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Consumption over the life cycle in Poland

Arkadiusz Florczak, Janusz Jabłonowski



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Abstract

The article attempts to verify the existence and strength of the buffer stock and precautionary savings' behaviours of households in Poland, with the use of the dynamic stochastic model of permanent income with life cycle hypothesis (PILCH). The theoretical part of the model relies heavily on Gourinchas & Parker [14], while numerical solutions are based on Carroll [3]. The model includes partial insurance of households against idiosyncratic risk. The data relies on two household surveys: on budgets (HBS) and wealth (HWS), with parametrisation based on the 1% sample from the social insurance administrative data. The results generally seem to confirm the initial presumption on doubtful reflection of the dynamic economic reality of the fast converging market economy in the applied version of the model. The reason may stem from the lack of sufficiently stable economic environment through at least one full working career path of the household generation. Polish households, in general, are not (yet) patient enough to create buffer stock behaviour based on financial means, so precautionary behaviour prevails. The detailed results for decomposed types of households show proof for buffer stock behaviour for high school graduates from richer regions, and specific professions.

JEL: C49, C61, D91

Keywords: consumption over life cycle, precautionary savings, household wealth survey, household budget survey, simulated method of moments, data matching, endogenous gridpoints.

Introduction

The article attempts to verify the existence and strength of the buffer stock and precautionary savings' behaviours of households in Poland, with the use of the dynamic stochastic model of permanent income with life cycle hypothesis (PILCH). The theoretical part of the model relies heavily on Gourinchas & Parker [14], while numerical solutions are based on Carroll [3]. The model includes a partial insurance of households against idiosyncratic risk, e.g. future income uncertainty, to maximise the expected utility of consumption.

In the theoretical justification of the PILCH we follow the rationale proposed by Guvenen [15], who, in pursuit of an answer to the consequences of the existence of heterogeneity in macroeconomics asks *how far are we from the complete markets?*. Or, more specifically, to what extent the partial insurance against the idiosyncratic risk affects the concept of complete markets? In principle, the PILCH should help to explain the changes in distribution patterns on aggregates analysing a partial insurance against the uncertainty, in case of lack of perfect risk sharing among agents. On the basis of data from the US, UK and Taiwan Deaton and Paxson [10] suggest the existence of heterogeneity in the life cycle for logs of income and growing inequalities that increase with age. The canonical part of the PILCH model relies on the assumption that the agents have free access to the assets and liabilities, which gives a lower level of insurance than perfect risk sharing. The analytical realisation of the model is fairly close to Carroll [3], where the households keep low financial means for precautionary purposes, labelled as the *precautionary behaviour*. Such an approach may help to explain the stock of savings accumulated by young households against the uncertainty of unemployment. However, after having built the initial precautionary stock of savings, impatience occurs, which should explain the hampered process of a further increase in savings that are kept relatively fixed despite advancing age and generally serve as negative shock absorbers. If things get permanently worse, the households reduce consumption, while the positive shocks to income are consumed immediately. With approaching retirement, households should increase their savings to smoothen the transition from full labour to reduced income flows from pensions, which is called *buffer stock behavior*. The model purposefully does not include the statutory pension system to distinguish voluntary savings only.

Since the range of calibration is limited to a single country on a rapid convergence process from central planning to a market economy, the context of the life cycle consumption path is affected by this transition process. Therefore, the data extended by the model may help to verify the existence of significant economic distortion in macroeconomic and social reality, i.e. the dynamic transformation of the economy from socialist central planning to an open market economy that started in early 90's. On the one hand, on the basis of the Social Diagnosis (2013), around 67% (with an observed increasing trend in recent years) of Polish households that declare any financial savings, justified them by precautionary motives. The comparable survey by Deutsche Bank from 2014 suggests 34%¹). On the other hand, the same Social Diagnosis reports the absence of savings in 41% of households, while 25% keep savings that do not exceed a single monthly salary, and the savings of 32% of the following households range between 1 and 3 salaries, with just 13% declaring savings between the semi-annual to annual salary level. Only 6% reports higher savings than the entire annual salary stock. The table below summarizes yet another survey on declarations of the saving motives.

¹The results do not seem different, since the Social Diagnosis allowed for multiple choice of motives

Table 1: Declared saving motives

Saving motive/year	2000	2003	2005	2007	2009	2011	2013	2015	av 2000-15
Reserve for unexpected events	75.6	72.1	70.6	65.8	61.1	61.5	66.0	63.7	67.1
For retirement	-	43.6	44.9	39.9	36.1	36.5	35.9	31.5	38.3
Health expenditure	44.7	39.6	42.1	34.5	29.2	25.0	29.6	27.1	34.0
Reserve for consumption expenditure	40.2	34.8	31.0	35.1	30.5	33.2	32.5	38.1	34.4
House/flat renovation	37.6	39.1	35.8	32.8	28.4	25.0	26.3	22.8	31.0
Securing children's future	35.0	31.2	30.9	28.7	25.0	23.2	23.1	18.7	27.0
Saving for leisure	30.6	29.3	31.3	29.5	26.4	23.5	24.7	22.7	27.3
To purchase durable goods	34.5	27.9	29.6	29.5	25.6	22.7	21.3	20.3	26.4
Other reasons	20.4	18.4	19.3	17.8	17.0	14.0	12.4	12.0	16.4
For fixed payments	20.2	20.8	18.5	15.5	14.1	12.0	12.7	16.6	16.3
No reason	11.3	9.9	12.2	8.7	11.4	10.1	9.7	9.7	10.4
Purchase of house/flat	11.8	12.1	10.5	8.7	8.7	6.4	5.6	4.1	8.5

Source: Social Diagnosis database.

The availability of the pilot Household Wealth Survey (HWS) ² in 2015 allowed for the estimation of the model with the very first representative sample of 3,500 Polish households. The results of the HWS draw the conclusion that around 95% of the entire net wealth of Polish households is stored in non-financial assets, while the financial means are generally low and kept for precautionary purposes. In addition to the Household Budget Survey (HBS)³, the data coverage in HWS helped in more reliable reflection of the reality in our model. The additional parametrisation based on the representative time invariant 1% sample of the social insurance system allowed for the detailed parametrisation of shocks to particular components of income. The existence of common variables in both datasets allowed for their matching by the nearest neighbours and creating an integrated dataset exercised for HWS and HBS comparability.

²Joint work of the Narodowy Bank Polski (NBP) and the Polish Central Statistical Office (CSO)

³Developed by the CSO since 1995, which provides a quarterly rotating sample of approx. 37,000 households.

The results generally seem to confirm the initial presumption on the poor reflection of the dynamic economic reality of the fast converging market economy in the PILCH model stemming from the lack of sufficiently stable economic environment through at least one full working career path of the household generation. Bearing in mind the first point, the model results are in line with the implicit conclusion of the HWS that Polish households, in general, are not (yet) patient enough to create the buffer stock behaviour based on financial means. The detailed results for decomposed types of households show proof for the buffer stock behaviour for high school graduates from richer regions, and specific professions. Nevertheless, generally, with further absence of patience for building voluntary financial savings, the insurance against shocks to permanent income will remain weak, which is worrying in terms of the inevitable ageing society process.

The paper is structured in the following way: after a current introduction, part 1 explains the model assumptions, part 2 develops the data description, part 3 covers the results, part 4 describes the doubts on the rationale, model and data, which is followed by the summary and finally concluded by the technical appendix.

1. Model description

1.1. Model framework

The discrete-time, life-cycle model of household consumption behaviour is considered as in [14]. Households live for N years and work for $T < N$ ⁴ years, and spend the remaining lifetime in retirement. At the beginning of every working year a household receives income $\{Y_t\}_{t \in \{1, \dots, T\}}$, which is of a stochastic nature.

Every year, right after receiving the salary, a household must decide how much to consume and how much to save for future utility from consumption or bequest. Households behave rationally, envisage perfectly their lifetime financial capacities and maximize the discounted expected utility from current and future consumption:

$$E \left[\sum_{t=0}^{N-s} \beta^t \cdot u(C_t, Z_t) + \beta^{N+1} \cdot V_{N+1}(W_{N+1}) \right], \quad (1)$$

where:

β - time rate preferences of household,

C_t - level of consumption at the moment t ,

Z_t - taste shifters which depends on household characteristics⁵,

V_{N+1} - additional utility derived from leaving bequest at moment of death,

$u(C_t, Z_t)$ - household's utility of consumption function,

W_{N+1} - bequest size (or equivalently, wealth at the moment of death i.e. $N + 1$).

Households allocate their savings at risk free financial asset with rate of return equal to $R - 1$.

1.2. Income process

A stochastic process that forms a household's income consist of permanent income component P_t and transitory shock U_t :

$$\begin{aligned} Y_t &= P_t \cdot U_t, \\ P_t &= G_t \cdot P_{t-1} \cdot N_t. \end{aligned} \quad (2)$$

⁴Values of N and T are predetermined. Though, it is easy to introduce varying probabilities of dying, however, it would have negligible effect on model since we focus only on the working period.

⁵For instance, size of household, number of children, household location etc.

for $t \in \{1, \dots, N\}$ where:

$\{G_t\}_{t \in \{1, \dots, N\}}$ - drift ⁶ of permanent income process at the moment t ,

$\{P_t\}_{t \in \{0, \dots, N\}}$ - stochastic process describing permanent income; it is a product of permanent income from the previous period t , drift G_t and transitory shock N_t . We consider P_0 to be fixed, but it can also be a random variable.

Table 2: Types of shocks to labour income

Permanent shock	Transitory shock
Uncertainty about promotions	Uncertainty about bonuses
Uncertainty about wage rises	Temporary unemployment
Health status shocks	Health status shocks
	Uncertainty about seasonal factors

As a result, the log of permanent income is AR(1) process with unit root⁷. There are distortions to the income process of a permanent and transitory nature:

$\{N_t\}_{t \in \{1, \dots, N\}}$ - shock to permanent income; consists of i.i.d. random variables which have a lognormal pdf-s with log-mean equal to 0 and log-standard deviation equal to σ_n^2 ,⁸

$\{U_t\}_{t \in \{1, \dots, N\}}$ - transitory shock to income; consist of i.i.d. random variables which have a mixed pdf-s; there exists a positive chance $p > 0$ of $\mu \cdot P_t$ income realization⁹ ($P(U_t = \mu \cdot P_t) = p$) and the rest of probability mass is lognormal, with log-mean equal to $\frac{-p}{1-p} \cdot \ln(\mu)$ ¹⁰ and log-standard deviation equal to σ_u^2 ,

⁶It catches predictable changes to income depending on household characteristics such as education level etc.

⁷It is a strong assumption, although, necessary to obtain a model with only one state variable.

⁸In another version of this model shocks to income have log-mean equal to $-\sigma_n^2/2$ (and this term applies to transitory shock) so expected values (on level terms) of these shocks are equal to one. In this paper all methods are presented with an assumption of log-like income process - it is easy to make changes to model solutions in order obtain results adequate to levels. In fact, we use a level-based model to perform an estimation on a basis of information about household wealth.

⁹ μ is interpreted as unemployment benefit in relation to permanent income.

¹⁰This correction guarantees that the expected shock to log-income will be equal to zero.

1.3. Vector of deterministic household characteristics

As in [14] we assume one dimensional characteristics vector Z_t which depends only on household size at the moment t .

1.4. Utility function

We assume that utility function is CRRA class with taste shifters $v(Z_t)$, which depends on the age of the household and is given by following formula:

$$u(C_t, Z_t) = v(Z_t) \cdot \frac{C_t^{1-\rho}}{1-\rho}. \quad (3)$$

1.5. Further assumptions

At the moment t households maximize expected discounted future utility from consumption on the basis of 1, subject to budget constraint:

$$W_{t+1} = R \cdot (W_t - C_t) + Y_{t+1}. \quad (4)$$

for $t \in \{0, 1, \dots, N - 1\}$. To make the problem tractable, the authors of [14] made additional assumptions. They assumed the existence of a second type of assets, which are retirement funds H_t . The accumulation of retirement funds is exogenous; households cannot use these assets to finance consumption before retirement. Furthermore, in the last working period there is no income risk, so $P_{T+1} = P_T$ and in this period households cannot borrow against future income. Illiquid wealth at the moment of retirement is proportional to the component of permanent income at the last working period i.e. $H_{T+1} = h \cdot P_{T+1}$.

Liquid assets at t will be denoted as X_t . Liquid wealth at the beginning of each period fulfils:

$$X_{t+1} = R \cdot (X_t - C_t) + Y_{t+1}. \quad (5)$$

The authors of [14] cut the optimization problem at the moment of retirement, assuming that future and present utility from consumption are equal to:

$$V_{T+1}(X_{T+1}, H_{T+1}, Z_{T+1}) = \kappa v(Z_{T+1}) \cdot (X_{T+1} + H_{T+1})^{1-\rho}, \quad (6)$$

where κ is a positive constant. Under the CRRA assumption, this functional form is exactly correct if time of death and/or asset returns are the only sources of uncertainty

after retirement (see Merton [24]). As a result of this assumption, optimal consumption at the moment of retirement will be a linear function of wealth. Finally, cash on hand at the beginning of life is strictly positive, i.e. $X_0 > 0$. Based on these assumptions, the household's problem at the moment $t \leq T + 1$ takes the following form:

$$V_t(X_t, P_t) = \max_{C_t, \dots, C_T} E_t \left[\sum_{s=t}^T \beta^{s-t} \cdot v(Z_s) \cdot u(C_s) + \beta^{T+1-t} \cdot \kappa \cdot v(Z_{T+1}) \cdot \frac{(\gamma_1 \cdot X_{T+1} + \overbrace{h \cdot \gamma_1}^{:=\gamma_0} \cdot P_{T+1})^{1-\rho}}{1-\rho} \right], \quad (7)$$

subject to budget constraint:

$$X_{t+1} = R \cdot (X_t - C_t) + Y_{t+1}, \quad X_{T+1} \geq 0.$$

The latter constraint means that household does not hold debt at retirement, which, combined with the characteristics of the income process, results in self-imposed liquidity constraint behaviour. A household in every moment of its working life does not incur debt in order to finance consumption because there is a positive probability of zero income realization for the remaining working time. Under no uncertainty about income and the bequest motive, marginal propensity to consume out of (liquid) wealth at the moment of retirement is equal to

$$\gamma_1 = \frac{1 - \beta^{\frac{1}{\rho}} \cdot R^{\frac{1}{\rho}-1}}{1 - \left(\beta^{\frac{1}{\rho}} \cdot R^{\frac{1}{\rho}-1}\right)^{N-T}}. \quad (8)$$

2. Model solution

2.1. Normalization

Problem (7) can be transformed to the following Bellman equation:

$$V_t(X_t, P_t) = \max_{C_t} v(Z_t) \cdot u(C_t) + v(Z_{t+1}) \cdot E_t [\beta \cdot V_{t+1}(X_{t+1}, P_{t+1})]. \quad (9)$$

In the given form, the problem has two state variables. The authors of [14] transform variables X_t, C_t by dividing them by the permanent income component P_t for every t . We denote them as $x_t := \frac{X_t}{P_t}$ and $c_t := \frac{C_t}{P_t}$. At the moment of retirement $T + 1$ we have:

$$\begin{aligned} V_{T+1}(X_{T+1}, P_{T+1}) &= v(Z_{T+1}) \cdot \frac{(\gamma_1 \cdot X_{T+1} + \gamma_0 \cdot P_{T+1})^{1-\rho}}{1-\rho} = P_{T+1}^{1-\rho} \cdot v(Z_{T+1}) \cdot \frac{(\gamma_1 \cdot x_{T+1} + \gamma_0)^{1-\rho}}{1-\rho} \\ &= P_{T+1}^{1-\rho} \cdot \hat{V}_{T+1}(x_{T+1}). \end{aligned} \quad (10)$$

If for $t + 1$, $V_{t+1}(X_{t+1}, P_{t+1}) = P_{t+1}^{1-\rho} \cdot \hat{V}_t(x_t)$ holds that it is easy to show that for t , $V_t(X_t, P_t) = P_t^{1-\rho} \cdot \hat{V}_t(x_t)$ and as consequence it is true for every t .

