^{sec}(t)$, which suggest that the time to default is more uncertain for less risky portfolios.

We overcome the problem of the long, uninterpretable times to default derived from the ETD measures by proposing a $CETD$ that focuses on the tail behavior of time to default. The dynamics of $CETD$ and VaR risk measures in all economic sectors are presented in Figures 9-12. We set plausible tolerance levels at $\alpha = 0.10$ and $\alpha = 0.05$ and calculate values at risk $VaR_{0.10}$ and $VaR_{0.05}$ in the time domain. Here, the times to default are much shorter, but they still rise above 100 quarters for good-quality loans for some sectors during more prosperous times. This result suggests relatively conservative lending policies on the part of banks toward nonfinancial firms in Poland. Such policies limit the risk of loan defaults in normal times. The $CETD$ measures confirm this finding and rarely fall below 10 quarters, even for the more conservative values of $\alpha = 0.05$.

A slightly different picture is revealed when observing the performance of worse quality loans from the substandard and doubtful categories (Figures 11 and 12). These loans are expected to default very quickly under an extreme scenario, as the $CETD_{0.05}$ fluctuates around one quarter over time for most economic sectors. In the stress scenario, the nonperforming loans are usually expected to default immediately. The differences between the values of *ETD* and *CETD* measures show the consequences of a likely scenario that is realized on average once in 10 or 20 quarters in the credit market. This finding is important because it suggests that nonperforming corporate loans may, in general, be treated as defaulted exposures in stress events.

The *CETD* measure calculated for loans from the normal category changes over time in a very similar way to the *ETD*, *VaR*, and $SP(4Y)$ measures of credit risk, where $SP(4Y) = 1 - PD(4Y)$ is the four-year survival probability of a loan in the investigated portfolio (cf., Figure 12). These measures can be used interchangeably as indices of credit risk. Interestingly, they do not perfectly match the developments of the nonperforming loan ratio (*NPLR*), a widely used statistic that assesses loan quality in empirical studies. One explanation of this result is that the *NPLR* is highly dependent on the dynamics of credit growth and the changing maturity structure of a loan portfolio.

5 Conclusions

This paper presents a new measure of extreme credit risk in the time domain, namely the conditional expected time to default. It has a clear interpretation and can be applied in a straightforward way to analyses of loan performance in time. We apply a novel method to compute the *CETD* with the use of Markov probability transition matrices that are widely used in survival analyses. Empirical study concerning corporate loans from 16 economic sectors in Poland confirms the usefulness of our new measure. *CETD* changed values in line with other measures of credit risk, including the survival probability, across industries and over the entire investigated sample. Using this measure, we were able to identify the riskiest sectors and the most turbulent periods in the Polish economy.

It is clear that the Markov process is only a rough approximation of the true process generating loan defaults. The main problem is that the Markov transition matrices are dependent on macro- and micro-economic fluctuations, and they are not constant in time or across economic sectors. We deal with these problems by estimating the transition matrices using the twelve-quarter rolling-window samples, with separate samples for different economic sectors. Nevertheless, future research may find suitable extensions for our simple model. The method of Górajski (2009) used to calculate distributions of time to default can also be used when transition matrices depend on external factors or when the sizes of the matrices are larger than in our example.

References

Anderson TW, Goodman LA (1957) Statistical inference about markov chains. The Annals of Mathematical Statistics pp 89–110

Andreeva G, Ansell J, Crook J (2007) Modelling profitability using survival combination scores. European Journal of Operational Research 183(3):1537–1549

Artzner P, Delbaen F, Eber JM, Heath D (1999) Coherent measures of risk. Mathematical Finance 9(3):203–228, DOI 10.1111/1467-9965.00068, URL http://dx.doi.org/10.1111/1467-9965.00068

Banasik J, Crook JN, Thomas LC (1999) Not if but when will borrowers default. Journal of the Operational Research Society pp 1185–1190

Bellotti T, Crook J (2013) Forecasting and stress testing credit card default using dynamic models. International Journal of Forecasting 29(4):563–574

Beran J, Djaïdja AYK (2007) Credit risk modeling based on survival analysis with immunes. Statistical Methodology 4(3):251 – 276, DOI http://dx.doi.org/10.1016/j.stamet.2006.09.001, URL http://www.sciencedirect.com/science/article/pii/S1572312706000487

Bharath ST, Shumway T (2008) Forecasting default with the merton distance to default model. Review of Financial Studies 21(3):1339–1369

Bluhm C, Overbeck L, Wagner C (2010) Introduction to credit risk modeling. Chapman & Hall

Bystrom H, Kwon OK (2007) A simple continuous measure of credit risk. International Review of Financial Analysis 16(5):508–523, URL http://ideas.repec.org/a/eee/finana/v16y2007i5p508-523.html

Carling K, Jacobson T, Lindé J, Roszbach K (2007) Corporate credit risk modeling and the macroeconomy. Journal of Banking & Finance 31(3):845–868

Charitou A, Dionysiou D, Lambertides N, Trigeorgis L (2013) Alternative bankruptcy prediction models using option-pricing theory. Journal of Banking & Finance 37(7):2329–2341

Cox DR (1972) Regression models and life-tables. Journal of the Royal Statistical Society Series B (Methodological) 34(2):pp. 187–220, URL http://www.jstor.org/stable/2985181

Crouhy M, Galai D, Mark R (2000) A comparative analysis of current credit risk models. Journal of Banking & Finance 24(1):59–117

Ebnöther S, Vanini P (2007) Credit portfolios: What defines risk horizons and risk measurement? Journal of Banking & Finance 31(12):3663–3679

Feng D, Gouriéroux C, Jasiak J (2008) The ordered qualitative model for credit rating transitions. Journal of Empirical Finance 15(1):111–130

Fernandes J (2005) Corporate Credit Risk Modeling: Quantitative Rating System And Probability Of Default Estimation. Finance 0505013, EconWPA, URL http://ideas.repec.org/p/wpa/wuwpfi/0505013.html

Fitch (2015) Global corporate finance 2014 transition and default study. Tech. rep., Fitch Ratings

Frydman H, Schuermann T (2008) Credit rating dynamics and markov mixture models. Journal of Banking & Finance 32(6):1062–1075

Glennon D, Nigro P (2005) Measuring the default risk of small business loans: A survival analysis approach. Journal of Money, Credit and Banking pp 923–947

Górajski M (2009) Reduction of absorbing markov chain. Annales UMCS, Mathematica 63(1):91–107

Jafry Y, Schuermann T (2004) Measurement, estimation and comparison of credit migration matrices. Journal of Banking & Finance 28(11):2603–2639

Kadam A, Lenk P (2008) Bayesian inference for issuer heterogeneity in credit ratings migration. Journal of Banking & Finance 32(10):2267–2274

Kiefer NM (1988) Economic duration data and hazard functions. Journal of Economic Literature 26(2):pp. 646–679, URL http://www.jstor.org/stable/2726365

Lambrecht B, Perraudin W, Satchell S (1997) Time to default in the uk mortgage market. Economic Modelling 14(4):485–499

Lando D (1994) Three essays on contingent claims pricing. Cornell University, May

Leland HE (2004) Predictions of default probabilities in structural models of debt. Journal of Investment Management 2:5–20

Moody's (2011) Corporate default and recovery rates, 1920–2010. special comment. Tech. rep., Moody's Investors Service

Narodowy Bank Polski (2015) Financial Stability Report. Narodowy Bank Polski, July

Pflug GC (2000) Some remarks on the value-at-risk and the conditional value-at-risk. In: Probabilistic constrained optimization, Springer, pp 272–281

