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#### Abstract

Modeling labor markets in a search and matching framework became a standard approach in DSGE studies. However, there is an expanding strand of literature arguing that similar frictions characterize the product market. When households are required to exert costly shopping effort in order to enjoy consumption, shifts in households preferences tend do have a larger impact on product and employment than in otherwise standard RBC model. We construct a general equilibrium model with frictions both in the labor and the product markets and confirm that in case of the US, preference shocks are the main driver of the business cycle. Moreover, we verify if presence of shopping frictions affects the relation between labor market institutions and unemployment, both in terms of its steady-state level and volatility. However, we find that most results are qualitatively in line with studies treating the product market as frictionless. Higher unemployment benefits and wage rigidity tend to increase variance of unemployment, while benefits also promote higher unemployment in the long run. Firing taxes contribute to lower level and volatility of unemployment. Surprisingly, while effects of recruitment cost on steady state allocation are comparable to the impact of firing cost, the former rises the volatility of unemployment in our simulations for the US.

JEL classification: D50, E02, E32, J65.

Keywords: Unemployment, Labor Market Institutions, Business Cycle.

#### 1. Introduction

There is a vast amount of research emphasizing a role of search and matching frictions to explain labor market dynamics (see the review by Pissarides (2000)). Because of heterogeneity in workers' skills and jobs characteristics combined with information asymmetry, matching an unemployed with a firm requires resource-consuming effort from both sides. Similar analogy can be made for the product market, where consumers search for goods best aligned to their needs and producers try to attract prospective buyers with marketing campaigns. As noted by Michaillat and Saez (2015) costs arising from such frictions can be sizable. In the US between 2003 and 2011 people on average spent 47 minutes daily on shopping for goods and services. Not only households, but also firms incur costs related to goods purchases - according to Michaillat and Saez (2015) the number of US workers occupied with buying and procurement is roughly equal to the number of workers involved in recruitment and interviewing of the job candidates. Finally the fact that a substantial share of sales stems from long-term relationships between sellers and customers, can be seen as evidence for the prevalence of switching costs or incomplete information in the market. Emergence of the evidence mentioned above was followed by the expanding range of studies incorporating shopping frictions into macroeconomic models. In this article we discuss factors shaping the labor market performance in the context of an economy characterized by frictions both in labor and product markets. Specifically, our focus is to evaluate the importance of labor market institutions like unemployment benefits, firing costs, costs to job creation and regulations contributing to wage rigidity. Linkages between labor market structural characteristics and unemployment dynamics are studied through the lenses of general equilibrium model calibrated to the US data. Apart from studying the responses of unemployment to shocks, we reassess the relative role of different type of shocks (technology and preferences) in explaining variance of the unemployment.

While we already mentioned basic evidence for the costs related to frictions in the product market, it is worth discussing the cyclical properties of shopping intensity by the customers. Kaplan and Menzio (2016) study an economy in which higher search intensity translates to lower prices paid by the customers. This combined with an observation that unemployed people tend to spend more time shopping than employed leads to counter-cyclical behavior of goods search and multiplicity of equilibria in the model. However, Petrosky-Nadeau, Wasmer, and Zeng (2016), using cross-sectional data for the US states, provide evidence of a positive correlation between shopping time and income. The notion of shopping effort procyclicality is supported by its aggregate decline with the onset of Great recession in the US. These empirical results are in line with predictions by models assuming that shopping effort is complementary to consumption (e.g. Bai, Rios-Rull, and Storesletten (2012), Petrosky-Nadeau and Wasmer (2015), Michaillat and Saez (2015)). We follow that approach, as in addition it enables us to treat frictions in product and labor markets in a symmetric manner.

Incorporating a frictional products market into a DSGE framework allows us to shed some new light on sources of business cycle volatility. As noted in Bai, Rios-Rull, and Storesletten (2012), the main source of macroeconomic fluctuations in RBC models are TFP innovations. In principle, output dynamics could be driven by shocks directly affecting the capital stock and employment, however this would lead to counter-cyclical movement in wages or require substantial volatility of capital. While TFP shocks are commonly associated with supply-side factors (i.e. technology), presence of product market frictions provides a linkage between measured TFP and demand, understood as shifts in households preferences. The most commonly studied example of such preference shifts are shocks to households' discount factor. Focusing on disturbances to the discount factor seems attractive as they may be interpreted as a short-cut for shocks to financial intermediation or households wealth in a more general setting (see Huo and Rios-Rull (2013)). Michaillat and Saez (2015) shows that in a model with goods market frictions preference shocks can explain the positive correlation between output and capacity utilization, while technology innovations are unable to account for that. We pursue that avenue and formally evaluate what part of variation in main macroeconomic aggregates can be explained by preference shocks.

Our model bears most resemblance to the one proposed by Michaillat and Saez (2015). They develop a theoretical model with symmetric search on goods and labor markets and perform comparative statics analysis to reveal sources of labor market fluctuations in the US. We depart from their approach in several ways. Firstly, we provide a richer description of the labor market, allowing for endogenous job destruction and accounting for the role of labor market institutions. Secondly, in Michaillat and Saez (2015) presence of nominal rigidities is essential for the ability of the model to match real data, which is related to the choice of search cost specification and pricing protocol. In their framework, allowing for price flexibility is equivalent to setting product market tightness as constant, i.e. independent from shocks. This is not the case in our approach, where tightness fluctuates even where prices are elastic. Thirdly, we analyze not only steady-state properties of the model but also business cycle volatility.

