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Demographics, monetary policy, and the zero lower bound

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Abstract

The recent literature shows that demographic trends may affect the natural rate of interest (NRI), which is one of the key parameters affecting stabilization policies implemented by central banks. However, little is known about the quantitative impact of these processes on monetary policy, especially in the European context, despite persistently low fertility rates and an ongoing increase in longevity in many euro area economies. In this paper we develop a New Keynesian life-cycle model, and use it to assess the importance of population ageing for monetary policy. The model is fitted to euro area data and successfully matches the age profiles of consumption-savings decisions made by European households. It implies that demographic trends have contributed and are projected to continue to contribute significantly to the decline in the NRI, lowering it by more than 1.5 percentage points between 1980 and 2030. Despite being spread over a long time, the impact of ageing on the NRI may lead to a sizable and persistent deflationary bias if the monetary authority fails to account for this slow moving process in real time. We also show that, with the current level of the inflation target, demographic trends have already exacerbated the risk of hitting the lower bound (ZLB) and that the pressure is expected to continue. Delays in updating the NRI estimates by the central bank elevate the ZLB risk even further.

JEL: E31, E52, J11
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1 Introduction

Many economies, developed and developing alike, are experiencing (or are soon expected to begin) a substantial demographic transition. Increasing longevity and sub-replacement fertility rates translate into ageing of societies, with the speed of this process varying between different countries. Ageing affects many aspects of economic activity, including aggregate output, pension system sustainability, structure and volume of fiscal expenditures or housing markets.

Recently, the demographics has also become of interest to central bankers and monetary economists, mainly because of its impact on the natural rate of interest (NRI), i.e. one of the key parameters affecting stabilization policies implemented by central banks. Economic theory predicts that a slowdown in population growth may translate into higher capital per worker, and hence into a decrease in the rate of return on capital. Increasing longevity lengthens the planning horizon of households, inducing them to save more, and thus exerting a further downward pressure on the NRI. Longer living households, dependent during their retirement largely on previously accumulated wealth and asset income, may prefer lower and more stable inflation rates, influencing politicians' and central bankers' preferences towards the inflation-output tradeoff. Demographic trends can also lead to changes in monetary transmission via their impact on asset distribution. Last but not least, for a given level of the inflation target, a lower NRI implies a lower average nominal interest rate, leaving less space for conventional monetary policy during slowdowns in economic activity, and thus increasing the risk of hitting the zero lower bound (ZLB) constraint.

In spite of the topic's importance, the impact of demographics on the monetary policy conduct has been addressed only in a limited number of papers to date. Kara and von Thadden (2016) calibrate a Blanchard-Yaari overlapping generations model to the euro area and project a decrease in the natural interest rate for this group of countries by 0.9 percentage points between 2008 and 2030. Consistently with the views expressed by some central bankers (e.g. Bean, 2004), they conclude that such slowly moving changes in the NRI are not important within the horizon that is relevant for monetary policy, and hence do not recommend any adjustment in its parameters. Carvalho et al. (2016) calibrate a similar model to the average of several developed countries, and simulate a more significant decline of the equilibrium interest rate (1.5 percentage points between 1990 and 2014). In contrast to Kara and von Thadden (2016), they argue that low and declining real interest rates do carry important challenges for the monetary authorities. An much stronger stance about the impact of demographics on monetary policy is taken by Eggertsson et al. (2017), who see it as one of the key forces behind the secular stagnation hypothesis. Gagnon et al. (2016) argue that in the US demography is virtually the sole culprit of the recent permanent decline in real GDP growth, rate of aggregate investment and safe asset yields, suggesting that this situation is

the "new normal". In all of these papers, however, the statements about monetary policy implications are qualitative rather than quantitative.

Other strands of the literature signalize that population ageing may affect the monetary policy effectiveness and objectives. Wong (2016) uses household-level data to show that older people are less responsive to interest rate shocks, and that the credit channel loses importance as societies age. Imam (2015) argues that monetary policy transmission channels weaken in older societies, with the exception of the wealth and expectation channels, which gain in importance. Using a dynamic panel data model, he shows that a demographic transition is associated with decreased monetary policy effectiveness. Bullard et al. (2012), Vlandas (2016) and Juselius and Takats (2015) analyse the impact of demography on social preferences and, as a result, on the inflation rate targeted by central banks, finding a negative relationship.

In this paper, we offer a quantitative model-based analysis of the impact of population ageing on monetary policy, focusing on the euro area. As evidenced by Figure 1, the European economy is currently undergoing a rapid drop in the number of people entering the working-age period of life, and the future fertility rates are projected to remain persistently low. Moreover, the mortality rates are consistently falling and the probability of reaching the retirement age is expected to increase from 83% in 1980 to almost 95% around 2080. These two forces reinforce each other in leading to a rapid increase in the old-age dependency ratio, which is projected to reach 70% by 2080 from well below 30% in the 1980s and 1990s.