Sarlija N, Bensic M, Zekic-Susac M (2009) Comparison procedure of predicting the time to default in behavioural scoring. Expert Systems with Applications 36(5):8778 – 8788, DOI http://dx.doi.org/10.1016/j.eswa.2008.11.042, URL http://www.sciencedirect.com/science/article/pii/S0957417408008610

Shumway T (2001) Forecasting bankruptcy more accurately: A simple hazard model*. The Journal of Business 74(1):101–124

Standard & Poor's (2015) Default, transition, and recovery: 2014 annual global corporate default study and rating transitions. Tech. rep., Standard & Poor's Ratings Services

Stefanescu C, Tunaru R, Turnbull S (2009) The credit rating process and estimation of transition probabilities: A bayesian approach. Journal of Empirical Finance 16(2):216–234

Stepanova M, Thomas L (2002) Survival analysis methods for personal loan data. Operations Research 50(2):277–289

Tong EN, Mues C, Thomas LC (2012) Mixture cure models in credit scoring: If and when borrowers default. European Journal of Operational Research 218(1):132–139

Virolainen K (2004) Macro stress testing with a macroeconomic credit risk model for Finland. Research Discussion Papers 18/2004, Bank of Finland, URL https://ideas.repec.org/p/hhs/bofrdp/2004_018.html

Appendix

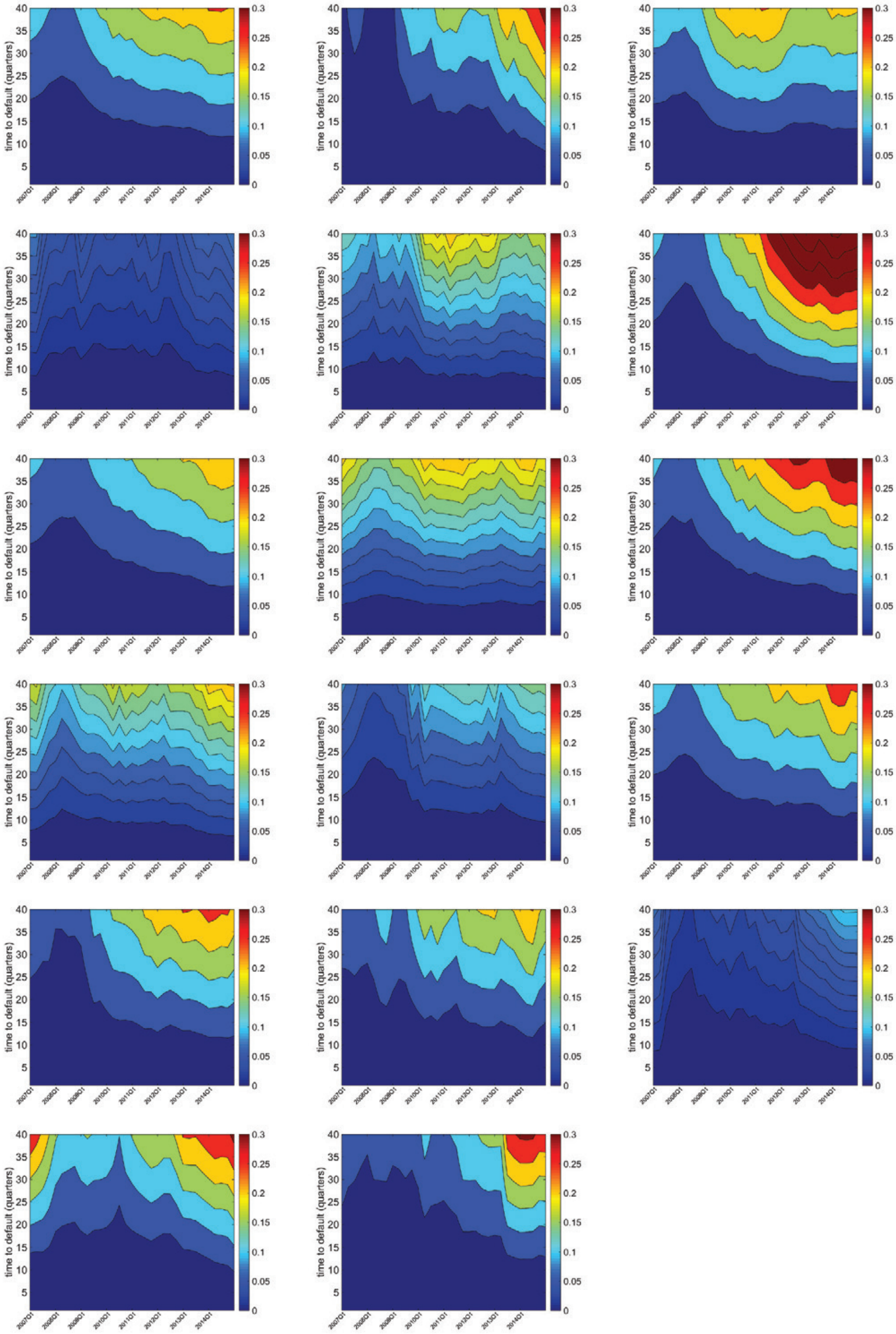

Figure 1: Time evolution of CDF for the time to default $\mathcal{T}_{j_0}^{sec}(t)$ for corporate loans in state $j_0 = 1$ and for all quarters $t \in [2007Q1, 2015Q1]$. Economic sectors $sec = 0, 1, \ldots, 16$ ordered by rows of the panel.
Source: own calculations