Finally, our paper is also related to the strand of studies embedding search and matching framework in the labor market into DSGE models (for the early contributions see Andolfatto (1996) and Merz (1995)). Search frictions give rise to equilibrium unemployment and allow for introduction of labor market institutions. Unemployment benefits act as a factor affecting the outside option of workers in wage negotiations. As noted in Costain and Reiter (2008) and Hagedorn and Manovskii (2008) higher benefits translate to lower share of total surplus from the match accruing to a firm, leading to lower steady state employment and its higher volatility over the business cycle. Zanetti (2011) introduces firing costs to the environment where job matches are affected by idiosyncratic productivity shocks as in Haan, Ramey, and Watson (2000). As opposed to unemploy-

<sup>&</sup>lt;sup>1</sup>Hagedorn, Karahan, Manovskii, and Mitman (2015) confirms relevance of unemployment benefits extensions to job creation using data for the US states.

ment benefits, increase in firing tax limits volatility of employment at the cost of higher variance of real wages. Surprisingly steady state employment rises with firing costs, as the effect of the lower job destruction rate dominates over the impact of hampered job creation. According to Abbritti and Mueller (2013) there is also a certain equivalence between firing costs and hiring costs, as both can be seen as rigidities constraining labor market flows. Authors contrast this class of rigidities with rigidities hindering wage adjustments that can be captured by the role of wage norms in the process of wage settlement (see Hall (2005)). As noted in Shimer (2005) wage rigidity can improve the ability of a baseline model to match the high volatility of the ratio between unemployment and vacancies observed in the data. Here, we depart from the literature by studying the relevance of institutions for labor market performance in a model allowing not only for frictions in the labor market but also in the market for products. In that way we depart from the environment in which business cycle is driven mainly by technology shocks and add additional channel of shocks propagation to the model (see Petrosky-Nadeau and Wasmer (2015)).

Our results may be summarized as follows. We confirm the findings by Michaillat and Saez (2015) that preference shocks tend to outperform technology innovations in terms of the ability to explain business cycle fluctuations in the US economy. Dominance of preference shocks is particularly stark when one focuses on labor market aggregates - 99% of variance in unemployment and wages can be attributed to preference shifts. Presence of product market frictions affects the way firms respond to shocks, for example positive technology shocks depress the equilibrium probability that products offered by a company are matched with customers, so firms reduce employment despite higher productivity of workers. However, while focusing on the effects of labor market institutions on steady state and business cycle volatility, the results are mainly in line with the literature neglecting shopping frictions. Higher unemployment benefits and wage rigidity tend to increase variance of unemployment, while benefits also promote higher unemployment in the long run. Firing costs contributes to lower variance of unemployment, while surprisingly the opposite holds for hiring costs. Finally, both firing an hiring costs exert a negative impact on unemployment in the long term. However, that result is mostly local by nature (i.e. it holds for small changes in parameters) as in the limit zero hiring costs imply no steady state unemployment in the model.

In the next section we present the setup of the model. Section 3 discusses calibration and estimation procedure. Dynamic properties of the model are presented in section 4, while in sections 5 and 6 we study relevance of labor market institutions for the steady state and business cycle volatility, respectively. Section 7 concludes.

#### 2. Model

#### 2.1. Households

There is a continuum of identical, infinitely-lived households of measure one. Each of them is populated by measure one of members. Households solve the following dynamic problem:

$$V(a, X) = \max_{c, v_g, a' > 0} \left\{ A_d \ln(c) - \kappa_g v_g + \beta \mathbb{E}_{X'|X} V(a', X') \right\}$$

$$c = q_g v_g \tag{1}$$

$$pc + a' + \tau = a(1+i) + w\tilde{N} + b(1-\tilde{N})$$
 (2)

where a is current stock of bond holdings, i is nominal interest rate, p is nominal price of goods, w - nominal wage,  $\tilde{N}$  - number of employed members, b - unemployment benefit,  $v_g$  - number of visits made by household,  $\tau$  - lump sum tax that finances unemployment benefits, c - consumption, a' - next period choice of liquid assets (i.e. bond holdings) and X - state variable  $X = \{N, A_d, A_z\}$  with  $A_d$  and  $A_z$  representing shocks to preferences and technology, respectively (moreover note that N is different from  $\tilde{N}$  and it will be defined later). In order to enjoy consumption, households have to excert shopping effort (i.e. visit the shop), which is costly in terms of utility (scaled by  $\kappa_g$ ). However, not all visits are successfull, as availability of goods depends on aggregate market tightness - an effect captured by a variable  $q_g \in [0,1]$  defined later. While the mechanism is simple it allows us to incorporate product market frictions into the otherwise standard DSGE model.

We use the constraints to obtain expressions for c and  $v_g$  and plug them into the Bellman equation. The FOC with respect to c and the envelope condition are:

$$-\frac{A_d}{c} + \frac{\kappa_g}{q_g} + p\beta \mathbb{E}_{X'|X} V_a(a', X') = 0$$

$$V_a(a, X) = (1+i) \left( \frac{\frac{A_d}{c} - \frac{\kappa_g}{q_g}}{p} \right)$$
 (3)

Combining these two equation yields the Euler equation:

$$1 = \mathbb{E}_{X'|X} \left( \Delta(X', X) \cdot (1 + i') \right) \tag{4}$$

where  $\Delta(X', X)$  is the stochastic discount factor defined as:

$$\Delta(X',X) = \beta \frac{p}{p'} \frac{\frac{A'_d}{c'} - \frac{\kappa_g}{q'_g}}{\frac{A_d}{c} - \frac{\kappa_g}{q_g}}$$
(5)

Equations 1-5 characterize the solution to household's problem. Observe that this characterization does not include the FOC associated with a' as the information contained

in this condition can be extracted from the Euler by using equation 3.

#### 2.2. Firms

Firms are identical. In terms of timing we assume that after the realization of shocks  $A_d$  and  $A_z$  and exogenous separations  $\sigma$ , firm makes decision about endogenous firings by setting a cutoff value  $\tilde{a}$  for the distribution of workers' idiosyncratic productivities, which are i.i.d and drawn repeatedly in each period. Next, it posts vacancies  $v_l$  that are filled at rate  $q_l$ . Hired workforce becomes active in the next period and the firm does not pay them salaries w in the current period. The cost of one vacancy is  $\kappa_l$  and it is expressed in terms of the foregone output capacity. Similarly, firing cost (one fired worker requires  $\phi$  goods spent on this process) is expressed in terms of foregone product capacity. To our knowledge, our study is the first to incorporate endogenous job destruction and presence of firing costs into a model with search in both product and labor market.<sup>2</sup> Firm's Bellman equation is:

$$J(n,X) = \max_{v_l,\tilde{a},n'} \left\{ p f_g A_z \left( (1-\sigma)n \int_{\tilde{a}}^{+\infty} a f(a) da - \kappa_l v_l - \phi(1-\sigma)n \int_{-\infty}^{\tilde{a}} f(a) da \right) - w(1-\sigma)n \int_{\tilde{a}}^{+\infty} f(a) da + \mathbb{E}_{X'|X} \Delta(X',X) J(n',X') \right\}$$