Therefore, the Bellman equation (9) can be written as:

$$\begin{aligned} P_t^{1-\rho} \cdot \hat{V}_t(x_t) &= \max_{c_t} P_t^{1-\rho} \cdot v(Z_t) \cdot u(c_t) + v(Z_{t+1}) \cdot E_t \left[\beta \cdot \underbrace{G_{t+1}^{1-\rho} \cdot N_{t+1}^{1-\rho} \cdot P_t^{1-\rho}}_{P_{t+1}^{1-\rho}} \cdot \hat{V}_{t+1}(x_{t+1}) \right] \Leftrightarrow \\ P_t^{1-\rho} \cdot \hat{V}_t(x_t) &= P_t^{1-\rho} \cdot \max_{c_t} v(Z_t) \cdot u(c_t) + v(Z_{t+1}) \cdot G_{t+1}^{1-\rho} E_t \left[\beta \cdot N_{t+1}^{1-\rho} \cdot \hat{V}_{t+1}(x_{t+1}) \right] \Leftrightarrow \\ \hat{V}_t(x_t) &= \max_{c_t} v(Z_t) \cdot u(c_t) + v(Z_{t+1}) \cdot G_{t+1}^{1-\rho} E_t \left[\beta \cdot N_{t+1}^{1-\rho} \cdot \hat{V}_{t+1}(x_{t+1}) \right]. \end{aligned} \quad (11)$$

A normalized version of budget constraints is presented below:

$$x_{t+1} = (x_t - c_t) \cdot \frac{R}{G_{t+1} \cdot N_{t+1}} + U_{t+1}, x_{T+1} \geq 0. \quad (12)$$

Thanks to normalization, the initial problem (7) was transformed to the one with only one state variable. It is easy to get the solution of (7) based on a solution of the normalized problem, as $c_t(X_t, P_t) = P_t \cdot c_t(x_t)$ and $V(X_t, P_t) = P_t^{1-\rho} \cdot \hat{V}_t(x_t)$.

2.2. First order condition

The first order condition of the Bellman equation (11), subject to budget constraint (12) takes the following form:

$$u'(c_t) = \frac{v(Z_{t+1})}{v(Z_t)} \cdot R \cdot \beta \cdot G_{t+1}^{-\rho} \cdot E_t \left[N_{t+1}^{-\rho} \cdot \hat{V}'(x_{t+1}) \right]. \quad (13)$$

The envelope theorem holds, hence $\hat{V}'(x_{t+1}) = u'(c_{t+1})$. Substituting this to (13) the Euler equation follows:

$$u'(c_t) = \frac{v(Z_{t+1})}{v(Z_t)} \cdot R \cdot \beta \cdot G_{t+1}^{-\rho} \cdot E_t \left[u'(N_{t+1} \cdot c_{t+1}) \right]. \quad (14)$$

It is assumed that in the last period of work a consumer doesn't face an income risk and experiences a liquidity constraint. Thus, the Euler equation for this period takes the form:

$$u'(c_T) = \max \left\{ u'(x_T), \beta \cdot R \cdot \frac{v(Z_{T+1})}{v(Z_T)} \cdot u'(c_{T+1}) \right\} \quad (15)$$

Using the consumption function at the moment of retirement one can obtain the following consumption function at the moment T :

$$c_T(x_T) = \begin{cases} \frac{A \cdot (\gamma_0 + \gamma_1 \cdot (x_T \cdot R + 1))}{1 + A \cdot \gamma_1 \cdot R}, & \text{for } x_T \geq A \cdot (\gamma_0 + \gamma_1) \\ x_T, & \text{in o.c.} \end{cases} \quad (16)$$

where $A = (\beta \cdot R \cdot \frac{V_{T+1}}{V_T})^{-\frac{1}{\rho}}$. From the moment $T - 1$ it is not possible to derive the closed form solution for the optimal rules consumption function. To obtain the consumption function, the authors of [14] performed a root finding calculation of equation (14) for a given grid of points and the consumption rule function from the next period. Finding the root of the Euler equation is, however, computationally burdensome, i.e. time consuming. Instead, we follow Carroll's methods presented in [5], [6] and [7]. He advocates the use of the so-called method of the endogenous grid points (EGPM), which takes into account some theoretical properties of the optimal consumption function.

2.3. Deriving optimal consumption rule

Starting from $T - 1$ period, optimal consumption rules are derived numerically by using a grid of fixed points $\{a_i\}_{i \in \{1, 2, \dots, k\}}$ and the optimal consumption rule from the next period (and its first derivative function).

2.3.1. Method of endogenous grid points

Let

$$a_t = x_t - c_t(x_t), \quad (17)$$

for every t and define function \mathbf{v}_t :

$$\mathbf{v}_t(a_t) := \frac{v(Z_{t+1})}{v(Z_t)} \cdot \beta \cdot G_{t+1}^{1-\rho} \cdot E \left[N_{t+1}^{1-\rho} \cdot \hat{V}(a_t \cdot \frac{R}{G_{t+1} \cdot N_{t+1}} + U_{t+1}) \right]. \quad (18)$$

Obviously $\mathbf{v}'_t(a_t)$ is equal to RHS of (14). Next, we define the function:

$$\mathbf{c}_t := (\mathbf{v}'_t)^{-\frac{1}{\rho}}. \quad (19)$$

Notice that:

$$u'(c_t(x_t)) = \mathbf{v}'_t(a_t) \Rightarrow c_t(x_t) = \mathbf{v}'_t(a_t)^{-\frac{1}{\rho}} = \mathbf{c}_t(a_t). \quad (20)$$

Values of $\mathbf{c}_t(a_i)$ evaluated at grid points $\{a_i\}_{i \in \{1,2,\dots,k\}}$ are equal to the values $c_{t,i}$ of the consumption function at points $x_{t,i} = a_i + \mathbf{c}_t(a_i)$ (hence the method's name). Given pairs $\{x_{t,i}, c_{t,i}\}$ one can, due to interpolation and extrapolation, obtain the optimal consumption function for every non-negative value x_t ¹¹. Carroll [8] argues that such endogenous representation of points should work reasonably well since these are densely located in parts of the function which exhibits heavy non-linearity. The quality of the solution using EGPM is basically the same as in the case of root the finding method, but the EGPM approach is much better in terms of numerical efficiency.

2.3.2. Slope at grid points

Given pairs of points $\{x_{t,i}, c_{t,i}\}$ it is possible to build consumption function c by linear interpolation. This approach, however, has some disadvantages:

- The linear interpolant will generally produce kinks in the objective function of the minimization step and slow down whole process, see Judd [18].
- The approximation to the consumption function might not have the desired accuracy in the parts of the function with the greatest non-linearity.

¹¹When savings converge to zero from the right then consumption also converges to zero which is the result of liquidity constraints. In practice this zero point is added to the grid and consumption is set at zero.

An approximation of the consumption function would be much more accurate ¹² when in addition to the values, also a marginal propensity to consume out of wealth (marginal propensity to consume later on or MPtC) at grid points would be considered.

At a_i the following equalities hold: $x_t(a_i) = a_i + \mathbf{c}_t(a_i)$ and $c(x_t(a_i)) = \mathbf{c}_t(a_i)$. Hence $x_t^a(a_i) = 1 + \mathbf{c}_t^a(a_i)$ and $\mathbf{c}_t^a(a_i) = c^x(x_t(a_i)) \cdot x_t^a(a_i)$. Substituting RHS of former equality in place of the multiplicand of the latter yields:

$$\mathbf{c}_t^a(a_i) = c^x(x_t(a_i)) \cdot (1 + \mathbf{c}_t^a(a_i)) \Leftrightarrow c^x(x_t(a_i)) = \frac{\mathbf{c}_t^a(a_i)}{1 + \mathbf{c}_t^a(a_i)}. \quad (21)$$

In order to acquire MPtC at grid point a_i , it is required to know the value of $\mathbf{c}_t^a(a_i)$.

The transformations needed to get $\mathbf{c}_t^a(a_i)$ are presented below:

$$\begin{aligned} u'(\mathbf{c}_t(a_i)) &= \frac{v(Z_{t+1})}{v(Z_t)} \cdot R \cdot \beta \cdot G_{t+1}^{-\rho} \cdot E \left[N_{t+1}^{-\rho} \cdot u'(c_{t+1}(a_i \cdot \frac{R}{G_{t+1} \cdot N_{t+1}} + U_{t+1})) \right] \Rightarrow \\ u''(\mathbf{c}_t(a_i)) \cdot \mathbf{c}_t^a(a_i) &= \frac{v(Z_{t+1})}{v(Z_t)} \cdot R^2 \cdot \beta \cdot G_{t+1}^{-\rho-1} \cdot E \left[N_{t+1}^{-\rho-1} \cdot u''(c_{t+1}(x_{t+1})) \cdot c_{t+1}^x(x_{t+1}) \right] \Rightarrow \\ \mathbf{c}_t^a(a_i) &= \frac{v(Z_{t+1})}{v(Z_t)} \cdot R^2 \cdot \beta \cdot G_{t+1}^{-\rho-1} \cdot E \left[u''(N_{t+1} \cdot c_{t+1}(x_{t+1})) \cdot c_{t+1}^x(x_{t+1}) \right] \cdot \frac{1}{u''(\mathbf{c}_t(a_i))}. \end{aligned} \quad (22)$$

Given functions c_{t+1} , c_{t+1}^x and values $\mathbf{c}_t(a_i) = c_t(x_t)$ it is straightforward to obtain $\mathbf{c}_t^a(a_i)$ by using (21), (22).

2.3.3. Shape preserving quadratic spline interpolation

In the next step we construct the optimal consumption function c_t and its first derivative function c_t^x given the trio of points $\{x_{t,i}, c_{t,i}, c_{t,i}^x\}$. Carroll has shown that the c_t function is C^2 , strictly increasing and strictly concave. It is then desirable for an approximation of the consumption function to be (at least) strictly concave, smooth and MPtC at grid points should be equal to $c_{t,i}^x$. All of these properties are preserved by the interpolation procedure proposed by L. Schumaker [28]. In their books, both Judd [18] and Ljungvist [22] emphasize the importance of this interpolation. As the result of this procedure, we acquire c_t and c_t^x functions.

2.3.4. Derivation of the expected value

In order to calculate the values $\{x_{t,i}, c_{t,i}, c_{t,i}^x\}$ it is imperative to know the expectation values at the RHS-es of equations (14), (22). There is no analytical solution to get

¹²At the same number of grid points - so cardinality of the grid can be reduced without sacrificing the quality of the solution.

the value of these integrals, so they have to be derived numerically. We follow the approach proposed in [14] and use Gauss-Hermite quadrature (GHQ). In short, GHQ gives value approximation to the following integral types:

$$\int_{\mathbf{R}^k} e^{-\sum_{i=1}^k x_i^2} f(x_1, \dots, x_k) d\mathbf{x} \simeq \sum_{i_1=1}^n \sum_{i_2=1}^n \dots \sum_{i_k=1}^n \omega_{i_1} \cdot \omega_{i_2} \cdot \dots \cdot \omega_{i_k} \cdot f(x_{i_1}, x_{i_2}, \dots, x_{i_k}). \quad (23)$$

where n is the order of quadrature, x_i are roots of n -th Hermite polynomial and ω_{i_j} are associated weights¹³.

The expectation values of RHS-es of equations (14), (22) in a form ready to apply GHQ are given below¹⁴:

$$E_t \left[u' (N_{t+1} \cdot c_{t+1}(x_{t+1})) \middle| U_{t+1} = 0 \right] = \int_{-\infty}^{+\infty} \frac{1}{\sqrt{\Pi}} \cdot u' \left(c_{t+1} \left(a_t \cdot \frac{R \cdot e^{-\sqrt{2} \cdot n \cdot \sigma_N}}{G_{t+1}} + \mu \right) \cdot e^{\sqrt{2} \cdot n \cdot \sigma_N} \right) \cdot e^{-n^2} \cdot dn, \quad (24)$$

$$E_t \left[u' (N_{t+1} \cdot c_{t+1}(x_{t+1})) \middle| U_{t+1} > 0 \right] = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \frac{1}{\Pi} \cdot u' \left(c_{t+1} \left(a_t \cdot \frac{R \cdot e^{-\sqrt{2} \cdot n \cdot \sigma_N}}{G_{t+1}} + e^{\sqrt{2} \cdot u \cdot \sigma_U} \right) \cdot e^{\sqrt{2} \cdot n \cdot \sigma_N} \right) \cdot e^{-(n^2+u^2)} \cdot dn \cdot du, \quad (25)$$

$$E_t \left[u'' (N_{t+1} \cdot c_{t+1}(x_{t+1})) \cdot c_{t+1}^x(x_{t+1}) \middle| U_{t+1} = 0 \right] = \int_{-\infty}^{+\infty} \frac{1}{\sqrt{\Pi}} \cdot u' \left(c_{t+1} \left(a_t \cdot \frac{R \cdot e^{-\sqrt{2} \cdot n \cdot \sigma_N}}{G_{t+1}} + \mu \right) \cdot e^{\sqrt{2} \cdot n \cdot \sigma_N} \right) \cdot c_{t+1}^x \left(a_t \cdot \frac{R \cdot e^{-\sqrt{2} \cdot n \cdot \sigma_N}}{G_{t+1}} + \mu \right) e^{-n^2} \cdot dn, \quad (26)$$

$$E_t \left[u'' (N_{t+1} \cdot c_{t+1}(x_{t+1})) \cdot c_{t+1}^x(x_{t+1}) \middle| U_{t+1} > 0 \right] = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \frac{1}{\Pi} \cdot u' \left(c_{t+1} \left(a_t \cdot \frac{R \cdot e^{-\sqrt{2} \cdot n \cdot \sigma_N}}{G_{t+1}} + e^{\sqrt{2} \cdot u \cdot \sigma_U} \right) \cdot e^{\sqrt{2} \cdot n \cdot \sigma_N} \right) \cdot c_{t+1}^x \left(a_t \cdot \frac{R \cdot e^{-\sqrt{2} \cdot n \cdot \sigma_N}}{G_{t+1}} + e^{\sqrt{2} \cdot u \cdot \sigma_U} \right) \cdot e^{-(n^2+u^2)} \cdot dn \cdot du. \quad (27)$$

In our numerical solution we used 12-th order of quadrature.