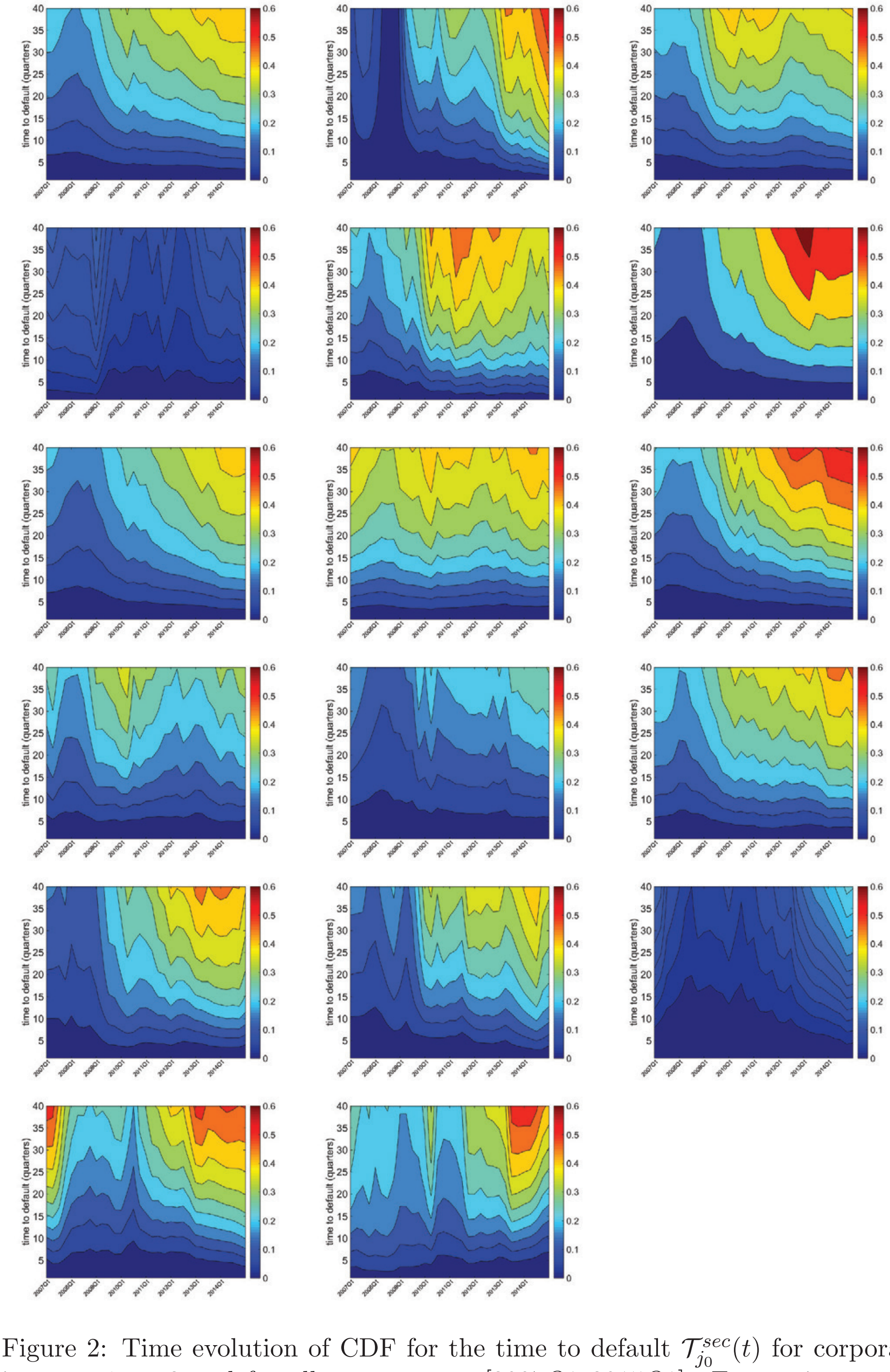

Figure 2: Time evolution of CDF for the time to default $\mathcal{T}_{j_0}^{sec}(t)$ for corporate loans in state $j_0 = 2$ and for all quarters $t \in [2007Q1, 2015Q1]$. Economic sectors $sec = 0, 1, \ldots, 16$ ordered by rows of the panel.
Source: own calculations

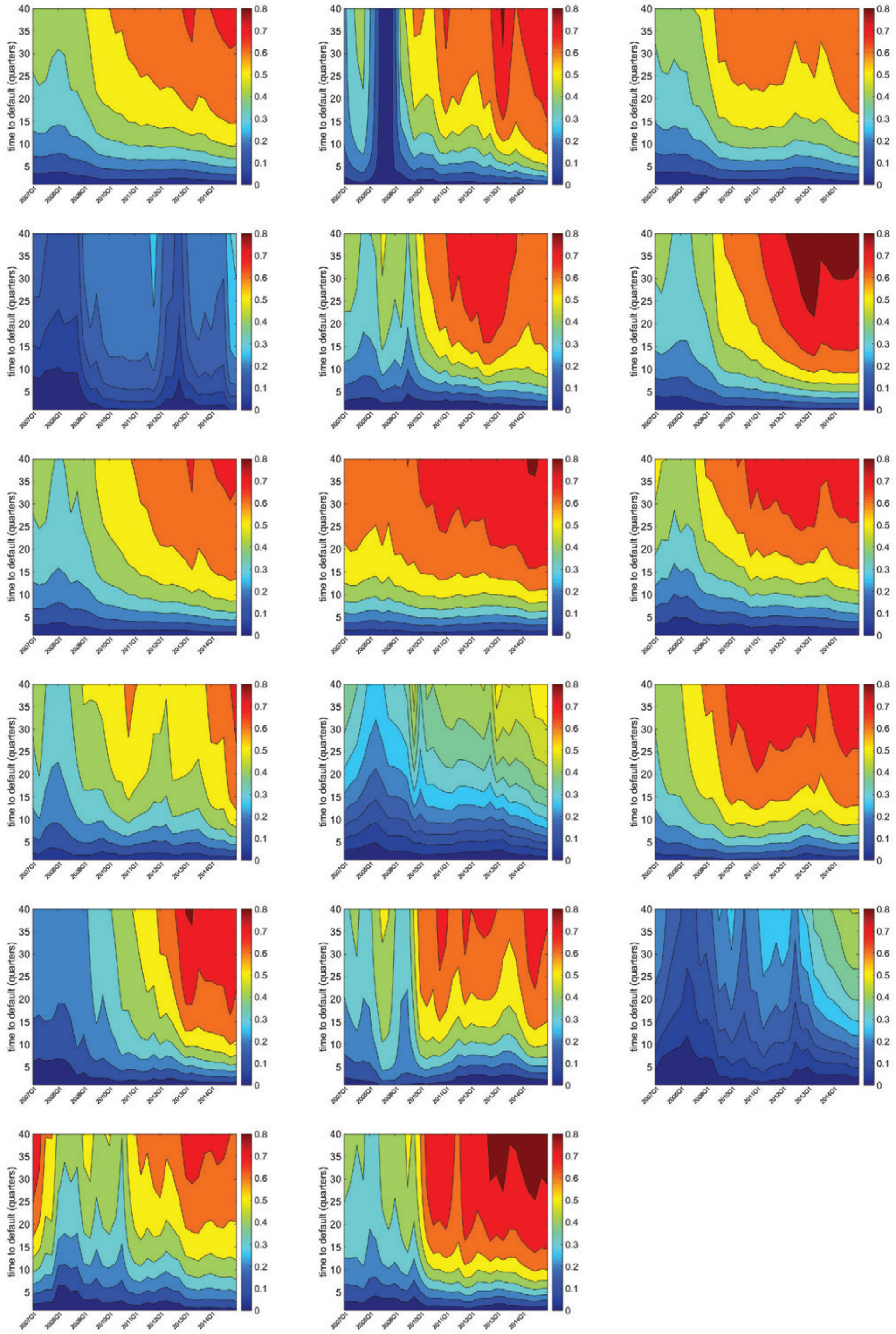

Figure 3: Time evolution of CDF for the time to default $\mathcal{T}_{j_0}^{sec}(t)$ for corporate loans in state $j_0 = 3$ and for all quarters $t \in [2007Q1, 2015Q1]$. Economic sectors $sec = 0, 1, \ldots, 16$ ordered by rows of the panel.
Source: own calculations