$$n' = (1-\sigma)n \int_{\tilde{a}}^{+\infty} f(a) da + q_l v_l$$
(6)

where f(a) is the probability density associated with the distribution of idiosyncratic productivities,  $f_g$  is the probability that good offered by the producer is sold, i.e. matched with a visiting customer. We plug 6 into the Bellman and calculate FOCs with respect to  $v_l$ ,  $\tilde{a}$ :

$$pf_g A_z \kappa_l = q_l \mathbb{E}_{X'|X} \Delta(X', X) J_n(n', X')$$
(7)

$$\tilde{a} = -\phi + \frac{w - \mathbb{E}_{X'|X}\Delta(X', X)J_n(n', X')}{pf_g A_z}$$
(8)

The envelope condition reads:

$$J_n(n,X) = pf_g A_z(1-\sigma) \left( \int_{\tilde{a}}^{+\infty} a f(a) da - \phi \int_{-\infty}^{\tilde{a}} f(a) da \right)$$
$$-w(1-\sigma) \int_{\tilde{a}}^{+\infty} f(a) da + (1-\sigma) \int_{\tilde{a}}^{+\infty} f(a) da \cdot \mathbb{E}_{X'|X} \Delta(X',X) J_n(n',X')$$

Let us rewrite it in a more interpretable form:

$$J_{n}(n,X) = \mathbb{P}\left(a > \tilde{a}\right) \cdot (1 - \sigma) \cdot \left\{ p f_{g} A_{z} \left( \mathbb{E}_{a} \left( a | a > \tilde{a} \right) - \phi \frac{\mathbb{P}\left(a \leq \tilde{a}\right)}{\mathbb{P}\left(a > \tilde{a}\right)} \right) \right\}$$

<sup>&</sup>lt;sup>2</sup>However, relevance of firing costs has been studied in papers abstracting from shopping frictions (see Zanetti (2011)).

$$-w + \mathbb{E}_{X'|X}\Delta(X',X)J_n(n',X')$$
 (9)

So the value of a job (to the firm)  $J_n$  takes into account its survival rate  $\mathbb{P}(a > \tilde{a}) \cdot (1 - \sigma)$ , productivity conditional on the survival rate  $\mathbb{E}_a(a|a > \tilde{a})$ , odds ratio between endogenous destruction and survival  $\frac{\mathbb{P}(a \leq \tilde{a})}{\mathbb{P}(a > \tilde{a})}$ , firing cost  $\phi$ , wage w, price p, productivity  $A_z$ , aggregate demand  $f_g$  and a sum of discounted future values of the job. Equations 6-9 characterize firm's solution.

#### 2.3. Government

Government consists of two branches: fiscal and monetary authority. The former collects taxes to finance unemployment benefits, aiming a balanced budget:

$$\tau = b(1 - \tilde{N})$$

and the latter sets the nominal interest rate:

$$i = \bar{i} + \Phi_p \cdot \frac{p - \bar{p}}{\bar{p}} \tag{10}$$

which for simplicity depends on the price level only, but it can be substituted by a Taylor-type rule.

#### 2.4. Price and wage setting

In an economy with flexible prices, the value of p is set at the optimal level derived from the planner's problem that optimizes the discounted stream of household's utilities with respect to the resource constraint and the law of motion for labor.<sup>3</sup> We choose the value of p so that the consumer's decision rule, i.e. FOC with respect to p boils down to the corresponding planner's FOC:

$$p_{opt} = \frac{A_d u'(c)}{\beta \mathbb{E}_{X'|X} V_a(a', X')} \cdot \frac{1}{1 + \frac{1}{x_g^{\alpha_g}}}$$

$$\tag{11}$$

where  $x_g$  is product market tightness which will be defined later.

Wage-setting follows the Nash Bargaining protocol. Bargaining takes place after the training of the new worker. The values of being employed and unemployed are:

$$V^{e} = w + \mathbb{E}_{X'|X} \Delta(X', X) \left\{ (1 - \sigma) \mathbb{P} \left( a > \tilde{a}' \right) V^{e'} + \left( 1 - (1 - \sigma) \mathbb{P} \left( a > \tilde{a}' \right) \right) V^{u'} \right\}$$

$$V^{u} = b + \zeta + \mathbb{E}_{X'|X} \Delta(X', X) \left\{ f_{l} \left( 1 - \sigma \right) \mathbb{P} \left( a > \tilde{a}' \right) V^{e'} + \left( 1 - f_{l} \left( 1 - \sigma \right) \mathbb{P} \left( a > \tilde{a}' \right) \right) V^{u'} \right\}$$

<sup>&</sup>lt;sup>3</sup>Resource constraint is of the following form:  $c = M_g(v_g, T)$  where T is aggregate capacity in the economy and  $M_g$  is the matching function on the product market.

where  $\zeta$  accounts for the non-monetary benefits of being unemployed (e.g. leisure or home production).<sup>4</sup> Notice that our timing convention requires that hiring rate enters the equation defining  $V^u$  and takes its present value. Firms' surplus is:

$$\tilde{J}_n = p f_q A_z \mathbb{E} \left( a | a > \tilde{a} \right) - w + \mathbb{E}_{X'|X} \Delta(X', X) J_n(n', X')$$

observe that  $\tilde{J}_n$  is measured after separations and takes into account the future value of job  $J_n$ . Notice that the relation between  $\tilde{J}_n$  and  $J_n$  can be expressed in the following way:

$$J_n = \left[ \mathbb{P}(a > \tilde{a}) \tilde{J}_n - p f_g A_z \phi \mathbb{P} \left( a \leq \tilde{a} \right) \right] (1 - \sigma)$$