2.3.5. Choice of grid points

Grid points were derived by using the triple exponential method as proposed in [6]. In addition, the grid was augmented by three points. When consumer assets converge to

¹³Detailed description in [18].

¹⁴They are straightforward results of integration by substitution.

zero from the right-hand side, then consumption also converges to zero as a consequence of self-imposed liquidity constraints. Furthermore, marginal propensity to consume converges to unity. It is then reasonable to add point $a = 0$ to the grid. For $a = 0$ we set $c(a) = 0$ and $c^x(a) = 1$. The second one is the point at which the optimal consumption function for the last working period exhibits a kink. The last added point is arbitrarily large and the optimal consumption function for arguments larger than this point was extrapolated linearly. ¹⁵

¹⁵There exist more sophisticated extrapolation methods, see [8].

3. Estimation of transitory and permanent shock standard deviations

In order to estimate the magnitude of permanent and transitory shocks, we followed the approach used in [21] and [4].¹⁶ Firstly, we remove the drift component g_t by performing standard Mincerian wage regression¹⁷ for the income process (2)¹⁸. Denote driftless log-income at the age t of i -th household (individual) as $y_{i,t}$.

Put $r_{i,k} = y_{i,t+k} - y_{i,t}$. Variation of $r_{i,k}$ is equal to:

$$\begin{aligned} \text{Var}(r_{i,k}) &= \text{Var}(y_{i,t+k} - y_{i,t}) = \text{Var}(p_{t+k} + \epsilon_{t+k} - p_t - \epsilon_t) = & (28) \\ \text{Var}(p_t + \eta_{t+k} + \eta_{t+k-1} + \dots + \eta_t - p_t + \epsilon_{t+k} - \epsilon_t) &= k \cdot \sigma_n^2 + 2 \cdot \sigma_u^2, \end{aligned}$$

where ϵ_t, η_t are shocks of permanent and transitory income shocks at the age of t , respectively. Because $y_{i,t}$ does not include drift then $E(r_{i,k}) = 0$ and as a consequence $v_{i,k} = r_{i,k}^2$ is an unbiased estimator of $\text{Var}(r_{i,k})$. In order to obtain estimates s_n^2, s_u^2 of σ_u and σ_n respectively, we regress (28) for each individual set of $v_{i,k}$. To obtain s_n^2, s_u^2 , it is necessary to have access to the income data in which the same households or individuals are present for at least three years.¹⁹ That condition ruled out the HBS set and alternative data was required. We used an administrative sample of individual data on social insurance spanning the 1999-2014 period. In order to reduce some noise, we discarded every individual, which did not fulfil any of the following criteria:

- the person must be on an employment contract or being registered as unemployed,
- the person must be present in the data for every year (16 consecutive years)²⁰ and in each of those years, they must pay social insurance contributions for at least eleven months²¹,
- age range must be between 25 and 60 years,

¹⁶For more complex methods, which take into account the autocovariances structure of the income process see Guvenen [16] for example.

¹⁷Unfortunately, we were not able to use information about the education level of individuals.

¹⁸With one exception that $p = 0$.

¹⁹Technically speaking, when individuals are observed for only three years one can just solve the system of two equations to get s_n^2 and s_u^2

²⁰This assumption is not necessary.

²¹We performed aggregation to annual levels in order to remove seasonality.

- the (implied) minimum of wage ²² (or in some cases mix of wage and unemployment benefits) realization must be higher than one fifth of the mean wage during that time.

For the remaining 17734 persons, we performed a calculation of s_n^2, s_u^2 . In order to obtain σ_n, σ_u we averaged results broken down by gender, NACE categories and voivodeship groups. ²³ Standard deviations for σ_n, σ_u were derived using the bootstrap procedure. Results of the estimation are different from those in [21] and can be found in Table 5. In general, we found both the permanent and transitory shocks to be weaker²⁴. We argue that the obtained permanent shock magnitudes are more realistic; however, weaker transitory innovations to the process of income might be results of sample differences (we consider the population of the most reliable workers and do not consider self-employed, those on job order contracts and the informal sector, which to some extent is included in household surveys). Such a parsimonious specification of labour income may be inadequate in the case of the Polish economy. Kolasa's findings [19] suggest that $AR(1)$ process which shapes log-income of permanent income do not have an unit root ²⁵. In other words, shocks may not be permanent and have only a persistent effect. Mis-specification of the labour income process may lead to an additional bias to relative risk aversion estimates. As shown in [12 REF], even if the real process is highly persistent (which corresponds to 0.97 coefficient of lagged $AR(1)$ variable) it leads to over/under-estimation of relative risk aversion by up to 30%. In addition, the bulk of the measured magnitude of permanent/persistent shocks to income may be in fact a result of endogenous mobility choices, see Low et al. [23] In order to minimize potential bias, we used quite long information of individuals income - it is easy to see, that if $p_t = \alpha \cdot p_{t-1}$ and $p_0 = \eta_0, E(\eta_0) = 0$ then $v_{i,k}$ are still unbiased estimates of $r_{i,k}$ and fully permanent process $\alpha = 1$ offers best linear approximation to nonlinear variance process of log-income in life cycle. This is the reason why estimated

²²Income was deflated by the CPI index.

²³Information for every payment consists of payer NACE category. NACE with the highest amount of payments during 1999-2014 represents each person NACE category. Information about voivodeship was derived using information about employee postal code at the beginning of presence in sample .

²⁴Bearing in mind the generally lower income declared by self-employed, entrepreneurs and the missing grey zone

²⁵Income specification of that kind is beyond the scope of this paper.

permanent shock magnitudes tends to be smaller as number of available $v_{i,k}$ grows²⁶. These magnitudes are however more consistent with observed Gini income indexes²⁷ for Poland and other countries (such as USA) than ones in [21]. Mis-specification of labour income process may lead to an additional bias to relative risk aversion estimates. As shown in [11] even if real process is highly persistent (which corresponds to 0.97 coefficient of lagged AR(1) variable) leads to over/under estimate of relative risk aversion by up to 30%. In addition, bulk of measured magnitude of permanent/persistent shocks to income may be in fact a result of endogenous mobility choices, see Low et al. [23].

²⁶Variance of log-income rises early in life cycle and then flattens or even decreases, although in some cases it begins to rise prior to retirement - consequently, if we use shorter sample than we will consider many individuals which just have started job careers and individuals which retire at some point or their age exceed 60. Age-dependent income variation process in Poland suggests that this case would yield higher estimates of magnitude of permanent shock to income. Increasing number of years at which the individuals are observed should improve the estimates as larger part of lifetime income is available.

²⁷There are other factors that may influence inequalities, for example demographic structure.

4. Income and consumption profiles

4.1. Methodology

In the HBS survey, households can participate up to two times in the same month, one year after another. Therefore, it is impossible to observe the same households in a long time span and this makes derivation of life-cycle income and consumption profiles harder. However, this problem can be dealt with by adaptation of the pseudo-panel strategy proposed by Deaton [9]. Instead of observing the same persons for several years, we can observe the same **cohorts** across surveys. But even in this approach, the generated pseudo-panel is too short to include the whole life cycle of any cohort. To (partly) overcome this problem, the approach pioneered by Deaton [10] is commonly used. In this approach the information about all the cohorts is used to decompose income and consumption into three effects:

- time effect - effect attributable to business and financial cycles, which may affect both income and consumption decisions,
- age effect - effect attributable to life-cycle consumption and saving motives which are influenced by age,
- cohort effect - effect describing heterogeneity across different cohorts such as distinct tastes or higher/lower life-cycle income profile which are a consequence of real economic growth.²⁸

Cohort and time effects have to be neutralized in order to obtain life cycle profiles for a given cohort group. The easiest way to make such a correction would be to regress income and consumption by the use of a set of dummies describing the year of the survey (time effect), age of the household head (age effect) and birth year of the household head (cohort effect). It is easy to see that the year of survey minus the birth year yields the age of the household head. Gourinchas and Parker [14] dealt with this problem by introduction of variable which catches time effects²⁹. In our approach we just create five-year cohort groups similar as in [19]. In addition, we get rid of the

²⁸This is the reason why the sample means across ages of household heads will lead to biased life-cycle profiles.

²⁹Monthly unemployment rates for U.S. states. The results of an analogous approach in the case of Poland were unsatisfactory.

variability of household size because of model assumptions. To sum up, consider the following regression models:

$$\ln(C_i) = \alpha_1 + \beta_1^T \cdot LOS_i + \beta_2^T \cdot GRKOH_i + \beta_3^T \cdot WIEK_i + \beta_4^T \cdot ROK_i + \epsilon_i, \quad (29)$$

$$\ln(Y_i) = \alpha_2 + \gamma_1^T \cdot LOS_i + \gamma_2^T \cdot GRKOH_i + \gamma_3^T \cdot WIEK_i + \gamma_4^T \cdot ROK_i + \psi_i. \quad (30)$$

where:

- $\ln(Y_i)$ - log of disposable income in real terms at the beginning of 1999³⁰ of i-th household,
- $\ln(C_i)$ - log of real consumption expenditure of i-th household,
- LOS_i - dummies describing the size of household (for $LOS_i > 10$ we put $LOS_i = 10$, we omitted $LOS_i = 1$),
- $GRKOH_i$ - dummies showing whether the head of the household is a member of five-year (except last one) span cohort group³¹
- $WIEK_i$ - age of the i-th household head (as dummies, age twenty three was omitted),
- ROK_i - dummies of the year when the interview took place for the i-th household (we omit 1999),
- ϵ_i, ψ_i - other factors which may influence expenditures and income (model residuals),
- $\beta_1, \beta_2, \beta_3, \beta_4, \gamma_1, \gamma_2, \gamma_3, \gamma_4$ - vectors of dummy coefficients.

Thereafter, on the basis of estimated coefficients of equations (29) and (30) corrected values of regressands are derived according to the following formulas³²:

$$\ln(\hat{C}_i) = \hat{\alpha}_1 + \hat{\beta}_1 \cdot L\bar{O}S_t + \hat{\beta}_3^T \cdot WIEK_i + \hat{\beta}_4 \cdot R\bar{O}K + \hat{\epsilon}_i, \quad (31)$$

³⁰Income data was deflated by monthly CPI data, see CSO.

³¹We consider the following cohort groups: 1936-1940, 1941-1945, 1946-1950, 1951-1955, 1956-1960, 1961-1965, 1966-1970, 1971-1975, 1976-1980, 1981-1985 and 1986-1991. We omit sixth (middle) cohort (1961-1965) so all the results represent profiles for that cohort.

³²Time effects are averaged, the same as household size. The cohort effect was corrected by omission of the cohort variable.

$$\ln(\hat{Y}_i) = \hat{\alpha}_2 + \hat{\gamma}_1 \cdot L\bar{O}S_t + \hat{\beta}_3^T \cdot WIEK_i + \hat{\gamma}_4 \cdot R\bar{O}K + \hat{\psi}_i, \quad (32)$$

where:

$$\hat{\beta}_1 \cdot L\bar{O}S_t = \frac{1}{I(t)} \sum_{i=1}^{I(t)} \hat{\beta}_1^T \cdot LOS_i, \quad \hat{\gamma}_1 \cdot L\bar{O}S_t = \frac{1}{I(t)} \sum_{i=1}^{I(t)} \hat{\gamma}_1^T \cdot LOS_i$$

$$\hat{\beta}_4 \cdot R\bar{O}K = \frac{1}{I(t)} \sum_{i=1}^{I(t)} \hat{\beta}_4^T \cdot ROK_i, \quad \hat{\gamma}_4 \cdot R\bar{O}K = \frac{1}{I(t)} \sum_{i=1}^{I(t)} \hat{\gamma}_4^T \cdot ROK_i.$$

To get corrected income and consumption profiles for the life-cycle, one simply averages equalities (31), (32) across different ages. The polynomial of order of five was used to get a smoothed version of profiles. Another product of estimation (29) and (30) are vector of expected income growth G defined as the first difference of smoothed income profile, vector of household characteristics Z_t and in consequence taste shifters vector $v(Z_t)$. We define them as: ³³:

$$Z_t = \frac{1}{I(t)} \cdot \sum_{i=1}^{I(t)} \hat{\beta}_1^T \cdot LOS_i, \quad (33)$$

$$v(Z_t) = e^{\rho Z_t}. \quad (34)$$

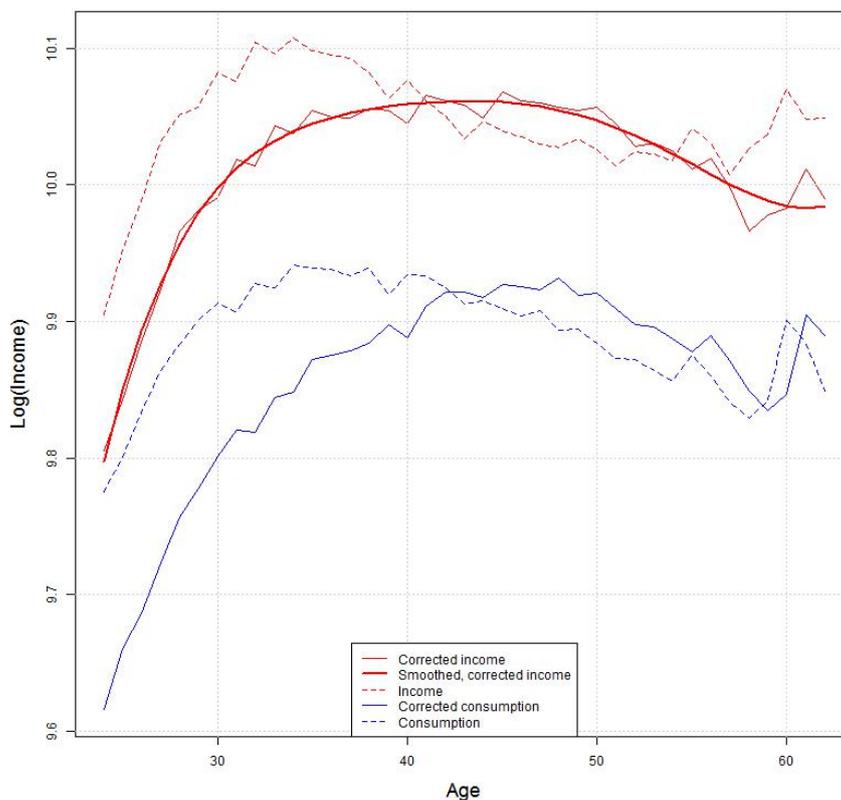
4.2. Different social groups profiles

To get profiles for different socioeconomic groups (gender, NACE ³⁴ sections categories and voivodeship) we add interaction terms for WIEK in equations (29) and (30). Below we present the product of the approach described in this chapter for all households:

³³One of (many) possible alternatives which are independent of the results of estimations (29), (30) may be a vector of average household size obtained by the use of equivalence of scale.

³⁴Actually, we used ISCO (International Standard Classification of Occupations) classification first digit as its content did not change much during 1999-2014 in contrast to PKD (the polish equivalent of NACE) sections during that period. We present the results only for those PKD sections which remain consistent during 1999-2014. Ideally, we would use the ISCO classification instead of NACE categories (for example, an IT specialist may work in almost every type of company); however, this information is not a part of ZUS data and as consequence, permanent and transitory shock magnitudes cannot be estimated for the ISCO classification.