Our analytical tool is a New Keynesian model with life-cycle features. While most of the literature models ageing using simplifying Blanchard-Yaari assumptions, we opt for modeling the demographic structure in full detail. In this respect, our model draws on the full-scale life-cycle framework pioneered by Auerbach and Kotlikoff (1987) and, in contrast to the Blanchard-Yaari approach, does not need to employ the Blanchard (1985) assumption of constant mortality risk while retired.[1] Our detailed setup also allows us to incorporate the age profiles of labor income and labor supply, estimated using the Household Finance and Consumption Survey (HFCS) for the euro area countries, and to closely match the resulting life-cycle patterns of consumption-savings decisions to this data. This is important, given that one of our goals is to investigate the effect of ageing on the real interest rate. Moreover, since our focus is on the monetary policy implications, the model includes real and nominal rigidities commonly used in the New Keynesian literature.

We believe that this rich structure allows us to deliver more reliable quantitative answers to questions that have already been discussed in the literature using more simplified approaches, including the assessment of the impact of demographic trends on the main

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[1]Heijdra and Romp (2008) show that embedding a realistic mortality risk within the Blanchard-Yaari framework has a major impact on how the model economy responds to shocks. However, their approach is tractable only with a small open economy assumption.

macrovariables, and in particular on the NRI and potential output growth. More importantly, we also use our model to address some novel issues. In particular, we check what happens if the monetary authority does not observe the NRI and potential output in real time, but instead learns about them only slowly. Finally, we assess how the demographic processes influence the probability of hitting the ZLB on the nominal interest rates. To our knowledge, we are the first to quantitatively relate demographics to the ZLB problem and consider the impact of imperfect central bank knowledge in this context.

Our findings suggest that, despite its glacial speed, the impact of ageing on the economy can be substantial from the monetary policy perspective. In particular, given the history and currently available projections, the equilibrium interest rate in the euro area is projected to decline due to demographic forces by more than 1.5 percentage points between 1980 and 2030. The growth rate of potential output declines by 0.6 percentage points over the same period. Updating the estimates of these two latent variables by the central bank in real time makes the demographic transition neutral for price stability. However, if the monetary authority learns about the impact of demographic processes on the NRI and potential output only slowly over time, the outcome is a prolonged period of below-target inflation. This deflationary bias may be sizable – over 0.3 percentage points for several years. Further, according to our simulations, demographic developments significantly increase the ZLB probability. Even if the NRI and potential output are perfectly observed by the monetary authority, the annual probability of hitting the constraint increases from just below 1% in the 1980s to over 4% in 2030. These numbers accumulate to a dramatic increase of the chance of hitting the ZLB over longer horizons: the probability rises from less than 4% for the whole 1980s decade to over 30% for the 2020s decade. Under the learning scenario, the ZLB risk increases even more - the annual probabilities rise to 4-5% already in 2020.

The rest of this paper is organized as follows. Section 2 lays out the model used in our analysis. Section 3 documents the construction of demographic input data and our calibration strategy. In Section 4 we present our simulation results, in particular pertaining to the natural interest rate, the zero lower bound and the role of central bank learning. In Section 5 we discuss how our main results are affected by the presence of a pension system. Section 6 concludes.

2 Model

We construct a New Keynesian model with overlapping generations as well as real and nominal rigidities. The model economy is populated by households facing age and time-dependent mortality risk, two types of producers, investment funds, and a monetary authority. Below we describe the problems facing each type of agents. While denoting real allocations, we employ the convention of using upper case letters for aggregates, and lower case letters for variables expressed in per capita terms.

2.1 Households

2.1.1 Optimization problem

Each household consists of a single agent, who appears in our model at the age of 20 and is assigned age index $j = 1$. Agents can live up to 99 years ($j = J = 80$), at each year subject to age and time-dependent mortality risk $\omega_{j,t}$. Hence, at each point in time, the model economy is populated by 80 cohorts of overlapping generations, with the size of cohort j denoted by $N_{j,t}$.