The bargaining process yields the following condition:

$$\lambda \tilde{J}_n = (1 - \lambda) \left[ V^e - V^u \right]$$

Workers' bargaining power is denoted by  $\lambda$ . Firm is not blamed for unsuccessful negotiations and hence we omit the cost  $\phi$  in the LHS of the expression above. Bargaining condition and the expression for  $\tilde{J}_n$  (in terms of  $J_n$  etc.) are used to obtain the value of wage:

$$w_{Nash} = \lambda p f_g A_z \mathbb{E} \left( a | a > \tilde{a} \right) + (1 - \lambda)(b + \zeta) \tag{12}$$

$$+\lambda \cdot \mathbb{E}_{X'|X}\left(\Delta(X',X)\left[f_lJ_n(n',X')-(1-f_l)p'f_a'A_z'\phi\mathbb{P}\left(a\leq \tilde{a}'\right)(1-\sigma)\right]\right).$$

Staggered wages are modeled as proposed in Hall (2005) and we resort to similar approach for the price of final good:

$$w' = (1 - \xi_w)w + \xi_w w'_{Nash} \tag{13}$$

$$p' = (1 - \xi_p)p + \xi_p p'_{out} \tag{14}$$

#### 2.5. Consistency conditions

In equilibrium the following consistency conditions must hold. Consistency between individual employment decisions of firms and the aggregate level of workers present in firms at the beginning of period requires:

$$n = N$$
.

The number of workers that take part in the production process and are paid wages is:

$$\tilde{N} = (1 - \sigma)N \int_{\tilde{a}}^{+\infty} f(a)da \tag{15}$$

<sup>&</sup>lt;sup>4</sup>In our treatment of  $\zeta$  we follow Zanetti (2011).

Product market tightness is:

$$x_g = \frac{v_g}{T}$$

where T is aggregate capacity in the economy which is defined as:

$$T = A_z \left[ \int_{\tilde{a}}^{+\infty} a f(a) da(1 - \sigma) n - \kappa_l v_l - \phi \int_{-\infty}^{\tilde{a}} f(a) da(1 - \sigma) n \right]$$
 (16)

Labor market tightness  $x_l$  is:

$$x_l = \frac{v_l}{1 - \int_{\tilde{a}}^{+\infty} f(a)da(1 - \sigma)n}$$
(17)

Rates associated with product/labor market frictions satisfy:

$$f_g = \frac{1}{\left(1 + \frac{1}{x_g^{\alpha g}}\right)^{\frac{1}{\alpha_g}}} \tag{18}$$

$$q_g = \frac{1}{\left(1 + x_g^{\alpha_g}\right)^{\frac{1}{\alpha_g}}} \tag{19}$$

$$f_l = \frac{1}{\left(1 + \frac{1}{x_l^{\alpha_l}}\right)^{\frac{1}{\alpha_l}}} \tag{20}$$

$$q_l = \frac{1}{\left(1 + x_l^{\alpha_l}\right)^{\frac{1}{\alpha_l}}} \tag{21}$$

where  $\alpha_l$ ,  $\alpha_g > 1$ . Resource constraints are:

$$c = f_q T (22)$$

and the resource constraint for labor market is captured by the law of motion for employment 6 aggregated over firms.

The economy is set in motion by two AR(1) processes for productivity and preferences:

$$\log A_d' = \rho_d \log A_d + \epsilon_d' \tag{23}$$

$$\log A_z' = \rho_z \log A_z + \epsilon_z' \tag{24}$$

The dynamic equilibrium in the model is described by equations: 1, 3-24. This is a system of 23 equations with 23 variables: w, n, a,  $q_l$ ,  $v_l$ , p,  $f_g$ ,  $A_z$ ,  $A_d$ , c,  $q_g$ ,  $\Delta$ ,  $J_n$ ,  $f_l$ , i, T,  $V_m$ ,  $x_g$ ,  $v_g$ ,  $x_l$ ,  $\tilde{N}$ ,  $p_{opt}$ ,  $w_{Nash}$ . Observe that in the characterization we omit: the resource constraint for assets (as it is a trivial condition), the budget constraint of household (that is implied by the combination of resource constraints for goods and

liquid assets and hence it conveys no additional information) and the definition of  $x_g$ . This condition can be obtain by dividing equations that define  $f_g$  and  $q_g$  (i.e., 18 and 19) to get:

$$\frac{f_g}{q_g} = x_g$$

and then we use the search constraint 1 together with 22 to get:

$$x_g = \frac{v_g}{T}$$

which is exactly the definition of product market tightness. This means that it carries no additional information and it can be excluded from the model.

#### 3. Parametrization

To analyze the properties of the model quantitatively we need to pin down the values of the parameters. We use data from the US and results from other studies to calibrate parameters determining the steady state allocation. Rest of the parameters are estimated with Bayesian methods.

There are ten calibrated parameters  $(\sigma, \beta, \alpha_L, \alpha_G, \kappa_L, \kappa_G, \phi, b, \zeta, \lambda)$ . Moreover so far we did not specify the distribution for idiosyncratic productivity shocks. For simplicity we assume an uniform distribution with support bounded by  $a_L = 0$  and  $a_H = 2$ . As commonly done in the literature we set the quarterly discount rate  $\beta$  to 0.99 (e.g. Smets and Wouters (2003)), and workers power in wage bargaining  $\lambda$  to 0.5 (e.g Zanetti (2011)). Matching function for the labor market is parametrized in line with Haan, Ramey, and Watson (2000), i.e.  $\alpha_L = 1.27$ . Following Hagedorn and Manovskii (2008) recruitment cost per filled vacancy ( $\kappa_L/q_L$ ) is equal to 4.5% of steady-state wage level. Level of unemployment benefits is chosen so that its ratio to steady state wages corresponds to data on replacement rates published by OECD. We denote the replacement rate of unemployment benefit as  $\nu$ , so  $b = \nu w^{ss}$ . Rest of the labor market parameters  $(\sigma, \phi, \zeta)$  are set to match long term average unemployment rate and flow rate out of unemployment (corresponding to the probability that unemployed finds a job) for the US data. Additionally, in order to differentiate between exogenous and endogenous job destruction rates we resort to estimates of the ratio between the two flows in Bukowski, Kowal, and Lewandowski (2011). We are left with parameters governing product market frictions. Lack of the data corresponding to disutility cost of shopping forces us to normalize  $\kappa_G$  to one. Still we can capture the relevance of frictions by setting the value of  $\alpha_G$  in order to match  $f_G$  with its empirical counterpart - capacity utilization in industry published by FED. Empirical values of variables matched in steady state are shown in table 1, while our calibration is presented in table 3.