Figure 1: Income and consumption profiles, raw and corrected for cohort born in 1961-1965, all households



Source: own calculations

As expected, corrected profiles are lower in the beginning of the career path and then become higher than ones obtained through cross-sectional means. Younger cohorts have a higher lifetime income path and the older income path is lower than representative the cohort group, which is the reason for differences between both raw and corrected profiles. Income peaks later than cross-sectional data suggest, and same applies to consumption (which tracks income). The results are consistent with Kolasa's [19] findings. Profiles for different socioeconomic groups can be found in Figure 19. Corrected data at an individual level was used to estimate the life-cycle model for HBS 1999-2014.

5. Structural estimation

5.1. Simulated method of moments

To get estimates of $\theta := (\rho, \beta, \gamma_0)$ we follow the [14] approach and use the simulated method of moments. We consider other parameters $\hat{\mu} := (\hat{R}, \hat{\sigma}_U^2, \hat{\sigma}_N^2, \hat{\omega}_{21}, \hat{\sigma}_{\omega_{21}}^2, \hat{p})$, income profile $\{\hat{Y}_t\}_{t \in \{1, \dots, T\}}$, predictable changes to income vector $\{\hat{G}_t\}_{t \in \{1, \dots, T\}}$ and taste shifts vector $\{v_{Z_t}\}_{t \in \{1, \dots, T\}}$ as given ³⁵. Denote $\hat{\chi} := (\{\hat{Y}_t\}_{t \in \{1, \dots, T\}}, \{\hat{G}_t\}_{t \in \{1, \dots, T\}}, \{v_{Z_t}\}_{t \in \{1, \dots, T\}})$. The simulated method of moments approach is to fit expected value of theoretical moments with empirical ones. In contrast to the generalized method of moments, theoretical moments cannot be obtained in an analytical way and have to be simulated. Therefore, theoretical moments consist of an additional error which must be taken into consideration.

In the case of the model presented in this paper, we fit the theoretical log-consumption path $\{\ln \hat{C}_t(\theta; \hat{\mu}, \hat{\chi})\}_{t \in \{1, 2, \dots, T\}}$ with empirical one ³⁶ $\{\ln C_t\}_{t \in \{1, 2, \dots, T\}}$ and we make such an exercise later on. Consider the following moment conditions:

$$\forall_{t \in \{1, \dots, T\}} g_t(\theta; \hat{\mu}, \hat{\chi}) := E[\ln(C_t) - \ln(\hat{C}_t(\theta; \hat{\mu}, \hat{\chi}))] = 0. \quad (35)$$

In our problem (35) is equivalent to the conditions:

$$\forall_{t \in \{1, \dots, T\}} g_t(\theta; \hat{\mu}, \hat{\chi}) := \frac{1}{I(t)} \sum_{i=1}^{I(t)} (\ln(C_{it}) - \frac{1}{J} \cdot \sum_{j=1}^J \ln(\hat{C}_{jt}(\theta; \hat{\mu}, \hat{\chi}))) = 0, \quad (36)$$

where C_{it} is the consumption of i -th surveyed household at the age of t , $I(t)$ is the number of households at the age of t , \hat{C}_{jt} is the consumption of j -th simulated model household at the age of t and J is the number of simulated model households ³⁷. As the number of moments is higher than the number of parameters, it is practically impossible to find the vector of θ satisfying (35). The concept of the generalized method is to minimize the function

$$g(\theta; \hat{\mu}, \hat{\chi}) \cdot W_T \cdot g(\theta; \hat{\mu}, \hat{\chi})^T, \quad (37)$$

where W_T is positive semidefinite $T \times T$ matrix and g is a vector function defined as $g(\theta; \hat{\mu}, \hat{\chi}) := (g_1(\theta; \hat{\mu}), g_2(\theta; \hat{\mu}, \hat{\chi}), \dots, g_T(\theta; \hat{\mu}, \hat{\chi}))$. We apply the following procedure

³⁵ γ_1 depends upon other parameters according to (8)

³⁶It is of course possible to choose another moments used in the estimation, on the basis of household wealth as in Cagetti [2].

³⁷ J is equal to 20000 in our simulations.

similar to the one in [14]:

- define W_T as identity matrix and find parameters $\hat{\theta}$ which minimizes the objective function (37),
- define $\Omega := E(g(\hat{\theta}; \hat{\mu}, \hat{\chi}) \cdot g(\hat{\theta}; \hat{\mu}, \hat{\chi})^T)$ and update the weighting matrix W_T according to $W_T = \Omega^{-1}$,
- obtain the new estimator $\hat{\theta}$ by minimizing the objective function with the updated W_T .

If $\lim_{I \rightarrow \infty} 1 + \frac{I}{J(I)} = \tau$, then MSM $\hat{\theta}$ is consistent and an efficient estimator of θ_0 for updated W_T . The random variable $^{38} \sqrt{I} \cdot (\hat{\theta} - \theta_0)$ is asymptotically normal and has the following variance-covariance matrix:

$$\hat{V} = \tau \cdot (\hat{D}^T \hat{D})^{-1} \hat{D}^T \hat{\Omega} \hat{D} (\hat{D}^T \hat{D})^{-1}, \quad (38)$$

where \hat{D} is the jacobian of g in $\hat{\theta}$, $\hat{\Omega} = E(g(\hat{\theta}; \hat{\mu}, \hat{\chi}) \cdot g(\hat{\theta}; \hat{\mu}, \hat{\chi})^T)$.

The MSM estimation provides a useful overidentification test. If the model is correct, then the following test statistic

$$\chi_{T-\#\theta} = I \cdot \tau \cdot g(\hat{\theta}; \hat{\mu}, \hat{\chi}) \Omega^{-1} g(\hat{\theta}; \hat{\mu}, \hat{\chi})^T$$

has a Chi-square pdf with $T - \#\theta$ degrees of freedom.

5.2. Simulated theoretical profiles

Equipped with model parameters θ , μ and optimal consumption functions for every working period of the household calculated according to 2, we perform Monte-Carlo integration to derive the theoretical consumption path.

5.3. Minimum of objective function

We perform minimization of (36) on parameter space $\Theta := \{\theta = (\beta, \rho, \gamma_0) : \beta \in [0.8, 0.999], \rho \in [0.05, 10], \gamma_0 \in [0.001, 0.9]\}$. In the first step, a standard evolutionary algorithm was used ³⁹ to find the starting point for the Nelder-Mead procedure used in

³⁸I is equal to $\frac{1}{T} \sum_{i=1}^T I(i)$.

³⁹To be more precise we used a differential evolution algorithm, for more information see [26]. In general, the algorithm should yield better results than a naive grid search; however, as the method is heuristic, it is not guaranteed that the obtained solution will be close to the global minimum.

the second step. We take advantage of modern CPU-s architecture and parallelize the first step calculations. The two-step estimation procedure described in 5 evaluates the objective function about 5000 times. As the result, estimation procedure takes from 3 to 10 minutes (and that depends, obviously, on the number of available cores and CPU specification)⁴⁰.

⁴⁰All solvers were written in R.

6. Data

The estimation relies on two household surveys: the Household Budget Survey (HBS) and the Household Wealth Survey (HWS). The HBS is a survey based on the monthly rotating sample of around 37,000 Polish households with a complete change of sample in quarterly periods. It covers detailed data on the household structure, life conditions, income and a precise breakdown of consumption and accumulation (in other words investment). In spite of the fact that the methodology seems quite consistent with the national accounts, the actual coverage of the accumulation (e.g. real estate purchases) barely corresponds to the accumulation account of the household sector⁴¹. In this respect, the raw age profile of the household sector that suggests a monthly saving rate of nearly 20% almost certainly misses the actual accumulation and does not correspond to the change in financial accounts of the sector of financial assets. The additional trouble to solve for this sector refers to a substantial difference between the aggregated result of financial and non-financial accounts - the financial accounts suggest growing savings, although much smaller than reweighted average 1000-1500PLN monthly per household, while the non-financial accounts show contradictorily a deficit over the past years. In line with the fruitful development of the Household Finance and Consumption Network (HFCN) managed by the European Central Bank, an opportunity appeared to receive an early results of the HWS, the very first wealth stock survey based on a sample of 3,500 Polish households, for the single year of 2013⁴². To face the well-known problems of large numbers of refusals reaching approx. 60%, multilevel imputation and supersampling was introduced to the most refusal-biased category of higher deciles of income and wealth. The summary of the survey provides proofs of the positive influence of these techniques on the level of representativeness. Below, the description of the data

⁴¹Our educated guess is that the household confronted with a voluntary, very detailed HBS questionnaire to be carefully filled in for the entire month, when the real estate has been purchased, built or renovated, will nearly certainly reject it, since the marginal cost in time spent on filling in the tables of, for example, eggs or rice consumption in busy times, is much smaller when compared with the small marginal revenue (e.g. frying pan) received in exchange. This might be a significant weakness of the questionnaire that mixes macroeconomically important variables, e.g. real estate purchase and a relatively less important one, e.g. detailed consumption of food products

⁴²A significant weakness of the single observation in time comes from the obvious fact that the change in stocks can't be caught with a single stock observation and needs to be estimated from the flows

is developed in detail.

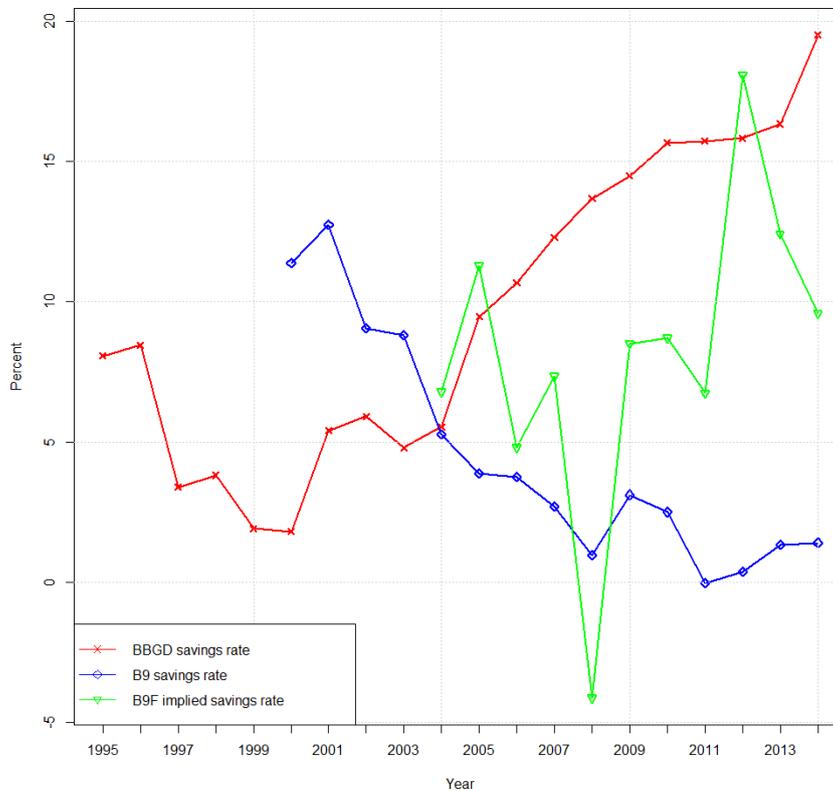
6.1. Household Budget Survey

In order to obtain “expected” permanent life cycle profiles of income and a corresponding empirical consumption profile, we use the Polish household budget survey (HBS) spanning the period 1999-2014, consisting of about 567,000 records at household level. To construct age profiles, we use information about the age of the household head (the head is defined as a person with the highest income within the household). Because we consider a model with labour income risk, we only consider working households (whose main source of income is labour income). We also discard households with any pension, self-employed and agricultural income. We consider households with the age of the head between 23 and 63 years⁴³ and in addition, we discard households with the oldest person at an age higher than 63. Finally, we discard households with no information about the head’s employer’s NACE category or type of job. We are left with about 173,000 observations. A small correction is made to households’ disposable income and expenditures: consumption made abroad is treated as household’s expenditures⁴⁴ and subtract the capital income from the disposable income⁴⁵.

⁴³In the model, households start life at the age of 24 (which is the typical age to complete tertiary education in Poland) and retire at the age of 62 (the effective working time is thus 39 years; there is a problem with attrition of households at ages 60+ which satisfy other conditions, see Table 7).

⁴⁴Money spent abroad is not considered in household expenditures since 2006, for the sake of comparability with the national accounts definition of consumption.

⁴⁵In the model these are considered as accrued interest and are not part of disposable labour income.

Figure 2: Household savings rate on a macro and micro basis

Source: HBS (BBGD), national accounts and financial accounts ⁴⁶

In the absence of even a temporary solution to these problems the data are taken as they are, bearing in mind the limitations listed above.

The HBS used the entire available range (1999-2014) for the estimation of the structural parameters of cover. Until the HWS dissemination, the HBS was the only demographically representative survey that reported both the revenues and the expenditures⁴⁷. Along with the standard setting in the literature, the self-appointed heads of each household, based on the highest income reported in the sampled month,

⁴⁶The financial accounts (B9F implied) savings rate is obtained by substituting net household lending/borrowing of financial accounts (B9F) into net lending/borrowing based on national accounts (B9) keeping the rest elements of the (gross) savings formula unchanged.

⁴⁷A comparable households' life conditions survey, the so-called EU-SILC, reports only the revenue side, alike Labour Force Survey.

were taken as the representatives of the entire household for the entire life-cycle.

The disposable income comprises mainly labour contract income, self-employment, profits from ownership of financial and non-financial wealth and social protection, on a net basis. The model, then, does not include the social security system and reflects the voluntary savings. The value of $\frac{\gamma_0}{\gamma_1}$ multiplied by the amount of the permanent income in the last working period sets the final stock of savings collected in the entire labour activity period. The γ_0 may also be interpreted as the consumption from the savings during retirement from voluntary savings.

The expenditures reported in the HBS survey are spent on the consumption of goods and services and remaining expenditures. The remaining expenditures reflect inter-vivos transfers and bequests, taxes, current payments, accumulation, and financial accounts, e.g. payments of instalments, including interest.

The expenditure, as mentioned above, theoretically includes all items required to build the complete set of national accounts expenditures, but the actual data does not seem to prove that assumption. For instance, reweighted cash-at-hand from the previous month gives on average just 10% of the aggregate, with an even worse result for non-financial assets net accumulation. That fact will be confronted later with the findings from the HWS.

With the absence of a more reliable spread over the life-cycle of accumulation, the standard setting for the cash result is taken: household revenues less expenditures. These may include, for example, the following:

- the difference between instalments paid (including interest) and new loans taken,
- the difference between rollover of deposits, as well as purchases and the sale of other financial assets,
- an increase in cash at hand,
- an increase in non-financial assets, including payments for the renovation fund, cost of renovation, including materials and services, purchases of real estate that are not related to the economic activity.
- expenditures made abroad,

- the sale of used consumption goods (with negative sign),
- a balancing item that mirrors the residual unexplained in the items above, calculated as a difference between revenues and expenditures.