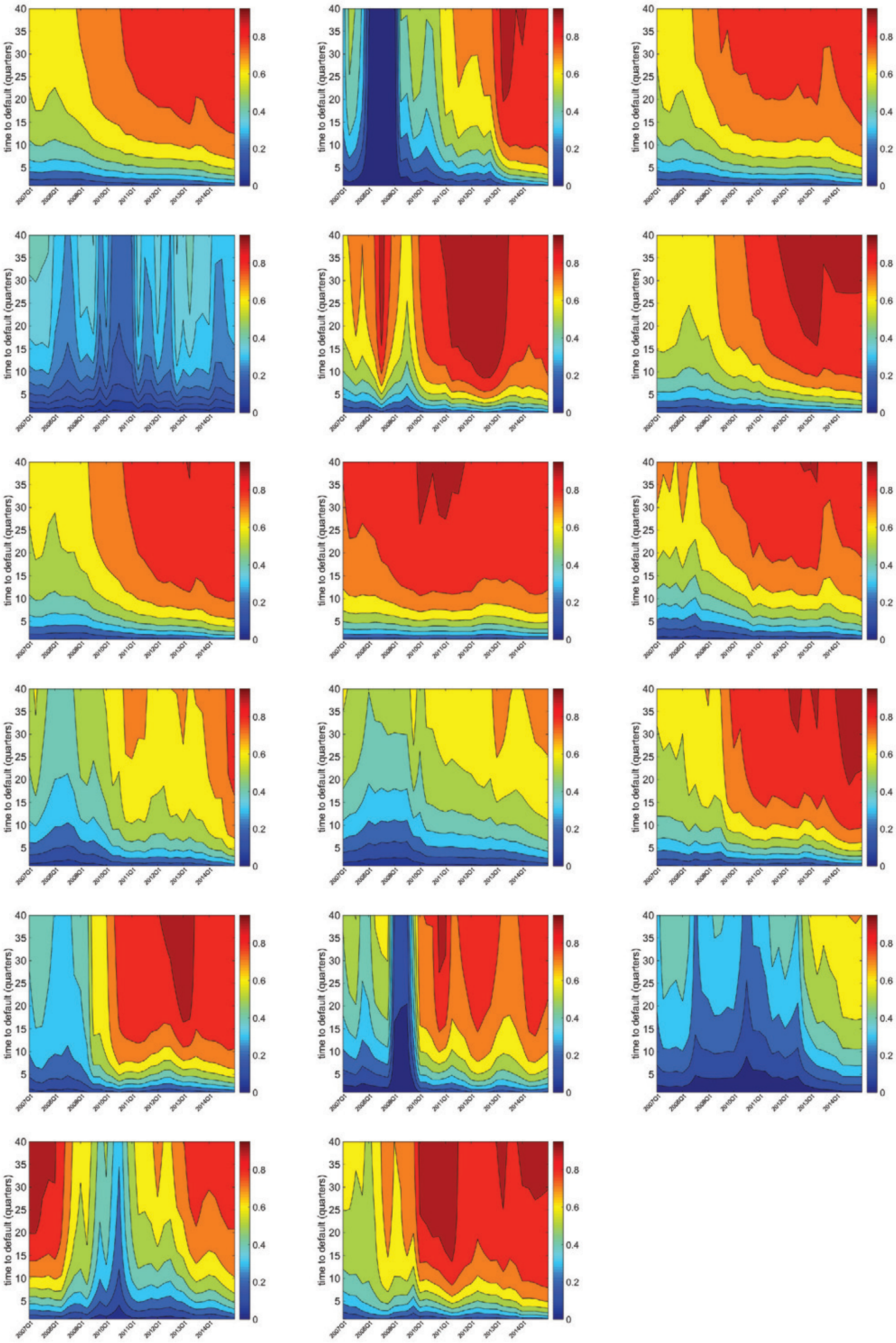

Figure 4: Time evolution of CDF for the time to default $\mathcal{T}_{j_0}^{sec}(t)$ for corporate loans in state $j_0 = 4$ and for all quarters $t \in [2007Q1, 2015Q1]$. Economic sectors $sec = 0, 1, \ldots, 16$ ordered by rows of the panel.
Source: own calculations

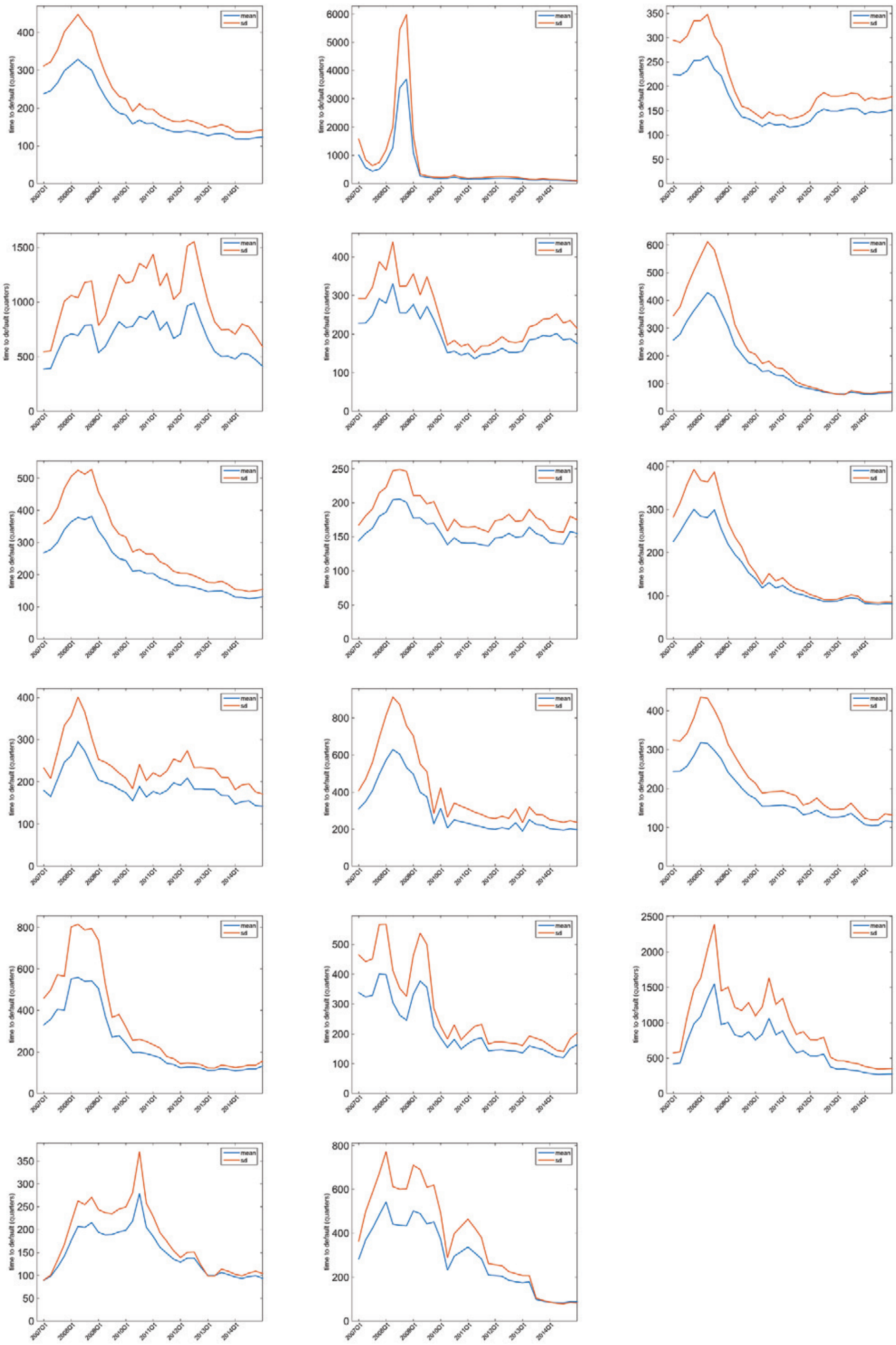

Figure 5: Time evolution of means and standard deviations of time to default $\mathcal{T}_{j_0}^{sec}(t)$ for corporate loans in state $j_0 = 1$ and for all quarters $t \in [2007Q1, 2015Q1]$. Economic sectors $sec = 0, 1, \ldots, 16$ ordered by rows of the panel.
Source: own calculations