Bayesian methods are used to estimate remaining seven parameters: four of them characterizing stochastic processes  $(\rho_z, \rho_d, \sigma_z, \sigma_d)$ , two capturing nominal rigidities  $(\xi_w \text{ and } \xi_p)$  and the parameter governing a reaction of interest rates to inflation dynamics  $(\Phi_p)$ . Unemployment rate and consumption are used as observable variables in the estimation

Table 1: Moments targeted in the steady state

Variable	Value for the US	Source	
u	6.1% FED		
$f_L$	57% Bukowski, Kowal, and Lewandov		
$\frac{\mathbb{P}(a \leq \tilde{a})(1 - \sigma)}{\sigma}$	0.85	Bukowski, Kowal, and Lewandowski (2011)	
$-\frac{\kappa_L}{q_L}/\frac{w}{pf_G}$	4,5%	Hagedorn and Manovskii (2008)	
$f_G$	80.7%	FED	

Table 2: Values of calibrated parameters

Parameter	β	λ	$\sigma$	$\alpha_L$	$\alpha_G$	$\phi$	$\kappa_L$	ζ	ν
Value	0.99	0.5	0.019	1.27	4.7	0.92	0.027	0.33	0.55

Table 3: Prior and posterior modes of estimated parameters

Parameter	$\Phi_p$	$\xi_p$	$\xi_w$	$ ho_z$	$\sigma_z$	$ ho_d$	$\sigma_d$
Posterior	0.58	0.31	0.29	0.80	0.004	0.89	0.011
Prior	1.13	0.33	0.3	0.5	0.07	0.5	0.07
Distribution	Normal	Beta	Beta	Beta	Inverse Gamma	Beta	Inverse Gamma

and the data set ranges from 1970 to 2016. Importantly, while the demand shock has the opposite impact on unemployment and consumption, after the technology shock both variables move in the same direction. As a result, our data allows us to identify both shocks. We use Kalman Filter in the estimation process and standard MCMC algorithm to obtain posterior densities for estimated parameters. Prior distributions are set in line with Smets and Wouters (2007), although we assume higher variance for  $\Phi_p$  to account for different interest rate rule in our paper. Results of the estimation are presented in table 3.

#### 4. Variance decomposition and impulse responses

Before investigating the role of labor market institutions, we discuss dynamic properties of the modeled economy. For the sake of brevity, our focus is limited to nine variables characterizing the allocation: consumption (c), productive capacity by firms (t), unemployment rate (u), productivity threshold for firing (a), vacancies  $(v_L)$ , nominal wages (w), prices (p) and tightness on both labor  $(x_L)$  and product markets  $(x_G)$ .

According to Bai, Rios-Rull, and Storesletten (2012) or Michaillat and Saez (2015), presence of product market frictions improves the ability of demand shocks to generate sizable business cycle fluctuations. We verify that proposition by looking at the asymptotic contributions of each shock to variances of discussed variables (4). The results confirm that labor market aggregates seem to be driven solely by innovations to preferences. Demand shocks are also a dominant force behind dynamics of other variables, except for the price level and product market tightness.

In order to clarify the differences between impact of preference and technology disturbances on the economy, we simulate and plot impulse responses with respect to positive shocks of each type (1 and 2). While both innovations trigger prolonged increase in consumption, the underlying mechanism behind that seems to differ across shocks. In terms of a technology shock, higher supply (t) leads to a drop in prices and a relief in product market frictions from a household's perspective (lower  $x_G = \frac{v_G}{t}$ ). This in turn contributes to increased search effort by customers and higher consumption. When the preference shock is considered, causality seems to run in the opposite direction. As the shock has direct positive impact on households' marginal utility of consumption, it makes households more willing to exert shopping effort. Firms observe higher prices and less congestion in the product market (higher  $x_G$ ) and respond with higher employment. Increased supply combined with an initial impact of shock on marginal utility of consumption, leads to higher consumption.

Interestingly while unemployment drops after an positive preference shock, it rises after a positive technology shock. To understand that, one should notice that firms' revenues are symmetrically affected by price, aggregate technology and product market frictions (through  $f_G$ ). After a preference shock both p and  $f_G$  move up, while  $A_z$  remains constant, so marginal revenue, defined as  $pf_GA_z$ , increases and firms face incentives to increase employment. The picture is less clear for a technology shock, after which increased productivity is accompanied by lower prices and lower  $f_G$ . For our baseline calibration, the latter effect dominates, so both marginal revenue and employment fall. Similar mechanism explains responses of wages as they are also a function of firm's marginal revenue.

Table 4: Unconditional variance decomposition for selected variables (%)

	С	t	u	a	$v_L$	W	p	$x_L$	$x_G$
$\epsilon_z$	11	37	1	0	4	1	66	1	50
$\epsilon_d$	89	63	99	100	96	99	34	99	50

Figure 1: Response of selected variables to a positive preference shock.

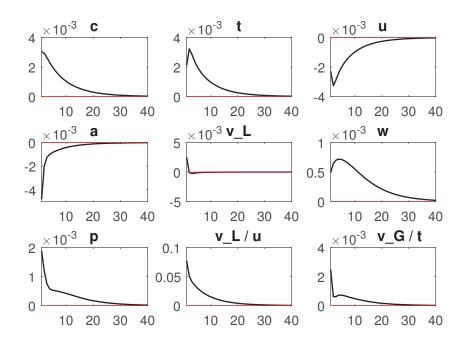
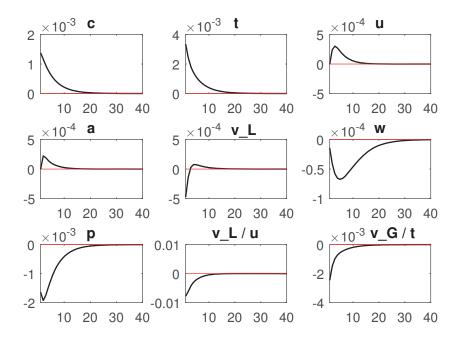


Figure 2: Response of selected variables to a positive technology shock.