The above-mentioned items are treated in the model as assets with rate of return R . There are some theoretical and practical implications related to the HBS data then:

- The age of the consumer is, in fact, the age of the head of the household, which with known but uncontrollable probability includes the pension revenues of the household. When the household head retires, the entire household retires in spite of the age composition.
- the actual households borrow and repay loans, which is by default not allowed in the model,
- in practice, the flows of financial and non-financial assets are barely present and very volatile in terms of reference to their aggregated counterparts, so in fact, they are useless for our purposes,
- any expenditures abroad are treated as savings.

It seems that the most relevant of all the above-mentioned points to deal with are the quality of data provided in questions related to financial and non-financial wealth. For (partly) unknown reasons, the reported values do not reflect their respective counterpart aggregates. The importance of the particular items to the big picture will be discussed in the chapter devoted to the results. The original dataset was limited to the heads of households aged between [23, 62], which reported incomes above zero, with an additional cut of all revenues from agricultural activity, where the negative income happened to be so large, that the net household revenues were negative too.

6.2. Household Wealth Survey and the integrated dataset

The completion of the very first household wealth survey for 2013 data allowed a much more profound insight into the structure of distribution of stocks and flows of the non-financial and financial accounts of the household sector. With extensively developed part on stocks, the HWS broadly overlaps the households' characteristics

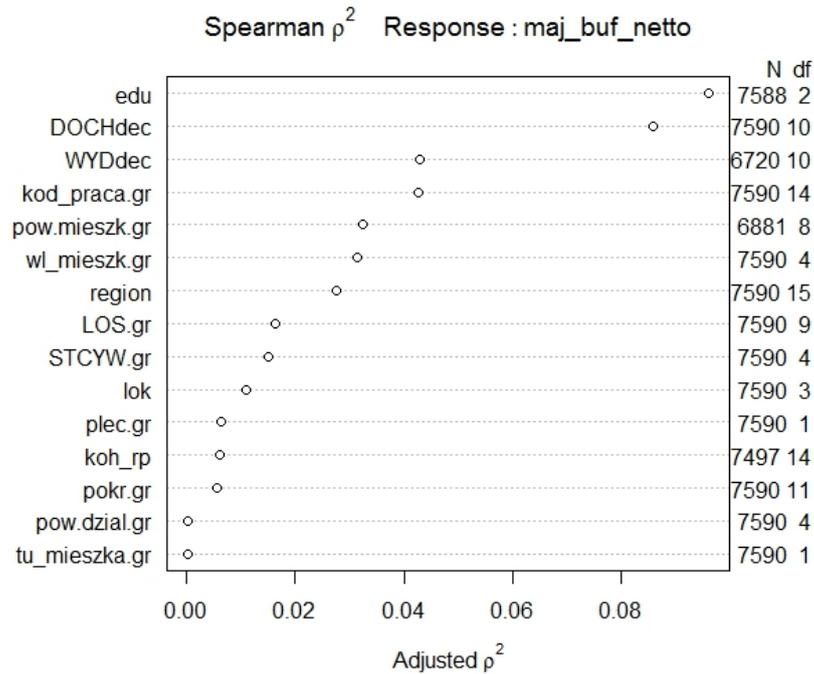
present in the HBS, including very similar categories of household's revenue and expenditures, which gave hope for potential matching of both panel datasets with the use of imputation, with particular focus on categories present in $v(Z_t)$.). The initial problem using combined HBS and HWS referred to their size: the HWS covers in the first version approx. 3,500 households and is available for only the single year of 2013, so any matching techniques that would impute HWS stocks to the respective households with the closest characteristics in HBS would mean that the donor (HWS) is 10 times smaller than the receiver (HBS), and in consequence a multiple use of the same variable. Obviously, a single year observation would also mean a selection bias with respect to an issue of time in the model.

It's fair to underline that the initial interpretation of the HWS put a question mark over the modelling and verification of existence and strength of the PILCH in Poland. In light of the previous available studies on the subject, the Polish households declared high concentration of wealth in financial non-means, especially real estate, and only a small part devoted for financial assets. However, at least in declarations of purposes of savings, the households put in first place unexpected events and in second or third place pension savings. It would mean that, if PILCH holds for Poland, the voluntary financial savings would grow in line with age and approaching retirement. Consequently, if indeed Polish households kept savings of 10-20% from each monthly salary, then their lifetime savings would make the compulsory pension schemes redundant. The data provided by the HWS confirmed the Social Diagnosis results, with a very modest PLN 8,000-12,000 median (approx. 2-3 salaries) for financial assets per household, compared with the nearly PLN 260,000 of subjective valuation for non-financial assets. The additional analysis of the cash result of the household, which is methodologically comparable with the HBS, revealed even higher "savings" per household⁴⁸, with, as in case of HBS, a lack of accumulation formation distributed over the life cycle. To conclude this part, it became evident that some important values are most probably missing from the households' expenditures, since their marginal propensity to consume does not meet the aggregated financial savings. Coming back to the data, the structure of the HWS is much broader than the HBS - altogether over 1300 items can be analysed, including questions for motives,

⁴⁸Counted as household's revenues less expenditures.

subjective valuation and plans for further economic management of wealth. The common categories present in both the HBS and HWS encompassed around 20 items, often in a common range of scale, which facilitated the matching process. The integration of the HWS and HBS was based on the StatMatch [12] package available in R that in principle follows the reduction of uncertainty in the statistical matching with the use of the correlation matrix. It is enough to say about the application that the nearest neighbours method (function *NND.hotdeck* proved the best mirroring for heterogeneity) was applied with Euclidean distance, unconstrained version. To allow for comparable distance measures, the quantitative variables, e.g. income, size of flat etc., were transformed into qualitative ones, i.e. deciles grouped into R factors with imposed order. The Spearman correlation table for repeated and equally scaled variables in both the HBS and the HWS served as a principle guide for the choice of the variables (donation classes) with the highest explanatory power, as presented on the chart below. The number of classes of donors was limited to the memory capacity, usually 2-3 donors divided into 2 to 15 factors each. For the net financial wealth, somewhat surprisingly, education usually ranked high. Since the HBS does not include nearly any information on the stocks of financial net wealth, only this category was imputed from the HWS to the HBS. Due to the high probability of repetition of imputed variables into the HBS, the smallest category of stocks, i.e. the financial wealth, was imputed⁴⁹. The application of the StatMatch in the current study can be regarded as a bit of a shortcut. The previous experiences with the unsupervised learning based on HBS and the cluster and correspondence analysis by Florczak, Jabłonowski and Kupc [13] gave a chance for much more exact control over the matching based on much larger number of the households' characteristics.

⁴⁹With such a dirty application of unsupervised machine learning, the imputation of the substantial non-financial wealth would be a wasted effort.



Notes: edu - education, DOCHdec - income decile, WYDdec - expenditures decile, kodpraca.gr - type of employment, pow.mieszk.gr - size of apartment, wlmieszk.gr - type of ownership of the apartment, region - voivodeship, LOS.gr - number of HH inhabitants, STCYW.gr - marriage status, lok - type of neighbourhood, plec.gr - gender, kohrp - age cohort, pokr.gr - family reference status, pow.dzial.gr - size of inhabited area, tumieszka.gr - habitat status.

Source: own calculations

Bearing in mind the low (varying mostly between 1-3 monthly salaries) precautionary savings in HWS held stable over the entire life cycle, compared with monthly continuous inflow of permanent income, should give an even better understanding of the actual management of available financial means by the households. Therefore, even repeated imputation from the HWS to 10 times larger HBS seemed low cost compared to the expected possible better comparability. Nevertheless, the original HWS served as well for the basis for the computation based on levels of permanent income and non-financial assets in the theoretical scenario, where the net wealth is regarded as fully liquid with a pejorative household valuation. There was a hope of catching the two extremes, keeping in mind that both of them are fully reliable: on the one hand, the HBS average profile based on logs of time series 1999-2014, where doubts refer to missing accumulation due to reasons specified above. On the other

hand, the version based on levels of combined average monthly income extended by the available, for a particular cohort, net precautionary financial wealth, where the consumption was estimated on the compound levels of available financial means for the original HWS and HBS for the given year with imputed net financial wealth from the HWS. On the extreme side of the latter scenario, the computation based on levels of income extended the net financial wealth by net non-financial wealth (non-financial gross wealth less remaining mortgage) to check the theoretical reaction of the model household on fully tradeable (mainly) real estate subjective value.

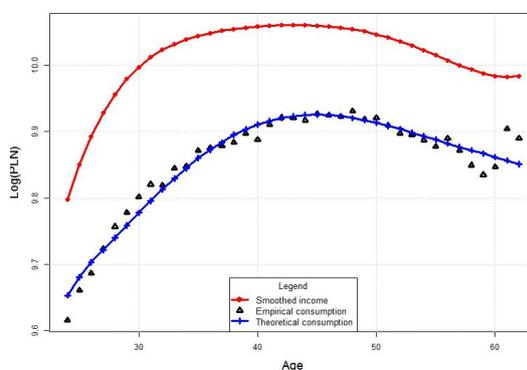
7. Comparison between the theoretical and empirical results

7.1. Simulation based on flows

The current section covers a comparison between the theoretical and empirical results. The slight upward redirection in the households' income path may be explained by the combined effect of the existence of a small number of still active households at the labour market, which may cause such effects at the end of estimates.

The following graph shows a comparison between the empirical and theoretical estimation of the model, in the most general setting for all households and regions, heads of the households aged 24-62.

Figure 3: Empirical consumption, income and fitted theoretical consumption for all households



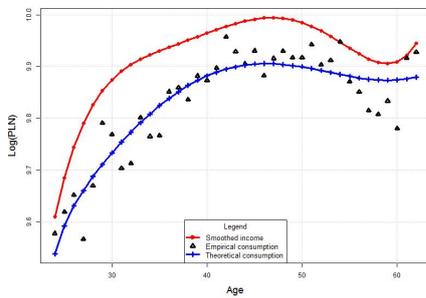
Source: own calculations

The model suits well the empirical profile of consumption (log), taken from the HBS as is⁵⁰. Interestingly, the income peak falls around the age of 45, and thereafter approaches the consumption, which flattens. In spite of the existence of significant regional differences in average salaries, the two following graphs show comparable

⁵⁰Doubts over the particular parts of modelling will be tackled separately

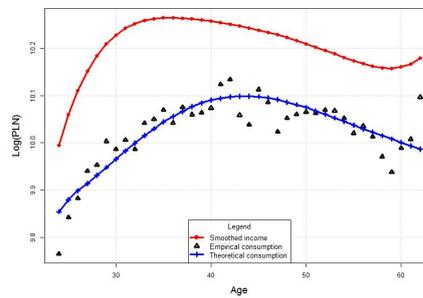
differences between poor ⁵¹ and rich regions, which may be explained by the adjustment of the consumption of the households to the presumed permanent income path. The general observed tendency is that in poorer regions the income approaches is that in poorer regions the income approaches the consumption faster than in richer regions. Additionally, in the richer regions the younger cohorts experience earlier income growth.

Figure 4: Podkarpackie voivodeship



Source: own calculations

Figure 5: Mazowieckie voivodeship



Source: own calculations

All charts for regional, NACE and gender versions can be found in appendix 16. The most important observations, which add new, important observations or differences from the related literature are listed below:

- based on HBS 1999-2014 data, the „buffer-stock” behaviour is not visible while the households in the majority keep a stable amount of their income through the lifetime and there is no evidence of „target wealth” level in HBS⁵². Households exhibit a high level of patience, which corresponds to β estimates of around 0.98. Precautionary wealth ⁵³ varies from 5% to 50% of the entire

⁵¹In terms of the average salary.

⁵²In the case of HBS we consider that all household wealth is liquid, a strong assumption which does not really hold in the case of housing (although housing wealth is liquid to some degree). Results based on financial stocks can be found in 7.2.

⁵³Precautionary wealth is defined as $\frac{1}{N-T} \cdot \sum_{t=1}^{N-T} \frac{wealth_{risk,t} - wealth_{no\ risk,t}}{wealth_{risk,t}}$, where $wealth_{risk,t}$ is an expected value of wealth at t-th working period in a model with present labour income risk and $wealth_{no\ risk,t}$ is an expected value of wealth in a model without labour risk (but with liquidity constraints) - it measures the strength of precautionary motive and is well defined since the presence

accumulated wealth and depends strongly on ρ parameter and magnitudes of permanent and transitory shocks to lifetime income path. In benchmark case (all households) precautionary wealth equals 14% of total wealth and is at lower bounds of precautionary motive strength reported in literature.

- low estimates of γ_0 indicate that households accumulate enough wealth to secure it's consumption at retirement through wealth de-cumulation by selling financial and non-financial (reverse mortgage) assets.
- there are differences on male and female preferences in terms of relative risk aversion, which is higher in the case of males. The model estimated for woman is statistically insignificant in terms of overidentifying restrictions test, contrary to the estimation performed for males and one of the possible reasons might be a problem with attrition of female-ruled households near retirement, see Figure 17.
- there is a substantial voivodeship heterogeneity across estimated parameters of relative risk aversion: in general, poorer voivodeships (in terms of mean salary) exhibit higher precautionary motive than richer ones (though, with a few exceptions). Estimates of relative risk aversion of voivodeships are in line with the literature in terms of macroeconomic interpretation of this parameter. Intertemporal elasticity of substitution of consumption (inverse of coefficient of relative risk aversion in the case of CRRA utility function) are lower for poor voivodeships, and a possible explanation for this is that poorer households consumption consists of a larger share of necessities which are inelastic. This relation between voivodships and ρ seems to be robust to unpublished sensitivity check's.
- there is some degree of heterogeneity across different NACE categories, with two groups with essentially the same relative risk aversion coefficient's within the group. The first group with higher ρ coefficient consists of mining, manufacturing, education and human health and social work activities while the second includes construction, wholesale and retail trade, transporting and storage, financial and insurance activities and real estate activities. It is thus harder to find a reason for such heterogeneity than in case of voivodeship, as economical characteristics

of liquidity constraints makes sure that wealth will be positive at every age.

of that groups are already heterogeneous (in terms of salary, type of job (blue or white collars) etc.).

The table below summarizes the main regional results, a full set of results can be found in Table 6.