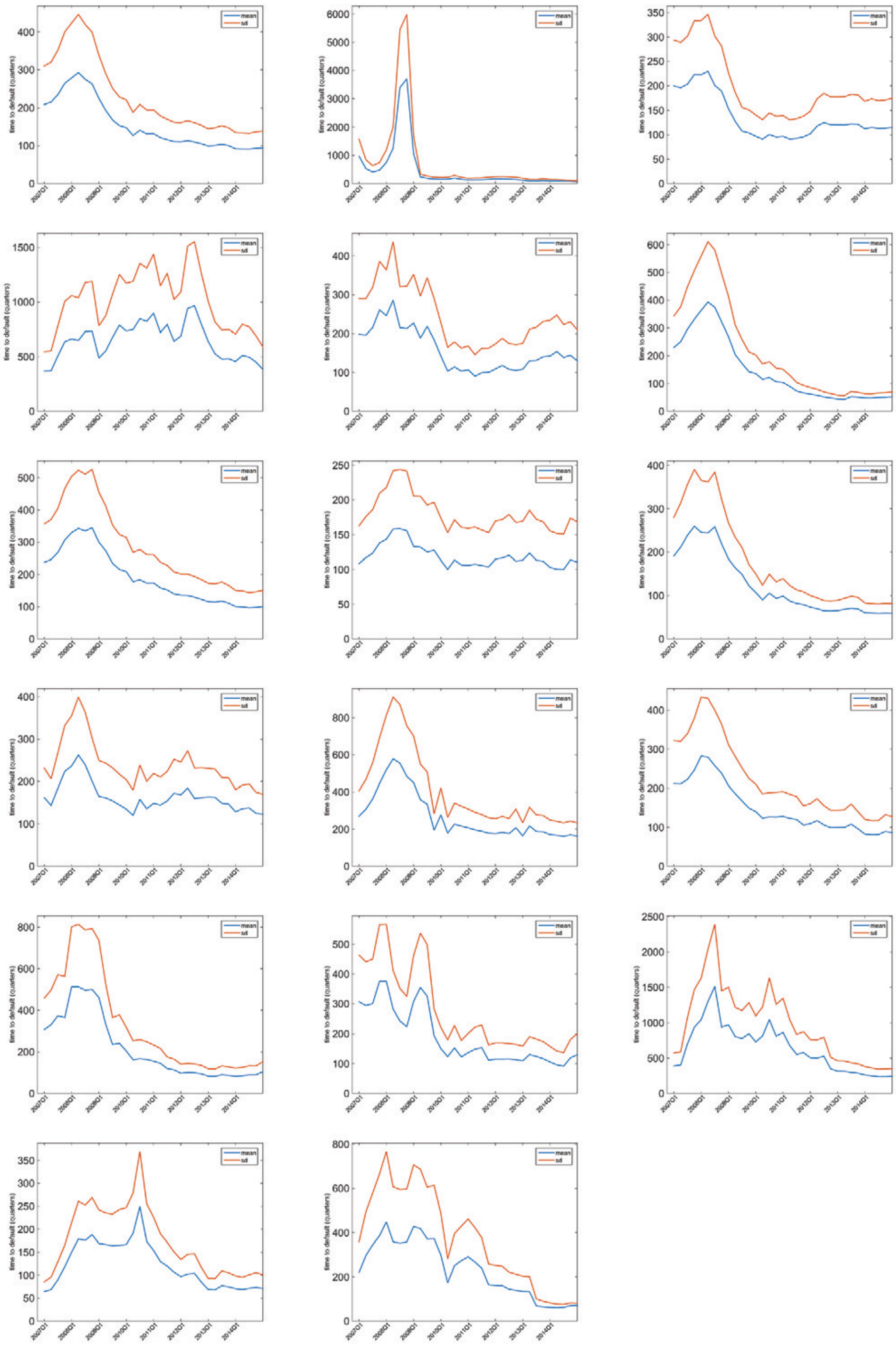

Figure 6: Time evolution of means and standard deviations of time to default $\mathcal{T}^{sec}_{j_0}(t)$ for corporate loans in state $j_0 = 2$ and for all quarters $t \in [2007Q1, 2015Q1]$. Economic sectors $sec = 0, 1, \ldots, 16$ ordered by rows of the panel.
Source: own calculations

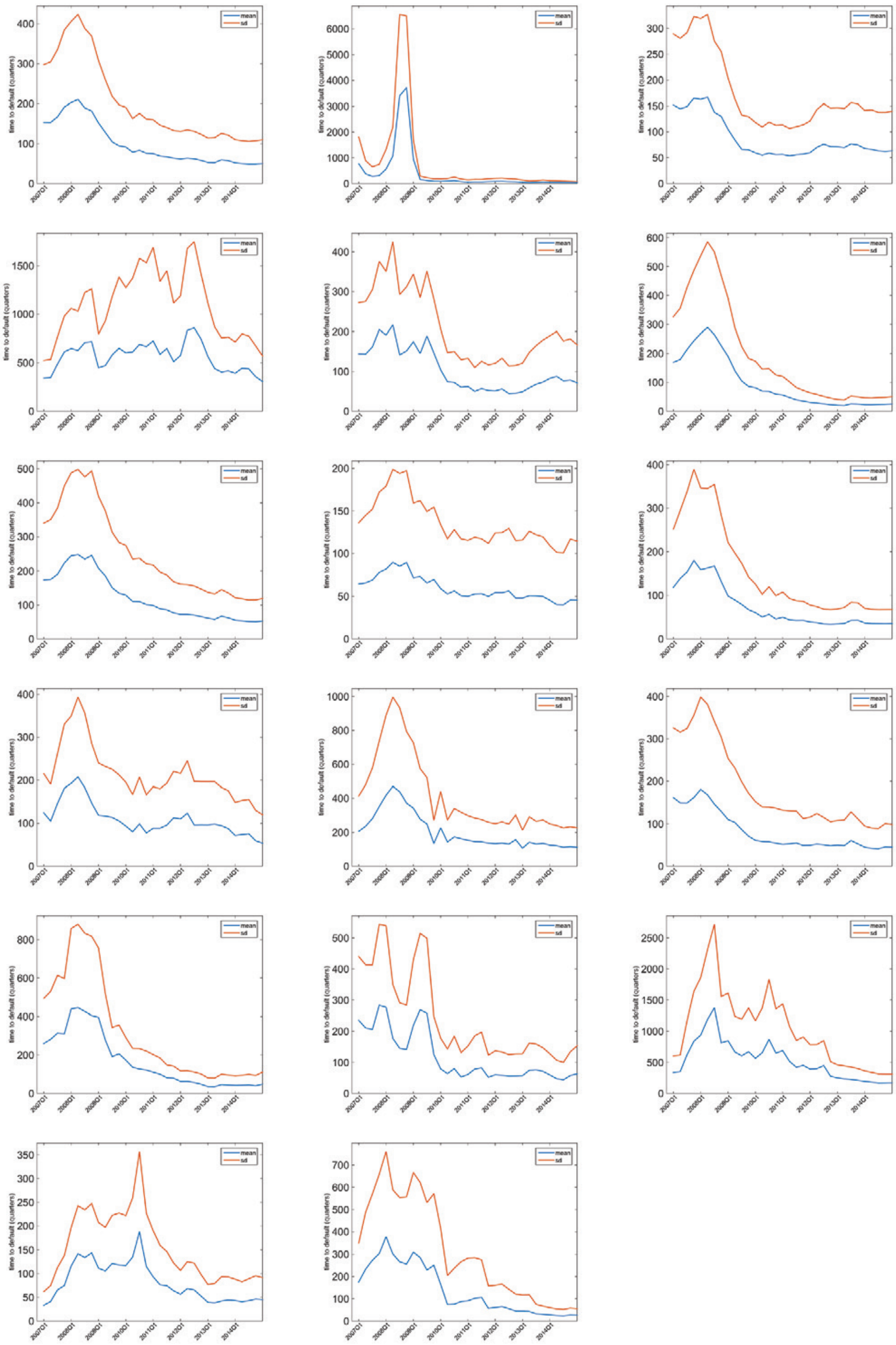

Figure 7: Time evolution of means and standard deviations of time to default $\mathcal{T}_{j_0}^{sec}(t)$ for corporate loans in state $j_0 = 3$ and for all quarters $t \in [2007Q1, 2015Q1]$. Economic sectors $sec = 0, 1, \ldots, 16$ ordered by rows of the panel.
Source: own calculations