A representative j-aged household maximizes her expected remaining lifetime utility that depends on consumption $c_{j,t}$ and hours worked $h_{j,t}$ according to

$$U_{j,t} = \mathbb{E}_t \sum_{i=0}^{J-j} \beta^i \frac{N_{j+i,t+i}}{N_{j,t}} \exp(\varepsilon_t^u) \left(\ln c_{j+i,t+i} - \phi_{j+i} \frac{h_{j+i,t+i}^{1+\varphi}}{1+\varphi} \right) \tag{1}$$

where β is the subjective discount factor, the ratio $N_{j+i,t+i}/N_{j,t}$ represents the probability of surviving for at least i more years, ε_t^u is a preference shock, ϕ_j is the age-dependent labor disutility parameter, and φ is the inverse of the Frisch elasticity of labor supply.

All households face the following budget constraint

$$P_t c_{j,t} + P_t a_{j+1,t+1} = W_t z_j h_{j,t} + R_t^a P_{t-1} a_{j,t} + P_t beq_t \tag{2}$$

where P_t denotes the aggregate price level, $a_{j,t}$ stands for the beginning-of-period t real stock of assets that are managed by investment funds and that yield the gross nominal rate of return R_t^a, W_t is the nominal wage per effective hour, while z_j represents age-specific labor productivity. Our model features exogenous retirement upon reaching the age of 64 ($j = JR = 45$), and hence we set $z_j = 0$ for $j \geq JR$. Finally, since most agents die before reaching their maximum age, they leave unintentional bequests, which are redistributed equally across all living agents in form of lump-sum transfers beq_t.

2.1.2   Demography and aggregation

In our model, the demographic processes are governed by changes in the size of the youngest cohorts $N_{1,t}$ and mortality risk $\omega_{j,t}$, both of which are assumed to be exogenous. Then, the total number of living agents N_t and the population growth rate n_{t+1} are given by

$$N_t = \sum_{j=1}^{J} N_{j,t} \quad \text{and} \quad n_{t+1} = \frac{N_{t+1}}{N_t} - 1 \tag{3}$$

where the number of agents in each cohort evolves according to

$$N_{j+1,t+1} = (1 - \omega_{j,t})N_{j,t} \tag{4}$$

To better capture the impact of expected demographic changes, we allow population growth in the steady state to differ from zero. As then the number of agents within each cohort becomes nonstationary, it is useful to define the size of cohorts relative to that of the youngest one

$$N_{j,t}^{rel} = \frac{N_{j,t}}{N_{1,t}} \tag{5}$$

and the growth rate of youngest agents $n_{1,t+1}$

$$n_{1,t+1} = \frac{N_{1,t+1}}{N_{1,t}} - 1 \tag{6}$$

This allows us to rewrite equations (3) and (4) in relative terms

$$N_t^{rel} = \sum_{j=1}^{J} N_{j,t}^{rel} \quad \text{and} \quad n_{t+1} = \frac{N_{t+1}^{rel}}{N_t^{rel}}(1 + n_{1,t+1}) - 1 \tag{7}$$

$$N_{j+1,t+1}^{rel} = \frac{(1 - \omega_{j,t})N_{j,t}^{rel}}{1 + n_{1,t+1}} \tag{8}$$

The aggregate allocations over all living households can be then expressed in per capita terms

as follows

$$c_t = \sum_{j=1}^{J} \frac{N_{j,t}^{rel} c_{j,t}}{N_t^{rel}} \tag{9}$$

$$h_t = \sum_{j=1}^{J} \frac{N_{j,t}^{rel} z_j h_{j,t}}{N_t^{rel}} \tag{10}$$

$$a_{t+1} = \sum_{j=1}^{J} \frac{N_{j,t}^{rel} a_{j+1,t+1}}{N_{t+1}^{rel}(1 + n_{1,t+1})} \tag{11}$$

$$beq_t = \sum_{j=1}^{J} \frac{\left[N_{j-1,t-1}^{rel} - N_{j,t}^{rel}(1 + n_{1,t}) \right] R_t^a a_{j,t}}{N_t^{rel}(1 + n_{1,t}) \pi_t} \tag{12}$$

where $\pi_t \equiv P_t / P_{t-1}$ is gross inflation.

2.2 Firms

There are two types of firms in our model economy – perfectly competitive final goods producers and monopolistically competitive intermediate goods producers, the latter indexed with i. Consistently with demographic processes in the household sector, the mass of each type of firms is tied to the size of population. To model the related entry or exit of firms, we assume that at the end of period t a randomly selected fraction $(N_{t+1}/N_t) - 1 = n_{t+1}$ of firms generate spin-offs (or die if population growth is negative). These spin-offs are identical clones of their parents. As intermediate good producers do not accumulate net worth, the price of their shares becomes zero when they die. Note that this firm demographics is relevant only for intermediate goods producers as final goods producers generate zero profits.