Table 3: Estimated preference parameters and shock magnitudes by region

Voivodeship	β	ρ	γ_0	γ_1	Rtest	Nobs	Prec.wealth	σ_n^2	σ_u^2
ALL	0.983 (0)	0.895 (0.23)	0.146 (0.01)	0.058	128.31 (0)	170390	0.14	0.0093	0.0238
DOLN	0.983 (0.001)	1.028 (0.698)	0.147 (0.031)	0.059	43.86 (0.173)	14094	0.15	0.0094	0.0234
KUJA	0.942 (0.015)	5.768 (0.856)	0.001 (0.033)	0.064	54.49 (0.025)	9021	0.42	0.0083	0.0267
LUBE	0.984 (0.006)	2.416 (1.155)	0.235 (0.043)	0.059	102.24 (0)	5883	0.29	0.0087	0.0197
LUBU	0.981 (0.007)	0.05 (1.325)	0.259 (0.047)	0.057	97.26 (0)	4385	0.06	0.0091	0.0213
LODZ	0.981 (0.006)	0.05 (0.943)	0.357 (0.036)	0.057	68.92 (0.001)	12617	0.07	0.0088	0.0241
MALO	0.981 (0.004)	0.05 (0.719)	0.205 (0.029)	0.057	75.12 (0)	12673	0.06	0.0096	0.0178
MAZO	0.981 (0.001)	0.061 (0.458)	0.001 (0.023)	0.058	97.9 (0)	27284	0.07	0.0109	0.0329
OPOL	0.843 (0.035)	9.305 (1.236)	0.001 (0.042)	0.069	47.18 (0.101)	3714	0.53	0.0082	0.0343
PODK	0.982 (0.007)	2.251 (1.128)	0.476 (0.042)	0.059	42.96 (0.198)	5799	0.39	0.0089	0.0143
PODL	0.868 (0.034)	8.368 (1.251)	0.001 (0.046)	0.068	42.77 (0.203)	4590	0.51	0.0080	0.0222
POMO	0.983 (0.001)	1.2 (0.593)	0.065 (0.03)	0.059	54.24 (0.026)	11265	0.17	0.0112	0.0242
SLAS	0.982 (0.003)	2.288 (0.507)	0.187 (0.022)	0.06	50.77 (0.052)	26938	0.28	0.0093	0.0204
SWIE	0.903 (0.026)	7.425 (1.115)	0.001 (0.037)	0.066	75.9 (0)	4594	0.53	0.0088	0.0258
WARM	0.985 (0.006)	3.587 (1.107)	0.001 (0.041)	0.059	48.84 (0.075)	6577	0.24	0.0076	0.0217
WIELK	0.981 (0.004)	0.095 (0.748)	0.072 (0.03)	0.057	42.26 (0.219)	13470	0.06	0.0084	0.0276
ZACH	0.971 (0.008)	3.312 (0.822)	0.001 (0.034)	0.062	81.52 (0)	7486	0.28	0.0089	0.0249

Notes: Standard error of estimated parameters and pvalue of overidentifying restrictions test (Rtest) in parenthesis.

Source: own calculations

It is easy to see that estimated ρ parameter varies dramatically in some cases. In fact, it is a hard task to estimate ρ and β simultaneously using the simulated method of moments presented in this paper. Robustness checks performed, for example, in [14] shows that the estimated ρ parameter exhibits high sensitivity to initial assumptions. In some cases, such as for voivodeships DOLN, OPOL, PODK, PODL, SLAS, WARM, WIELK the model is statistically correct at 5 percent level of significance according to overidentifying restrictions test (Rtest) statistic. It might be needed to choose different estimation technique (like Bayesian estimation, exogenous calibration of preference parameters either ρ or β) or to expand the initial model ⁵⁴. One of the possible reasons for the volatility of ρ parameter may be attrition of households near their 60's resulting from the high variance of empirical profiles.

7.2. Simulation based on stocks

7.2.1. Precautionary motive

The preceding chapter, devoted to a simulation based on a comparison of flows,⁵⁵ assumed that the integral of field between lifetime incomes and expenditure stands for stock of savings available at the doors of retirement, collected in the labour activity period. The actual stocks of savings collected by the households were not confronted, which may cause an over- or underestimation of actual economic behaviours. The availability of the HWS data and encouraging work by Carroll [3] led us to the extension of the simulation by the stocks (levels) of wealth. To somehow tackle, on the one hand, the very modest average financial wealth, and on the other hand, the very significant real-estate self-defined value, the distinction between these two is introduced as follows:

- net wealth = net financial buffer wealth + net non-financial wealth⁵⁶,
- net financial buffer wealth = (HH gross financial wealth – HH current financial debt),
- net non-financial wealth = (HH gross non-financial wealth – HH mortgage debt).

⁵⁴For example, the introduction of housing as an asset.

⁵⁵Transactions in revenues (income) and expenditures (consumption).

⁵⁶Cars, which represent a significant part of the wealth reported in the HWS are regarded as fully amortised when purchased, and, therefore, regarded as pure consumption.

On top of the above, a mighty household head assumption is eased for a more individual approach: the decision power is based on the individual contribution weight w of any household member, who brings in any non-social security income. The assumption behind this relies not only on a presumed more democratic process of decision making by household members, but also by some practical observations of the individuals across developed countries, where nearly 40-50% of marriages divorce or decide earlier for a marriage settlement⁵⁷, e.g.:

$$w = I_{Mhh}/I_{hh} \quad (39)$$

where w stands for the individual share of economic decision power, I_{Mhh} is individual income brought in in a given month by a given member M of that household, and I_{hh} is household total income.

Consequently, each household is spread over the respected members' cohorts, with individual endowment of income and adequate, household related, portion of net (financial) wealth. Finally, the input to the model is listed below:

- indiv. net wealth = indiv. net financial buffer wealth + indiv. net non-financial wealth
- indiv. net financial buffer wealth = (HH gross financial wealth – HH current financial debt)* w
- indiv. net non-financial wealth = (HH gross non-financial wealth – HH mortgage debt)* w

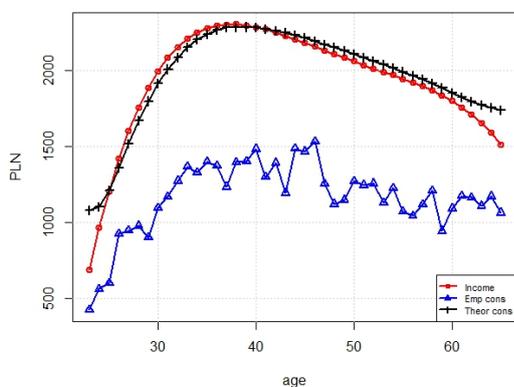
With the above proposed solution we tried to overcome the unification that stems from the use of the OECD equivalence scale⁵⁸ that in our results unified households' individual consumption in spite of heterogeneity in individual income and further characteristics.

⁵⁷To the authors' best knowledge, this is the first time such an approach has been applied, although, some more complex systems based on members declarations can be imagined.

⁵⁸From the CSO website: The scale used for the calculation of the households' expenditure level and determination of poverty thresholds (relative and subsistence minimum), in order to eliminate the effect of the socio-demographic composition of household on the maintenance cost. According to this scale, weight 1 is attributed to the first person in the household at the age of 14 or more, weight 0.7 - to any other person at the age 14 or more, weight 0.5 - to every child under 14 years old.

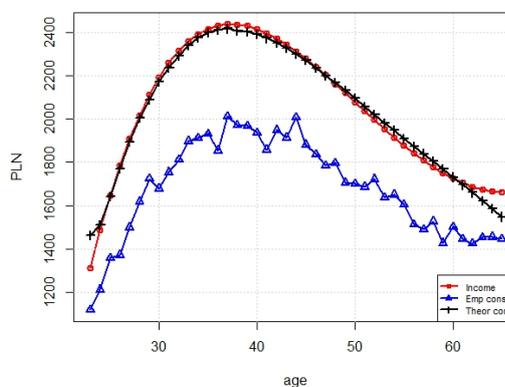
The option where the total net wealth is regarded as fully liquid seems the extreme scenario, in which the household, at any moment in life can start cashing it in in the form of for example, a reversed mortgage, where the financial institution gradually becomes the owner of the real estate upon request⁵⁹. The charts below show the comparison between the HWS based simulation and integrated dataset (HBS \cap HWS) in the scenario, where the model estimates consumption on the basis of income extended by the individually available net financial wealth:

Figure 6: HWS



Source: own calculations

Figure 7: HBS \cap HWS



Source: own calculations

The parameters, as expected, *need* to overcome the imposed motives: precautionary and the buffer stock, when compared with the actual consumption, so the β varies around 0.5. The findings from this exercise seem valuable, and confirm to a large extent the observations by Carroll that the young households tend to create modest, precautionary stocks of financial means at the beginning of the career path to insure themselves against idiosyncratic risk of, for example longer unemployment periods. However, they seem to be impatient to keep increasing their financial wealth even when facing the approach of retirement. This is most probably why the theoretical consumption is attached to the stock of financial means until almost the very end of labour activity. In this respect the households, on average, may believe in an acceptable

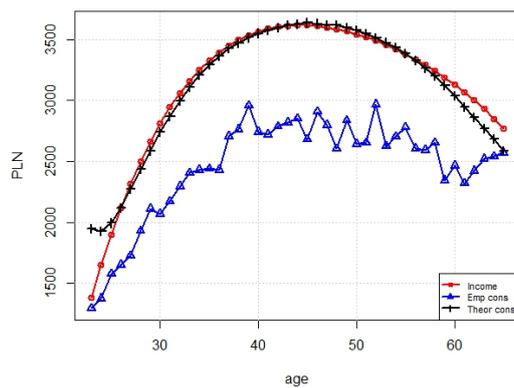
⁵⁹It is then a theoretically possible composition of the part of the real estate market, where the households do not take a mortgage, which would mean that the owners of the real estates are among other sectors of the economy e.g. financial corporations or general government.

pension from the general pension system or accept a dramatic reduction of consumption in response to any pension offered. The model confirms the summary of the HWS: the precautionary motive is dominant, with a yet unknown distribution of the buffer stock motive.

7.2.2. Buffer stock motive

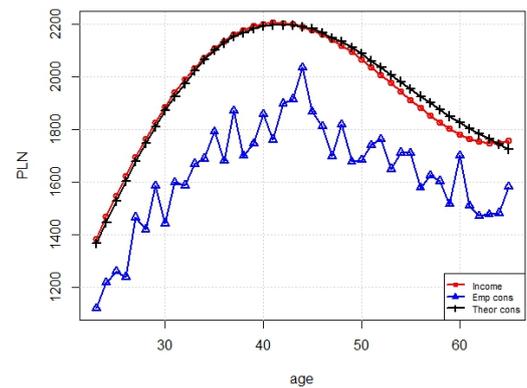
Since the buffer stock is quite a rare type of behaviour, the HWS in its current version is too small to create the required size of the subset for, for example, top level managers with top deciles of incomes or wealth. Nevertheless, some signs of buffer stock behaviour can still be found when simulating the trimmed datasets, e.g. for education:

Figure 8: Higher education



Source: own calculations

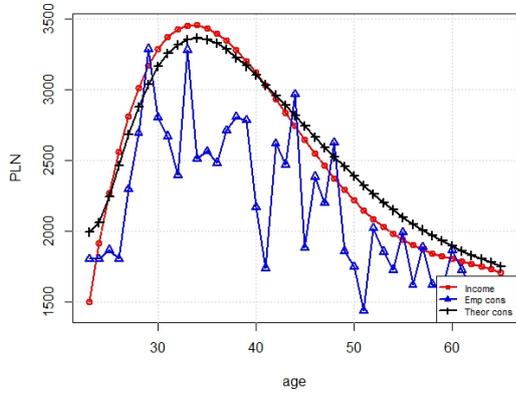
Figure 9: Mid education



Source: own calculations

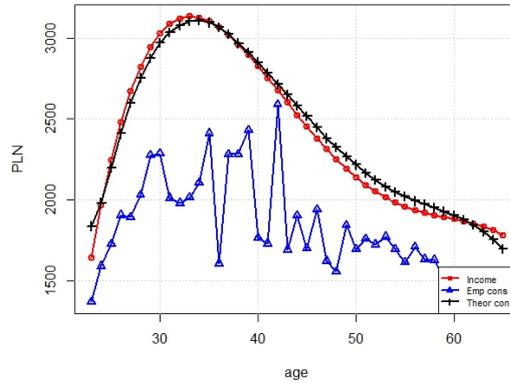
...household composition:

Figure 10: One person without children



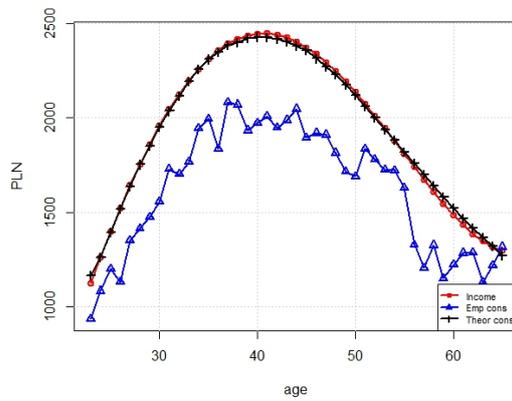
Source: own calculations

Figure 11: Two persons without children



Source: own calculations

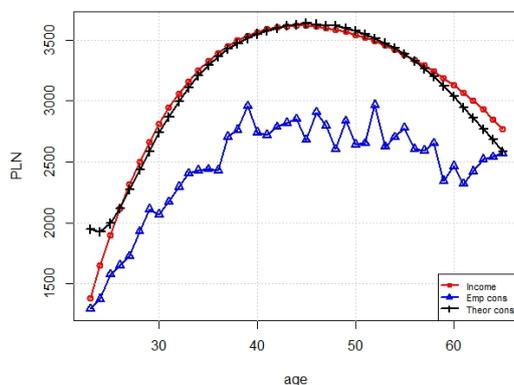
Figure 12: Two persons with children



Source: own calculations

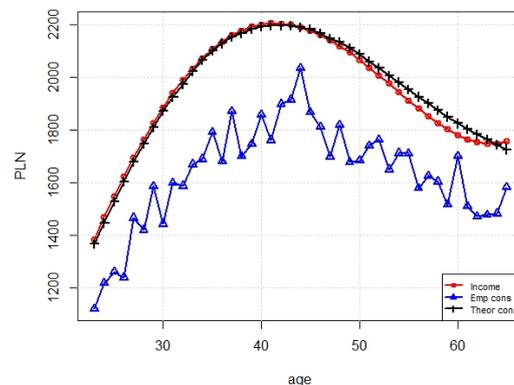
...region:

Figure 13: Mazowieckie



Source: own calculations

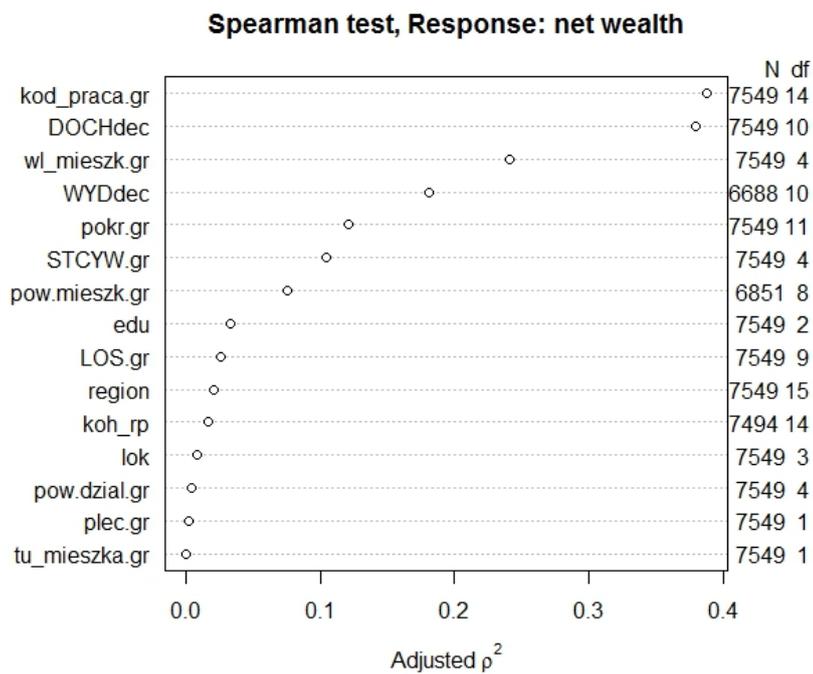
Figure 14: Podkarpackie



Source: own calculations

7.3. Modelling with the use of the net wealth stocks

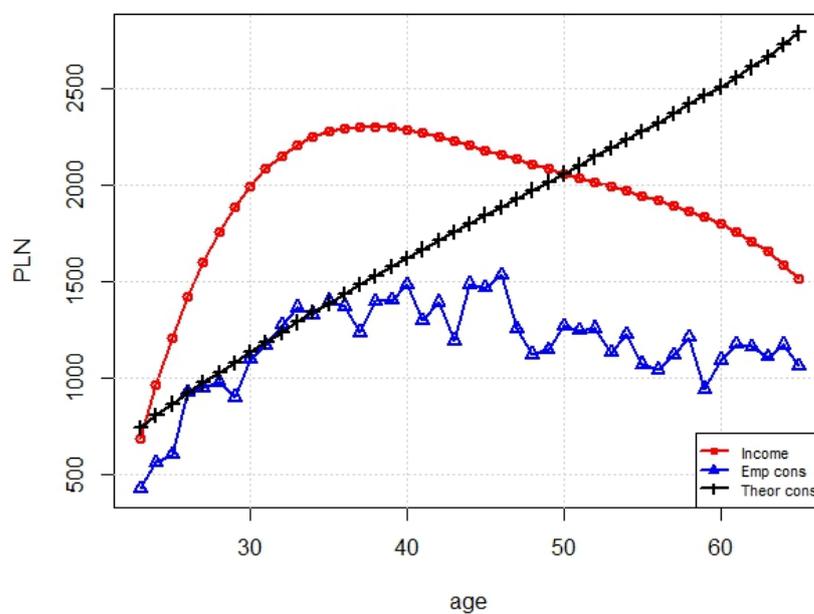
The graph below presents a theoretically and practically discussible assumption of full liquidity of the entire net wealth, additionally with subjective valuation declared by households in the HWS. If it was so, then the households could start cashing their durable property as if it was fully liquid throughout their entire life. The graph presents the modelling results based on HWS only, since the imputation of substantial amounts of net wealth from smaller data donor (HWS) to larger data receiver (HBS) bears the risk of unsupervised errors. The graph below suggests a different composition of regressands for the net wealth than in case of net financial wealth, with a surprisingly high position of the type of job, and more expected further categories, e.g. income (DOCHdec) or type of ownership of the apartment or house (wl_mieszk.gr):



Source: own calculations

The diverging theoretical consumption from the empirical consumption suggest that Polish households do not consider their non-financial wealth as a source of consumption smoothing.

Figure 15: Profiles on the basis of net wealth



Source: own calculations

7.4. Summary of the modelling

Although the model suits empirical consumption well, it brings a worrying picture: the precautionary motive dominates, but for the buffer stock motive observed for the well-educated members of the households in richer regions. The non-financial assets as a predominant source of wealth are not cashed to smoothen consumption over the life cycle, as suggested by the model. The discount factor ranges within the values reported in the literature, though, the risk aversion fluctuates and in some cases exceeds both the upper and lower values present in the literature.

8. Doubts on the robustness of the rationale, the model and the data

8.1. Doubts about the rationale of the approach

When looking at Figure 1, the question occurs if indeed the current 45 year old cohorts assume that their professional career will mirror that of their predecessors, i.e. 45's mean a peak in function of their permanent income path? The slope of the function suggests that they should give up their development and refrain from the ambitions towards less paid occupation. In fact it does not reflect a single generation life cycle pattern but a path of all generations through the dynamic process of convergence of the Polish economy. More generally, in our opinion it is highly doubtful if the life cycle model in the proposed realisation is applicable to the developing economies on the steep path of convergence to the desired level, while all generations seem affected by this dynamic process simultaneously. The older generations grew up in a different economic reality of the communism era, upon the collapse of which their cumulated financial wealth held in Polish zloty evaporated, and they have been left with (or even without) mainly non-financial wealth and endowed with a different educational level and professional experience than their descendants. It is possible that the herding driven by the motive of catching up to the expected life conditions overcomes the buffer stock behaviour.

The question arises then as to how fast the „wave” of high income growth observed for the current generations in their 40's will persist in the future of rapidly ageing Polish society? With an expected high rate of return of education that formed the human capital, these generations probably expect that their career path will last longer. If so, then the question arises as to whether the flattened, prolonged high permanent income path will translate into consumption? If it detaches from the permanent income, then the next questions follow: will an appetite for financial assets occur, and if so, then at which share of risk-free ones? Or will the households keep their devotion to accumulation and petrify the shallow Polish financial market? The strong assumption on households' life-time perfect foresight and rationality were questioned in international and Polish literature, e.g. by Bańbuła [1]. The pure

rationality in the applied method of PILCH does not recognise behavioural aspects, e.g. herding, changes in taste and moods, experience weighted affection, bad or good luck affecting consumption and labour - these, and many more, actually observed phenomena escape the utilitarian functional world.

8.2. Adequacy of the applied model solution

Taking into account the obtained results and the HWS summary, which concludes that 95% of Polish households' wealth is represented by non-financial wealth and the average (not very different from the median) financial wealth per household amounts to nearly 1-3 monthly salaries in spite of age, suggests serious consideration of an option of at least two types of assets to choose from in the life cycle: the financial, used clearly for precautionary purposes should be extended by some reliable valuation of the non-financial assets, particularly the real estate. Such a development of the model would require a second state variable (it would not be possible to perform normalization as in 2.1) and second control variable (choice to allocate savings between financial and real estate assets) or apply the adjusted solutions of Krueger et al [20], which would significantly complicate and lengthen the numerical solution or require to significantly change the model. Furthermore, a method of valuation of the non-financial assets, and foremost, the probability to achieve success in cashing it at the desired value and time, is beyond the scope of the planned work over the current project. Nevertheless, such a calculation seems inevitable if the replacement rate ⁶⁰ in the future drops to 15-20% (see Jabłonowski et al [17]), then the Walrasian auctioneer shall include such assets in households' life-cycle planning.

With regard to technicalities, the ρ and β can be computed with use of the Bayesian approach, which should improve their precision and decrease reliance on data. More attention may be placed on ρ , while it approaches the higher margin suggested in the literature. On the basis of the authors' previous internal work, the high aversion to risk that is revealed in increasing the share of the liquid part of financial wealth in the total financial wealth, may be explained by the consequences of the global crisis in 2008 that caused a significant devaluation of financial wealth stored in for example, share investment funds. The theory of experience weighted affection (EWA) may be

⁶⁰Regarded as the ratio between the average pension to average salary in the economy

helpful in finding out if such (negative) experiences change temporarily or permanently the attitude to investment structure and consumption to investment ratio.

8.3. Quality of the data

The current section covers further thoughts based on experience on the possible reasons for the most probably inadequate reflection of the households' heterogeneity that affects significantly the complete markets approach in Poland. The methodology of the HBS includes two-level sampling with first level stratification. The units of sampling at the first level are local research points (LRP). The entire country is divided into LRP that meet the requirement of concentration of a min. of 250 in towns and 150 households in villages. The second stage sampling units are apartments. Before the first level they are stratified according to the following order: regions, then in line with capital cities in the regions, groups of cities of comparable number of inhabitants, and finally communes of villages. The allocation of the number of flats at the first stage sampling among strata is proportional to the number of flats in that strata. In each LRP a draw of 24 flats is performed (2 each month) plus the backup sample of 150 flats, stored in random order. In the case of a lack of possibility to complete the interview or statistical booklet by the preselected household, another one is drawn from the backup sample.

Among all pre-selected households to be approached for the first time ever in the HBS in 2013 only 36.7% were successfully interviewed. The percentage decreases in line with the growing size of the cities. To overcome this problem, specific weights are introduced: at the first stage the weights rely on reversed probability of the selection that are set in two-stages sampling. Further, on the basis of the National Census the stratification is introduced on basis of the class of the place of residence (town, village) and size of the household. The obtained weights are multiplied by such amounts to finally meet the aggregate of the population from the National Census.⁶¹

Since the HBS weights do not include any verification of the age of the inhabitants in drawn households, but on their number in such a household, the demographic structure (size of annual cohorts) is inconsistent with the National Census. The literature (for example, see Myck et. al [25]) brings some expert solutions on correction of the original

⁶¹The weights for the years 2005-2011 relied on the 2002 National Census, the weights for later years relied on the National Census 2011.

weights to meet the National Census in size and structure. However, from the point of interest of this paper, if the main cause of refusal to participate in the HBS interview refers to the sensitive information on, for example income, then there is a risk of the existence of significantly biased estimators. When comparing with, for example HWS stocks of self-defined value of financial or non-financial assets or liabilities, it becomes quite clear that it is nearly certain. In fact, the HWS is probably realised in the same way as the HBS, which may cause comparable problems, e.g. HWS covers 60% of total financial liabilities, and 40% of financial assets, but the HBS reflects just 10% of total cash in the pockets of the households recorded in the national accounts. Perhaps, the marginal cost (in time spent, and sensitive information revealed) of filling in the detailed household consumption booklet is bigger and still growing when compared with the marginal revenue of satisfaction and some small remuneration, then the risk of biased estimators may cause the unknown misinterpretation.

The estimation of variance of the shocks to permanent and transitory components of income based on ZUS misses three significant categories of households: self-employed, entrepreneurs and the grey zone. The first two groups are allowed to choose how much gross income to declare, and the social security data suggest that they declare the allowed minimum. The grey zone seems reliably untouched by any of the analysed statistics considered in this report.

9. Conclusions

Although, the Polish households declare that they save for retirement and initial analysis based on HBS 1999-2014 flows confirm this, a closer analysis of the wealth structure suggests that they seem to rely on non-financial means. Despite the discussible rationale, model and data, the precautionary motive of savings and liquidity constraints cannot be neglected. Even wealthier, better educated households from regions with higher average permanent incomes do not increase the voluntary savings to smoothen consumption over the life cycle.

With regard to Guvenen's inistially asked question *to what extent does partial insurance against idiosyncratic risk affect the concept of complete markets?* we think that due to the short period of existence of the market economy and high propensity of Polish households to consume (or invest?) in real estate and bettering the life conditions households seem quite homogenous, but their insurance against idiosyncratic risk is objectively weak. In this respect, the serious shocks to unemployment or reduction in salaries would result in nearly immediate liquidity constraints and significant limitations in consumption of all age cohorts.

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A. Appendix

A.1. Tables

Table 4: Table of other fixed parameters

Parameter	Value	Source	Estimated model
R	1.02	Rubaszek [27]	Both
p	0.01	-	Both
μ	0.2	<u>Un. benefit</u> mean wage	Both
$E(x_0)$	$\exp(-2.8)$	Gourinchas, Parker [14]	Consumption
$sd(x_0)$	$\exp(1.78)$	Gourinchas, Parker [14]	Consumption
$N - T$	20	-	Both

Notes: In case of „wealth-based” estimation, expected cash on hand and standard deviation were evaluated using the HWS data.

Table 5: Estimates of transitory and permanent shocks to income

Scope	σ_n^2	σ_u^2	$sd(\sigma_n^2)$	$sd(\sigma_u^2)$	Nobs
F a ALL	0.0097	0.0016	0.0018	0.0057	60
F b ALL	0.0152	-0.0071	0.0026	0.0036	78
F c ALL	0.0083	0.0283	0.0007	0.0045	962
F d ALL	0.0052	0.0077	0.0013	0.0031	104
F e ALL	0.0054	0.0099	0.0015	0.0043	71
F f ALL	0.0100	0.0246	0.0032	0.0106	71
F g ALL	0.0086	0.0330	0.0011	0.0039	668
F h ALL	0.0119	-0.0025	0.0015	0.0041	336
F i ALL	0.0162	0.0203	0.0062	0.0150	67
F j ALL	0.0119	0.0289	0.0032	0.0147	82
F k ALL	0.0107	0.0156	0.0010	0.0043	432
F l ALL	0.0079	0.0095	0.0018	0.0045	204
F m ALL	0.0116	0.0358	0.0012	0.0056	364
F n ALL	0.0127	0.0211	0.0034	0.0112	54
F o ALL	0.0046	0.0231	0.0004	0.0018	1122
F p ALL	0.0089	0.0281	0.0004	0.0018	2149
F q ALL	0.0077	0.0169	0.0005	0.0014	1549
F r ALL	0.0083	0.0205	0.0018	0.0049	158
F s ALL	0.0071	0.0397	0.0022	0.0092	74
F NA ALL	0.0108	0.0128	0.0012	0.0044	379
F ALL ALL	0.0085	0.0224	0.0002	0.0009	8984
M a ALL	0.0087	0.0181	0.0012	0.0064	185
M b ALL	0.0102	0.0072	0.0009	0.0020	514
M c ALL	0.0095	0.0273	0.0006	0.0020	2506
M d ALL	0.0050	0.0082	0.0006	0.0022	534
M e ALL	0.0043	0.0238	0.0011	0.0056	249
M f ALL	0.0091	0.0477	0.0013	0.0055	466
M g ALL	0.0143	0.0389	0.0013	0.0043	722
M h ALL	0.0072	0.0174	0.0006	0.0025	824

M i ALL	0.0136	0.0084	0.0043	0.0102	33
M j ALL	0.0096	0.0461	0.0022	0.0131	181
M k ALL	0.0148	0.0196	0.0028	0.0076	151
M l ALL	0.0096	0.0259	0.0028	0.0097	151
M m ALL	0.0162	0.0278	0.0025	0.0080	190
M n ALL	0.0113	0.0411	0.0031	0.0148	88
M o ALL	0.0082	0.0272	0.0011	0.0039	472
M p ALL	0.0143	0.0291	0.0012	0.0040	588
M q ALL	0.0128	0.0239	0.0022	0.0050	278
M r ALL	0.0048	0.0286	0.0021	0.0097	92
M s ALL	0.0118	0.0289	0.0042	0.0085	49
M NA ALL	0.0135	0.0109	0.0015	0.0040	477
M ALL ALL	0.0101	0.0252	0.0003	0.0009	8750
ALL a ALL	0.0089	0.0140	0.0010	0.0047	245
ALL b ALL	0.0109	0.0053	0.0009	0.0017	592
ALL c ALL	0.0092	0.0276	0.0004	0.0018	3468
ALL d ALL	0.0051	0.0081	0.0006	0.0018	638
ALL e ALL	0.0046	0.0208	0.0008	0.0045	320
ALL f ALL	0.0093	0.0447	0.0012	0.0051	537
ALL g ALL	0.0115	0.0361	0.0008	0.0029	1390
ALL h ALL	0.0086	0.0116	0.0006	0.0022	1160
ALL i ALL	0.0153	0.0163	0.0042	0.0101	100
ALL j ALL	0.0103	0.0408	0.0017	0.0100	263
ALL k ALL	0.0118	0.0166	0.0011	0.0038	583
ALL l ALL	0.0086	0.0165	0.0016	0.0050	355
ALL m ALL	0.0132	0.0330	0.0011	0.0043	554
ALL n ALL	0.0118	0.0335	0.0022	0.0102	142
ALL o ALL	0.0057	0.0243	0.0004	0.0018	1594
ALL p ALL	0.0100	0.0283	0.0004	0.0015	2737
ALL q ALL	0.0085	0.0180	0.0006	0.0016	1827
ALL r ALL	0.0070	0.0235	0.0014	0.0045	250
ALL s ALL	0.0089	0.0354	0.0022	0.0064	123
ALL NA ALL	0.0123	0.0118	0.0010	0.0029	856
ALL ALL ALL	0.0093	0.0238	0.0002	0.0007	17734
ALL ALL DOLN	0.0094	0.0234	0.0006	0.0021	1298
ALL ALL KUJA	0.0083	0.0267	0.0008	0.0030	941
ALL ALL LUBE	0.0087	0.0197	0.0007	0.0026	908
ALL ALL LUBU	0.0091	0.0213	0.0010	0.0035	449
ALL ALL LODZ	0.0088	0.0241	0.0006	0.0038	1164
ALL ALL MALO	0.0096	0.0178	0.0006	0.0017	1485
ALL ALL MAZO	0.0109	0.0329	0.0006	0.0021	2459
ALL ALL OPOL	0.0082	0.0343	0.0011	0.0048	482
ALL ALL PODK	0.0089	0.0143	0.0008	0.0021	992
ALL ALL PODL	0.0080	0.0222	0.0010	0.0037	477
ALL ALL POMO	0.0112	0.0242	0.0010	0.0034	858
ALL ALL SLAS	0.0093	0.0204	0.0005	0.0017	2420
ALL ALL SWIE	0.0088	0.0258	0.0010	0.0043	497
ALL ALL WARM	0.0076	0.0217	0.0008	0.0031	635
ALL ALL WIELK	0.0084	0.0276	0.0006	0.0023	1446
ALL ALL ZACH	0.0089	0.0249	0.0009	0.0033	690
ALL ALL NA	0.0092	0.0165	0.0009	0.0030	533

Notes: A negative value should be interpreted as insignificantly different from zero. M,F means male and female accordingly, a lower case letter describes the NACE category and NA stands for not available.