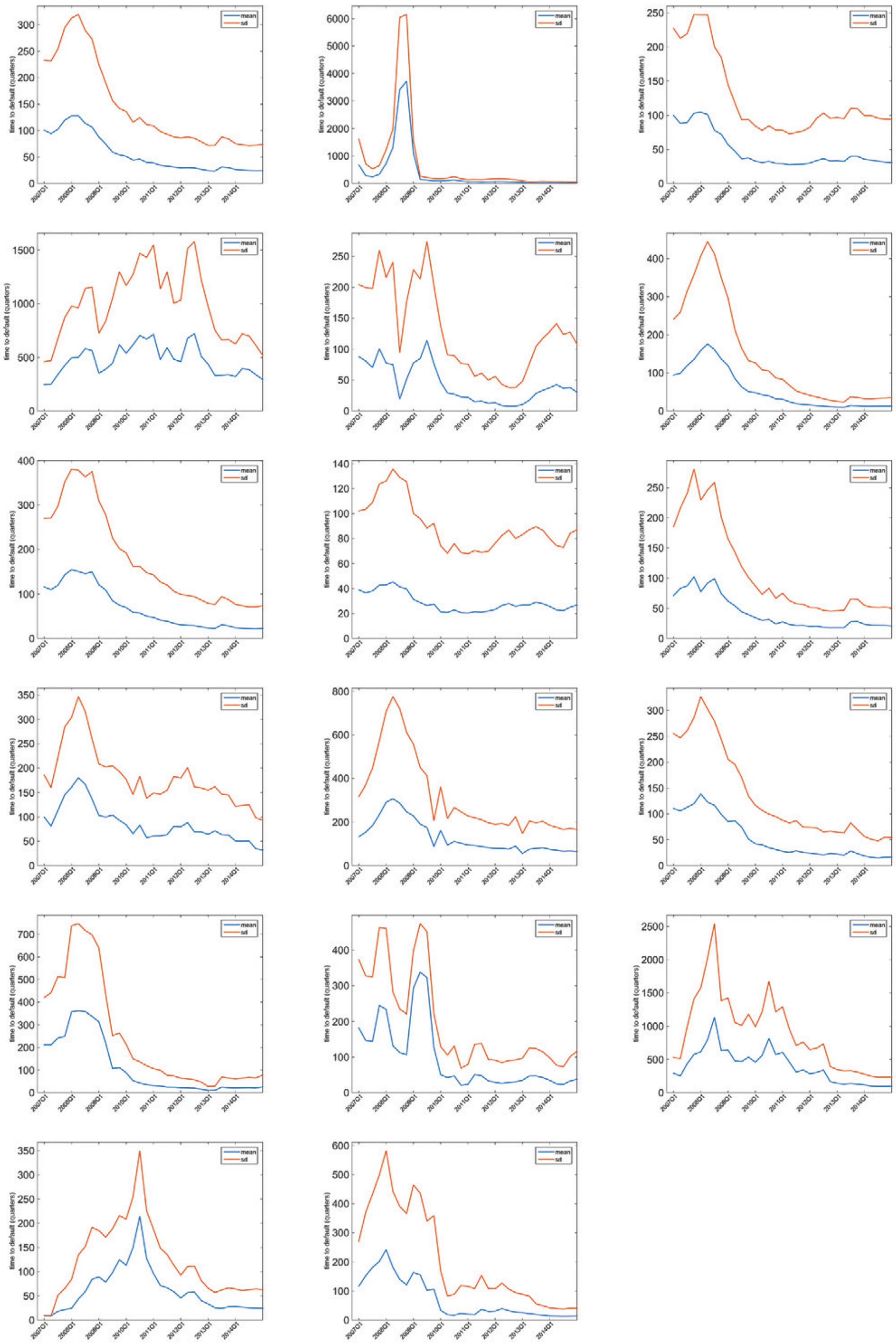

Figure 8: Time evolution of means and standard deviations of time to default $\mathcal{T}_{j_0}^{sec}(t)$ for corporate loans in state $j_0 = 4$ and for all quarters $t \in [2007Q1, 2015Q1]$. Economic sectors $sec = 0, 1, \ldots, 16$ ordered by rows of the panel.
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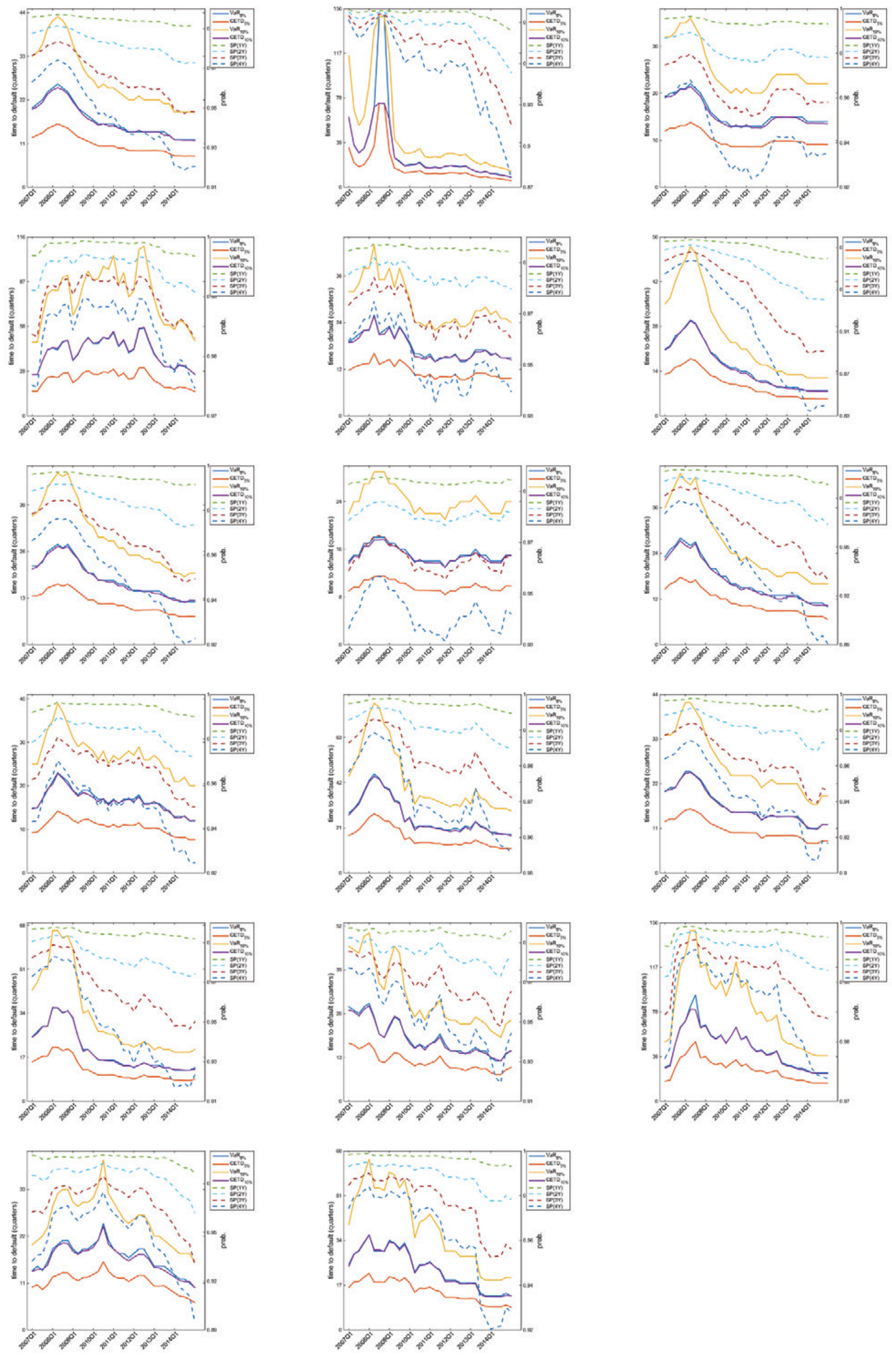