### 5. Steady state analysis of the impact of labor market institutions

In this section we use numerical methods to perform comparative statics exercises concerning the impact of labor market institutions on steady state values of economic variables. More precisely, we investigate the influence of changes in: labor market frictions  $\kappa_l$  (interpreted as how effective the labor market institutions are at reducing the firm-level costs of recruiting workers), unemployment benefits  $\nu$  and the firing cost  $\phi$ .

First, let us study the impact of the size of vacancy posting  $\cot \kappa_l$ . Results are presented in Figure 3: intuitively, increase in  $\kappa_l$  reduces the number of posted vacancies. At the same time, higher cost of hiring new workers makes firms cut endogenous firing (i.e., there is a decrease in  $\tilde{a}$ ). This happens as they recognize that replacing fired workers with newly hired ones becomes more costly. The net impact of those two processes on employment is positive: steady state value of N is an increasing function of  $\kappa_l$  in the neighborhood of the steady state. A decrease in firing causes a drop in  $\mathbb{E}(a > \tilde{a}|a)$  -average productivity of active workers which in turn imposes a downward pressure on wages w (see equation 30). The resulting cut in payroll expenditures raises the marginal value of a worker  $J_n$ . Vacancy filling rate  $q_l$  is affected by two opposite forces: on the one hand, lower unemployment (higher N) decreases the pool from which firms recruit new workers - this tends to decrease  $q_l$ . On the other hand, lower aggregate number of posted vacancies tends to increase  $q_l$ . As it can be seen, the latter force dominates the former.

Let us turn to the effects of changes in the replacement rate of unemployment benefits  $\nu$  (see Figure 4). The wage-setting formula 30 indicates that higher  $\nu$  leads to a rise in wages w (as the value of outside option for unemployed workers increases and thus strengthens their position in wage negotiations). Higher wages increase labor costs and lower marginal value of worker faced by firms  $J_n$  which amplifies job destruction (higher

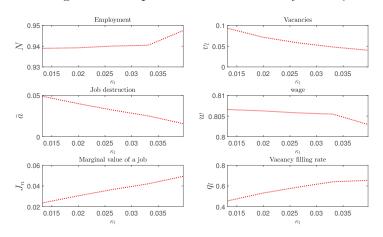
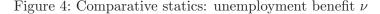
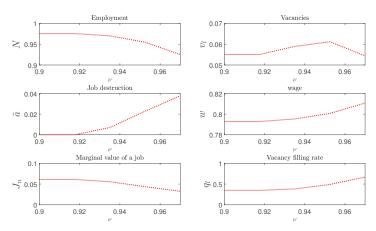


Figure 3: Comparative statics: vacancy cost  $\kappa_l$ 





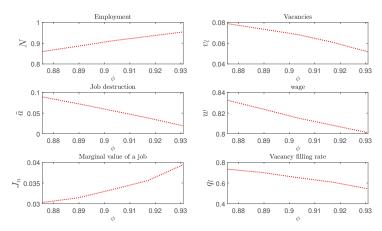
 $\tilde{a}$ ). The latter raises  $\mathbb{E}(a > \tilde{a}|a)$  and hence wages rise even further. A dynamic increase in  $\tilde{a}$  is associated with a decrease in employment which has a positive effect on vacancy filling rate  $q_l$ . The latter decreases the effective costs of vacancy posting and hence the amount of hiring activities undertaken by firms (and measured by  $v_l$ ) remains relatively stable despite the drop in the marginal value of a job.

Finally, let us concentrate on the effects of changes in firing costs  $\phi$ . Equation that determines the threshold value  $\tilde{a}$  is (derived in the Appendix):

$$\tilde{a} = a_H - \left[ 2\left(a_H - a_L\right) \left(\phi + \frac{\kappa_L}{\left(1 - \sigma\right)\beta q_l}\right) \right]^{\frac{1}{2}}$$
(25)

where  $a_L$  and  $a_H$  are bounds on the support of the uniform distribution from which idiosyncratic productivity a is drawn. Equation 25 shows that the direct impact of the increase in  $\phi$  is a drop in endogenous firing. This imposes a downward pressure on  $\mathbb{E}(a > \tilde{a}|a)$  which has a negative effect on wages. Those two opposite effects influence  $J_n$  but the former dominates the latter which leads to a rise in the marginal value of a worker. Higher value of firing costs mean that hiring new workers becomes riskier as it is more costly to fire them as their individual productivity level deteriorates which translates into a drop in the number of posted vacancies  $v_l$ . This effect has a weaker influence on employment than the cut in job destruction so N tends to increase with  $\phi$ . Lower unemployment decreases the probability of hiring workers  $q_l$ . Firms recognize this effect (it is the only general equilibrium effect captured by equation 25) so they are more cautious when firing workers - job destruction is decreased even further.

Figure 5: Comparative statics: firing cost  $\phi$ 



#### 6. Labor market institutions and unemployment fluctuations

In this section we evaluate the importance of labor market institutions for the cyclical fluctuations of unemployment. We consider four policy parameters: unemployment benefit replacement rate  $\nu$ , firing cost  $\phi$ , the vacancy cost  $\kappa_L$  and wage flexibility  $\xi_w$ .

Table 5 present elasticities of mean and variance of unemployment to the policy parameters. To understand the importance of each policy instrument for the response of unemployment to shocks, in Table 6 we report the elasticity of cumulative increase of unemployment at different periods after the shock. The elasticities are computed for a negative demand shock and a positive productivity shock, as these disturbances lead to an increase of total unemployment. It is important to note that elasticities are computed locally. Hence, they inform us about the consequences of a small reform around the steady state.

Unemployment benefits replacement rate  $\nu$  has strong, positive impact on both level and volatility of employment. As we noted before, higher replacement rate leads, via the wage bargaining, to higher wages and lower profits, which reduces vacancy creation and increases job destruction. High replacement rate also reduces the match surplus and leads to more stable wages. Since wages adjust less, firms instead rely on cyclical adjustment of employment. This impact of benefits on unemployment volatility is explored by Costain and Reiter (2008) and Hagedorn and Manovskii (2008). In Table 6 we see that  $\nu$  substantially increases the magnitude of response of unemployment to the demand shock. With respect to the productivity shock, the response of unemployment is increased, but also delayed.