Source: own calculations, on the basis of ZUS sample.

Table 6: Results of structural estimation of life-cycle consumption/savings model (on the basis of HBS consumption)

Scope	β	ρ	γ_0	γ_1	Rtest	Nobs	Prec.wealth
F ALL ALL	0.981 (0.002)	0.05 (0.48)	0.241 (0.016)	0.057	538.22 (0)	57968	0.06
M ALL ALL	0.978 (0.002)	2.616 (0.234)	0.021 (0.011)	0.06	44.49 (0.157)	112422	0.27
M,F ALL ALL	0.983 (0)	0.895 (0.23)	0.146 (0.01)	0.058	128.31 (0)	170390	0.14
M,F ALL DOLN	0.983 (0.001)	1.028 (0.698)	0.147 (0.031)	0.059	43.86 (0.173)	14094	0.15
M,F ALL KUJA	0.942 (0.015)	5.768 (0.856)	0.001 (0.033)	0.064	54.49 (0.025)	9021	0.42
M,F ALL LUBE	0.984 (0.006)	2.416 (1.155)	0.235 (0.043)	0.059	102.24 (0)	5883	0.29
M,F ALL LUBU	0.981 (0.007)	0.05 (1.325)	0.259 (0.047)	0.057	97.26 (0)	4385	0.06
M,F ALL LODZ	0.981 (0.006)	0.05 (0.943)	0.357 (0.036)	0.057	68.92 (0.001)	12617	0.07
M,F ALL MALO	0.981 (0.004)	0.05 (0.719)	0.205 (0.029)	0.057	75.12 (0)	12673	0.06
M,F ALL MAZO	0.981 (0.001)	0.061 (0.458)	0.001 (0.023)	0.058	97.9 (0)	27284	0.07
M,F ALL OPOL	0.843 (0.035)	9.305 (1.236)	0.001 (0.042)	0.069	47.18 (0.101)	3714	0.53
M,F ALL PODK	0.982 (0.007)	2.251 (1.128)	0.476 (0.042)	0.059	42.96 (0.198)	5799	0.39
M,F ALL PODL	0.868 (0.034)	8.368 (1.251)	0.001 (0.046)	0.068	42.77 (0.203)	4590	0.51
M,F ALL POMO	0.983 (0.001)	1.2 (0.593)	0.065 (0.03)	0.059	54.24 (0.026)	11265	0.17
M,F ALL SLAS	0.982 (0.003)	2.288 (0.507)	0.187 (0.022)	0.06	50.77 (0.052)	26938	0.28
M,F ALL SWIE	0.903 (0.026)	7.425 (1.115)	0.001 (0.037)	0.066	75.9 (0)	4594	0.53
M,F ALL WARM	0.985 (0.006)	3.587 (1.107)	0.001 (0.041)	0.059	48.84 (0.075)	6577	0.24
M,F ALL WIELK	0.981 (0.004)	0.095 (0.748)	0.072 (0.03)	0.057	42.26 (0.219)	13470	0.06
M,F ALL ZACH	0.971 (0.008)	3.312 (0.822)	0.001 (0.034)	0.062	81.52 (0)	7486	0.28
M,F b ALL	0.981 (0.005)	1.753 (0.9)	0.113 (0.046)	0.06	81.49 (0)	6710	0.24
M,F c ALL	0.983 (0.001)	1.463 (0.434)	0.113 (0.018)	0.059	55.71 (0.019)	39589	0.18
M,F f ALL	0.981 (0.004)	0.05 (0.659)	0.141 (0.026)	0.057	106.38 (0)	19477	0.07
M,F g ALL	0.981 (0.003)	0.05 (0.545)	0.185 (0.029)	0.057	96.63 (0)	19608	0.08
M,F h ALL	0.984 (0.002)	0.684 (0.776)	0.182 (0.03)	0.058	61.43 (0.005)	14640	0.10
M,F k ALL	0.981 (0.004)	0.05 (0.839)	0.213 (0.046)	0.057	120.47 (0)	4937	0.07
M,F l ALL	0.981 (0.017)	0.05 (1.969)	0.197 (0.063)	0.055	71.54 (0)	2103	0.05
M,F p ALL	0.981 (0.002)	1.601 (0.627)	0.075 (0.029)	0.06	127.63 (0)	13397	0.19
M,F q ALL	0.985 (0.001)	1.571 (1.014)	0.167 (0.039)	0.058	17.29 (0.996)	9503	0.19

Notes: Standard deviations in parantheses and p-value in case of overidentifying restrictions test (Rtest). Standard deviations at 0 are equivalent to be smaller than 0.001.

Source: own calculations.

Table 7: Number of households broken down by age and gender of household head

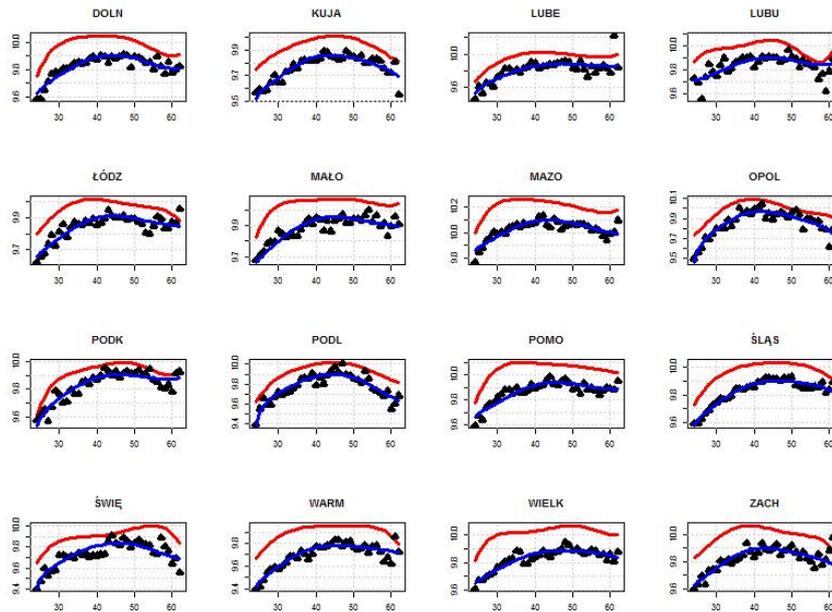
Age	N	Nm	Nk	Age	N	Nm	Nk
23	1977	1292	685	43	5539	3483	2056
24	2655	1798	857	44	5640	3573	2067
25	3390	2287	1103	45	5719	3480	2239
26	3858	2625	1233	46	5575	3393	2182
27	4387	3134	1253	47	5698	3392	2306
28	4931	3553	1378	48	5730	3357	2373
29	5082	3728	1354	49	5523	3202	2321
30	5353	3892	1461	50	5415	3070	2345
31	5334	3971	1363	51	5123	2977	2146
32	5376	3937	1439	52	4840	2843	1997
33	5434	3990	1444	53	4537	2642	1895
34	5367	3875	1492	54	4169	2409	1760
35	5287	3725	1562	55	3486	2151	1335
36	5308	3799	1509	56	2710	1799	911
37	5358	3822	1536	57	2178	1447	731
38	5228	3594	1634	58	1727	1190	537
39	5349	3597	1752	59	1312	931	381
40	5380	3502	1878	60	792	600	192
41	5368	3509	1859	61	465	371	94
42	5454	3514	1940	62	313	260	53
				63	203	179	24

Notes: N - number of households, Nm - number of households with male head, Nk - number of households with female head.

Source: HBS 1999-2014.

A.2. Figures

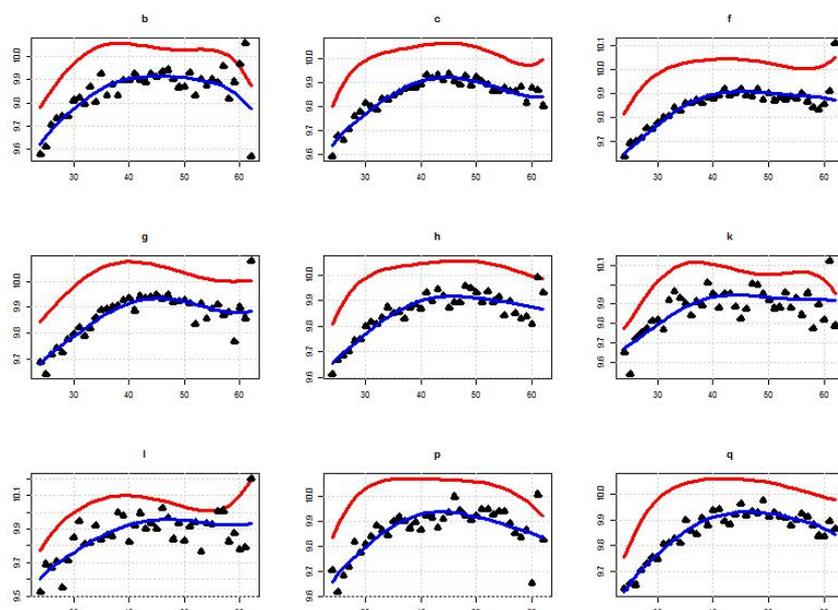
Figure 16: Voivodeship profiles, estimation based on HBS 1999-2014



Red line - smoothed income profile, blue line - theoretical, fitted consumption, black triangles - empirical consumption. All results presented on log-levels.

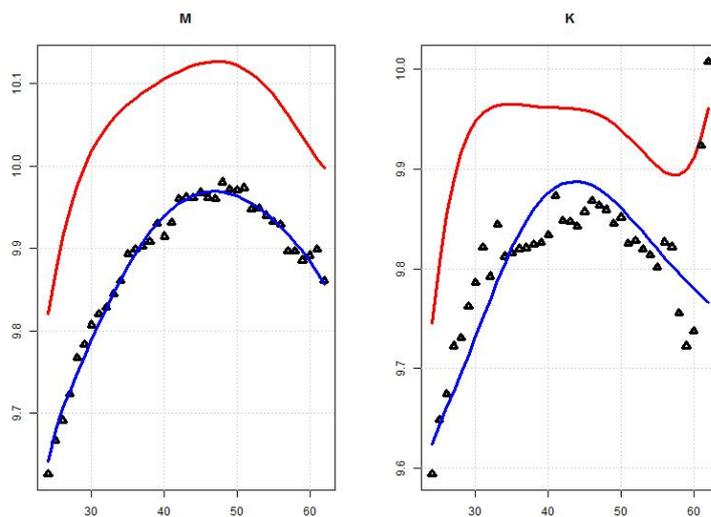
Source: own calculations

Figure 17: NACE profiles, estimation based on HBS 1999-2014



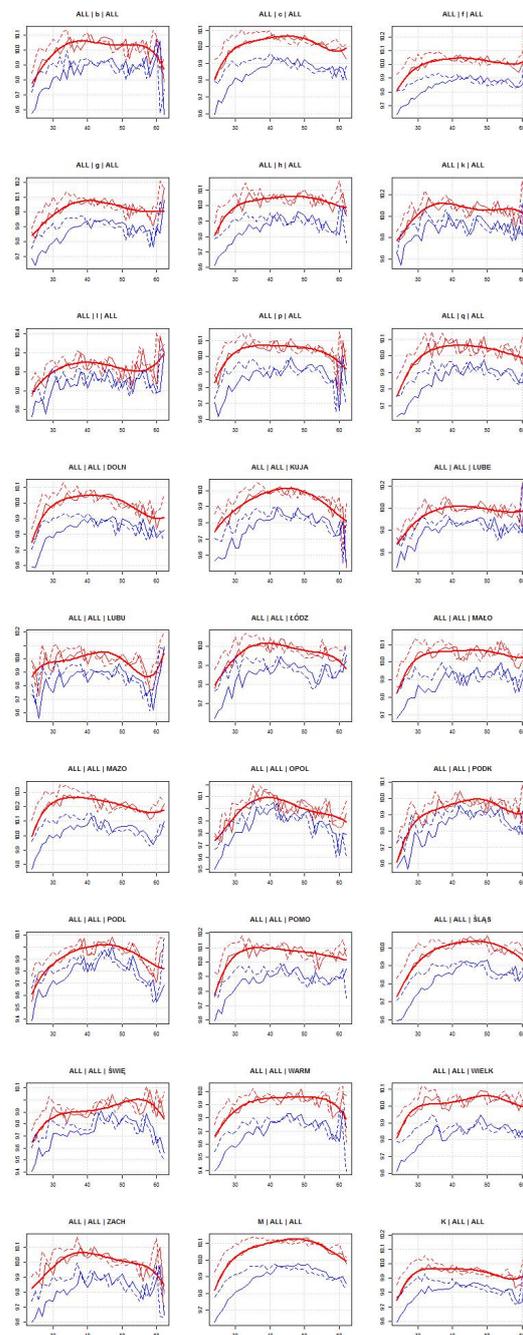
Source: own calculations

Figure 18: Gender profiles, estimation based on HBS 1999-2014



Source: own calculations

Figure 19: Income and consumption profiles, raw and corrected for cohort effects



Notes: Red solid line - smoothed corrected income, red line - corrected income, dashed red line - raw income, blue line - corrected consumption, dashed blue line - raw consumption.

Source: own calculations

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