Figure 9: Time evolution of $VaR_\alpha(\mathcal{T}^{sec}_{j_0})(t)$, $CETD_\alpha(\mathcal{T}^{sec}_{j_0})(t)$ and $SP(kY)_{j_0}(t)$ for corporate loans in state $j_0 = 1$ and the tolerance levels $\alpha = 5\%, 10\%$, years $k = 1, 2, 3, 4$ and for all quarters $t \in [2007Q1, 2015Q1]$. Economic sectors $sec = 0, 1, \ldots, 16$ ordered by rows of the panel.

Source: own calculations

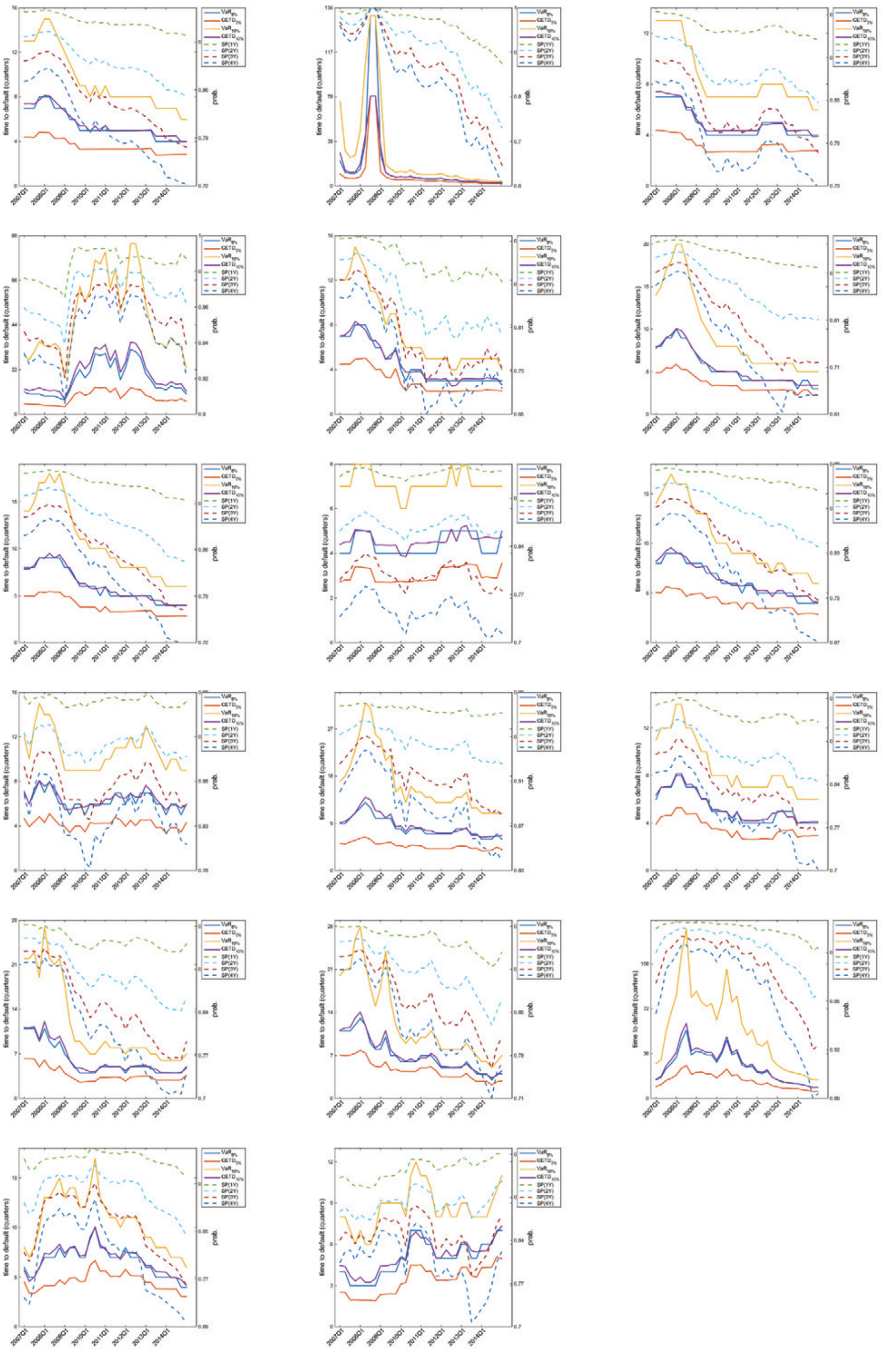

Figure 10: Time evolution of $VaR_\alpha(\mathcal{T}_{j_0}^{sec})(t)$, $CETD_\alpha(\mathcal{T}_{j_0}^{sec})(t)$ and $SP(kY)_{j_0}(t)$ for corporate loans in state $j_0 = 2$ and the tolerance levels $\alpha = 5\%, 10\%$, years $k = 1, 2, 3, 4$ and for all quarters $t \in [2007Q1, 2015Q1]$. Economic sectors $sec = 0, 1, \ldots, 16$ ordered by rows of the panel.