Firing cost  $\phi$  has a sizable, negative impact on unemployment level and a smaller in magnitude, negative impact on unemployment volatility, in line with fidings of Zanetti (2011). The firing cost reduces both the mean firing and the mean hiring rate, which, for our calibration, leads to higher employment. The firing cost interacts differently with different shocks. It reduces the long-run unemployment response to the demand shock and substantially increases the response to the productivity shock. Recall that the demand shock accounts for almost all unemployment fluctuations. The moderation of the demand shock fluctuations hence dominates the magnification of the other shock, yielding a negative impact on overall volatility of unemployment.

Wage flexibility  $\xi_w$  does not affect the mean unemployment, but it reduces its variance, as in Shimer (2005). Furthermore, in Table 6 we see that more flexible wages delay the unemployment response for both shocks. That is in line with the argument that wage adjustments are a substitute for employment adjustments.

The higher vacancy cost  $\kappa_L$  modestly reduces the mean of unemployment at the expense of volatility. Table 6 shows that volatility increases after both types of shocks, yet the impacts are minuscule and very delayed. It is worth pointing out that elimination of

Table 5: Elasticity of mean and variance of unemployment w.r.t. policy parameters

	E(U)	Var(U)
$\nu$	13.844	11.040
$\phi$	-16.441	-2.399
$\kappa_L$	-0.137	0.119
$\xi_w$	0.000	-0.118

Table 6: Elasticity of cumulative unemployment rise w.r.t. policy parameters

		2 quarters	8 quarters	40 quarters
ν	$\begin{array}{c} \operatorname{demand\ shock\ (-)} \\ \operatorname{productivity\ shock\ (+)} \end{array}$	1.169 -2.932	2.885 0.955	3.837 1.762
$\phi$	demand shock (-) productivity shock (+)	0.076 $10.538$	-1.008 3.447	-1.530 3.049
$\kappa_l$	demand shock (-) productivity shock (+)	0.001 -0.001	0.002 0.000	0.002 0.001
$\xi_w$	demand shock (-) productivity shock (+)	-0.044 -0.077	-0.026 -0.029	$0.027 \\ 0.034$

the entire vacancy cost, if feasible, would yield no unemployment in the steady state. Therefore, a small reduction of  $\kappa_L$  have the opposite impact on unemployment level than a sufficiently large reduction. This curious result highlights the difference between the local, small reform approach followed in this section and large policy changes.

In contrast to Abbritti and Mueller (2013), we find that vaccancy posting cost  $\kappa_L$  and the firing cost  $\phi$  have the opposite impact on unemployment volatility. Moreover, in Table 6 we see that the asymmetry appears after a demand shock. The unemployment fluctuations in our model are driven mostly by the demand shocks, which can explain why our results differ from the literature focusing on search frictions in the labor market alone.

#### 7. Conclusions

In this paper we propose a novel, dynamic, macroeconomic model incorporating search frictions in both product and labor market. We find that a TFP shock, a typical source of fluctuations in the Real Business Cycle literature, accounts only for accounts only for 11% of variance of consumption and 1% of variance of unemployment. The remaining volatility is driven by a preference shock affecting contemporaneous marginal utility from consumption.

The model allows us to study the impact of labor market institutions on the level and volatility of unemployment. Presence of product market frictions affects the way firms respond to shocks, for example positive technology shocks depress the equilibrium probability that products offered by a company are matched with customers, so firms reduce employment despite higher productivity of workers. However, while propagation mechanisms of the shocks and the source of unemployment fluctuations are different than in the standard labor search model, the implications for the labor market institutions are almost unchanged. Higher unemployment benefits and wage rigidity tend to increase variance of unemployment, while benefits also promote higher unemployment in the long run (i.e. steady state). Firing costs contributes to lower variance of unemployment, while surprisingly the opposite holds for hiring costs. Finally, both firing an hiring costs exert a negative impact on unemployment in the long term. However, that result is mostly local by nature (i.e. it holds for small changes in parameters) as in the limit zero hiring costs imply no steady state unemployment in the model.

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#### A. Steady state computation

First notice, that stationary equilibrium allocation does not depend on the price level p. To see that, observe that, in the steady state, firm's decisions and firm's surplus depend on real wages  $\frac{w}{p}$  and on the real marginal value of a filled vacancy  $\frac{J_n}{p}$ . The same is true for household's decisions: in the stationary equilibrium, p does not appear in the formula for the stochastic discount factor  $\Delta$ . This implies that we are free to standardize the steady state value of p. We set p=1. As it has been mentioned in the core text, equilibrium in the dynamic model is described by 24 equations that contain 24 variables. In the static model, some of them become parameters (technology levels  $A_d=1$ ,  $A_z=1$ ) and some are equal to their perfectly rigid counterparts (i.e.,  $w=w_{Nash}$  and  $p=p_{opt}$ ). On the top of that, there are some variables that are given by simple, explicit formulas (like  $\bar{N}$ , T and b). All this means, that we can reduce the analyzed steady state system of equations to 15 equations with 15 variables  $(n, \tilde{a}, q_l, v_l, f_g, J_n, w, \Delta, f_l, i, c, V_m, q_g, v_g, x_g$  and  $x_l$ , notice that p=1):

$$n = (1 - \sigma) \cdot \mathbb{P}(a > \tilde{a}) \cdot n + q_l v_l \tag{26}$$

$$-f_q \kappa_l + q_l \Delta J_n = 0 \tag{27}$$

$$\tilde{a} = -\phi + \frac{w - \Delta J_n}{f_q} \tag{28}$$

$$J_{n} = \mathbb{P}(a > \tilde{a}) \cdot (1 - \sigma) \cdot \left\{ f_{g} \left[ \mathbb{E}(a|a > \tilde{a}) - \frac{\mathbb{P}(a \leq \tilde{a})}{\mathbb{P}(a > \tilde{a})} \right] - w + \Delta J_{n} \right\}$$
(29)

$$w = \lambda f_g \mathbb{E} \left( a | a > \tilde{a} \right) + (1 - \lambda) \nu w + \lambda \Delta \left( f_l J_n - (1 - f_l) f_g \phi \mathbb{P} \left( a \le \tilde{a} \right) (1 - \sigma) \right)$$
 (30)