Source: own calculations

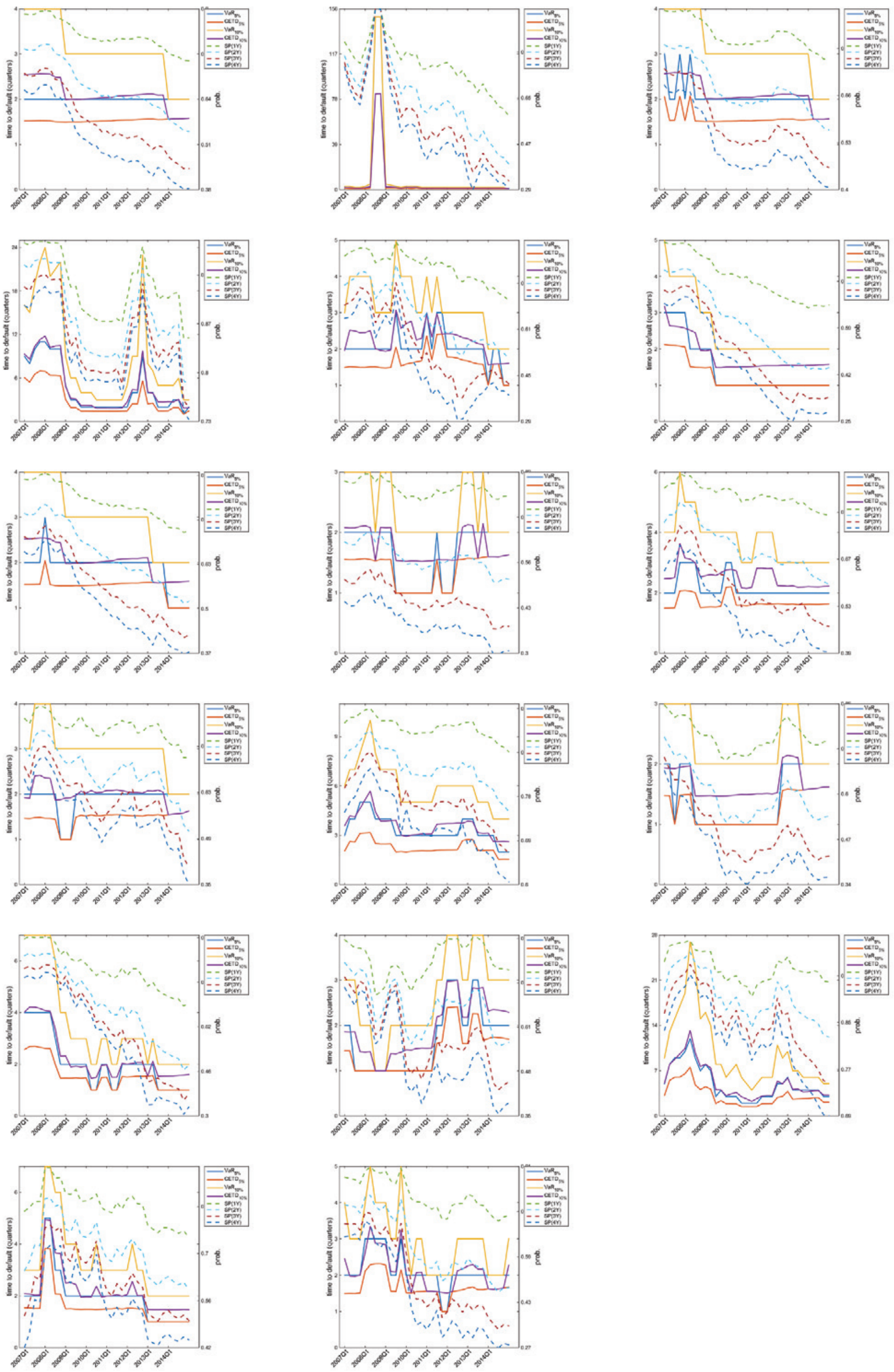

Figure 11: Time evolution of $VaR_{\alpha}(\mathcal{T}_{j_0}^{sec})(t)$, $CETD_{\alpha}(\mathcal{T}_{j_0}^{sec})(t)$ and $SP(kY)_{j_0}(t)$ for corporate loans in state $j_0 = 3$ and the tolerance levels $\alpha = 5\%, 10\%$, years $k = 1, 2, 3, 4$ and for all quarters $t \in [2007Q1, 2015Q1]$. Economic sectors $sec = 0, 1, \ldots, 16$ ordered by rows of the panel.
Source: own calculations

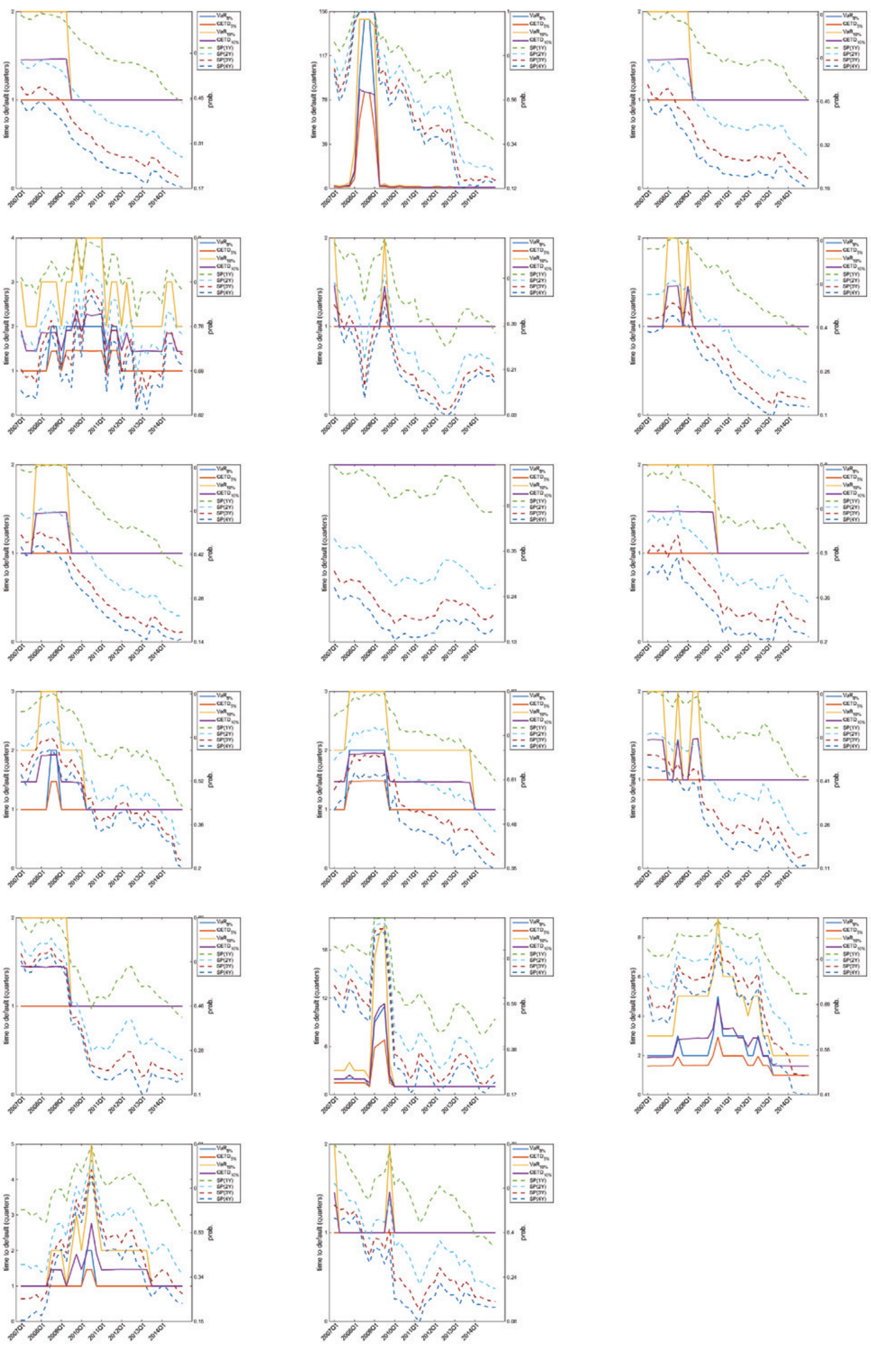

Figure 12: Time evolution of $VaR_\alpha(\mathcal{T}_{j_0}^{sec})(t)$, $CETD_\alpha(\mathcal{T}_{j_0}^{sec})(t)$ and $SP(kY)_{j_0}(t)$ for corporate loans in state $j_0 = 4$ and the tolerance levels $\alpha = 5\%, 10\%$, years $k = 1, 2, 3, 4$ and for all quarters $t \in [2007Q1, 2015Q1]$. Economic sectors $sec = 0, 1, \ldots, 16$ ordered by rows of the panel.
Source: own calculations

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