$$1 = \Delta(1+i) \tag{31}$$

$$\Delta = \beta \tag{32}$$

$$c = q_g v_g \tag{33}$$

$$V_a = (1+i)\left(u'(c) - \frac{\kappa_g}{q_g}\right) \tag{34}$$

$$1 = \frac{u'(c)}{\beta V_a} \frac{1}{1 + \frac{1}{x_a^{\alpha g}}} \tag{35}$$

$$c = f_g \cdot \left( \int_{\tilde{a}}^{+\infty} a f(a) da \cdot (1 - \sigma) \cdot n - \kappa_l v_l - \phi \mathbb{P} \left( a \le \tilde{a} \right) (1 - \sigma) n \right)$$
 (36)

$$x_l = \frac{v_l}{1 - \mathbb{P}(a > \tilde{a}) \cdot (1 - \sigma) \cdot n} \tag{37}$$

$$q_l = \frac{1}{\left(1 + x_l^{\alpha_l}\right)^{\frac{1}{\alpha_l}}} \tag{38}$$

$$f_l = \frac{1}{\left(1 + \frac{1}{x_1^{\alpha_l}}\right)^{\frac{1}{\alpha_l}}} \tag{39}$$

$$q_g = \frac{1}{\left(1 + x_g^{\alpha_g}\right)^{\frac{1}{\alpha_g}}} \tag{40}$$

$$f_g = \frac{1}{\left(1 + \frac{1}{x_q^{\alpha g}}\right)^{\frac{1}{\alpha_g}}}.\tag{41}$$

All variables in the system take their steady state values. Our strategy is the following. First, we guess two numbers  $q_l$  and  $q_g$  (the range of this guess is constrained to the interval [0,1] thanks to the Den Haan - Ramey - Watson specification of matching technologies). Second, we use all equations (except for equations 30 and 35) to compute the values of the remaining variables. Finally, we plug those values into 30 and 35 and we check whether these conditions hold with equality. If not, we pick another pair  $q_l$  and  $q_g$  and repeat the procedure.

Let us be more specific about the way we calculate the values of all variables given the probability of a successful visit  $q_g$  and the vacancy filling rate  $q_l$ . First, it is straightforward to see, that we can obtain  $x_l$  and  $x_g$  given  $q_l$  and  $q_g$  from equations 38 and 40. Next, we use  $x_l$  and  $x_g$  together with equations 39 and 41 to get  $f_l$  and  $f_g$ . The value of  $\Delta$  is taken from 32. Variables  $f_g$ ,  $q_l$  and  $\Delta$  are used for calculation of  $J_n$  from equation 27. In what follows, we use equations 28 and 29 and the values of calculated variables to get  $\tilde{a}$  and w. More precisely, we calculate w from 28 and we substitute the resulting expression for w in equation 29. After some simple manipulations we obtain:

$$\mathbb{P}(a > \tilde{a}) \cdot \left[ \mathbb{E}(a|a > \tilde{a}) - \tilde{a} \right] = \phi + \frac{J_n}{(1-\sigma)f_q}. \tag{42}$$

Since we know the values of  $J_n$  and  $f_g$  then the equation above can be solved numerically for  $\tilde{a}$  if the distribution of a is specified. To obtain some analytical, intuitive formula for  $\tilde{a}$ , we will assume that  $a \sim U(a_L, a_H)$  (i.e., a has a uniform distribution with a support  $[a_L, a_H]$ ). Under this assumption, equation 42 we are able to pin down the value of  $\tilde{a}$  in an explicit way:

$$\tilde{a} = a_H - \left[ 2(a_H - a_L) \left( \phi + \frac{J_n}{(1 - \sigma) f_g} \right) \right]^{\frac{1}{2}}.$$
 (43)

Let us make a short digression here. First, observe that if we used the formula 27 to substitute for  $\frac{J_n}{f_g}$  in equation above, then we obtain the cut-off rule associated with endogenous firing that is expressed only in terms of parameters and the aggregate variable  $q_l$ :

$$\tilde{a} = a_H - \left[ 2 \left( a_H - a_L \right) \left( \phi + \frac{\kappa_L}{\left( 1 - \sigma \right) \beta q_l} \right) \right]^{\frac{1}{2}}. \tag{44}$$

Second, notice that 44 will be useful for obtaining some restrictions on parameters which

would guarantee that  $\tilde{a} \in [a_L, a_H]$ . It is easy to see that  $\tilde{a} < a_H$  always holds. Inequality  $\tilde{a} > a_L$  is equivalent to:

$$\frac{a_H - a_L}{2} - \phi > \frac{\kappa_l}{\beta q_l (1 - \sigma)}.$$
(45)

Since  $q_l$  is always positive then the LHS of the inequality above cannot be negative which gives us the first restriction on parameters:

$$\frac{a_H - a_L}{2} - \phi > 0.$$

Since  $q_l$  is lower than 1 then 45 implies the second restriction on parameters:

$$\frac{\kappa_l}{\beta \left(1 - \sigma\right) \left[\frac{a_H - a_L}{2} - \phi\right]} < 1.$$

Let us go back to the derivation of w. We use  $\tilde{a}$  calculated from equation 43 and plug it into 28 to obtain w.

In what follows, we will use equations 26 and 37 to derive the values of  $v_l$  and n. We use 26 to get a formula for  $v_l$  which is then plugged into 37 (substitution for  $v_l$ ):

$$n = \frac{1}{\frac{1 - (1 - \sigma)\mathbb{P}(a > \tilde{a})}{x_l q_l} + \mathbb{P}(a > \tilde{a})(1 - \sigma)}$$

Since we already know the values of  $\tilde{a}$ ,  $x_l$  and  $q_l$  then it means that we know n, too. We use 26 to derive the number of vacancies posted:

$$v_l = \frac{1 - (1 - \sigma)\mathbb{P}(a > \tilde{a})}{q_l} \cdot n.$$

Given the variables whose values are already known, we compute c out of 36, we use c and  $q_g$  to get  $v_g$  from 33. Euler equation 31 is used for deriving i which coupled with c and  $q_g$  gives us the value of  $V_a$  from 34. This means that we have calculated all the values of variables and we are in position of checking whether equations 30 and 35 are satisfied.