2.2.1 Final goods producers

Final goods producers purchase intermediate goods $y_t(i)$ and produce a homogenous final good y_t according to the following CES aggregator

$$y_t = \left[\frac{1}{N_t} \int_0^{N_t} y_t(i)^{\frac{1}{\mu_t}} \mathrm{d}i \right]^{\mu_t} \tag{13}$$

where $\mu_t = \exp(\varepsilon_t^\mu) \mu$ can be interpreted as a stochastic gross markup resulting from imperfect substitution between intermediate varieties, and whose steady state value is $\mu \geq 1$. The solution to a representative final goods producer's profit maximization problem implies the

following demand function for intermediate goods

$$y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{\frac{\mu_t}{1-\mu_t}} y_t \tag{14}$$

and the associated aggregate price index is given by

$$P_t = \left[\frac{1}{N_t}\int_0^{N_t} P_t(i)^{\frac{1}{1-\mu_t}}\,\mathrm{d}i\right]^{1-\mu_t} \tag{15}$$

2.2.2 Intermediate goods producers

Intermediate goods producers hire capital and labor, and produce differentiated output according to the Cobb-Douglas production function

$$y_t(i) = \exp(\varepsilon_t^z)k_t(i)^\alpha h_t(i)^{1-\alpha} \tag{16}$$

where k_t denotes physical capital and ε_t^z is a productivity shock. They face demand schedules given by equation (14), and set their prices subject to the Calvo friction, with θ representing the probability of not receiving the reoptimization signal, in which case prices are fully indexed to steady state inflation π. Intermediate goods producers are risk neutral, i.e. they use the nominal risk-free rate to discount expected future profit flows. The reoptimizing firms hence maximize

$$\mathbb{E}_0 \sum_{t=0}^\infty \left(\prod_{s=0}^t R_s\right)^{-1} \theta^s \left[P_t(i)\pi^s - MC_{t+s}\right] y_{t+s}(i) \tag{17}$$

where MC_t is nominal marginal cost consistent with production function (16).

2.3 Investment funds

Perfectly competitive and risk-neutral investment funds use households' savings to buy and manage a portfolio of assets, transferring every period the earned gross return back to households. The portfolio consists of physical capital K_t, bonds B_t and claims on intermediate goods producing firms (shares) $D_t(i)$.

A representative investment fund maximizes the expected present value of future gross returns

$$\mathbb{E}_0 \sum_{t=0}^\infty \left(\prod_{s=0}^t R_s\right)^{-1} \left[\begin{array}{c} \left[R_{t+1}^k + (1-\delta)\,Q_{t+1}\right] K_{t+1} + R_t P_t B_{t+1} \\ + \int_0^{N_{t+1}} \left[P_{t+1}F_{t+1}(i) + (1+n_{t+2})P_{t+1}^d(i)\right] D_{t+1}(i)\mathrm{d}i \end{array}\right] \tag{18}$$

where δ is the capital depreciation rate, R_t denotes the gross nominal risk-free rate, R_{t+1}^k is

the nominal rental rate on capital, Q_{t+1} is the nominal price of a unit of capital, $D_t(i)$ stands for the number of shares issued by intermediate goods producing firm i. These shares are traded at the end of period t at price $P_t^d(i)$ and yield real dividends $F_t(i)$.

The nominal balance sheet of investment funds at the end of period t can be written as

$$P_t A_{t+1} = Q_t(1-\delta)K_t + P_t I_t + P_t B_{t+1} + \int_0^{N_{t+1}} P_t^d(i)D_{t+1}(i)\mathrm{d}i \tag{19}$$

where I_t denotes investment in physical capital, which accumulates according to

$$K_{t+1} = (1-\delta)K_t + \exp(\varepsilon_t^i)\left[1 - S_k\left(\frac{I_t}{I_{t-1}}\right)\right]I_t$$

where ε_t^i is a shock to efficiency of investment and $S_k(\bullet)$ describes investment adjustment costs. We use the following functional form

$$S_k\left(\frac{I_t}{I_{t-1}}\right) = \frac{S_1}{2}\left(\frac{I_t}{I_{t-1}} - (1+n_t)\right)^2 \tag{20}$$

where $S_1 \geq 0$, as it ensures that adjustment costs are zero in the steady state even if population size is non-stationary.

Since we assume that all revenue from asset management is transferred back to households, the ex-post rate of return on assets R_t^a is implicitly given by

$$R_t^a P_{t-1} A_t = \left[R_t^k + (1-\delta)Q_t\right]K_t + R_{t-1}P_{t-1}B_t \tag{21}$$
$$+ \int_0^{N_t}\left[P_t F_t(i) + (1+n_{t+1})P_t^d(i)\right]D_t(i)\mathrm{d}i$$

2.4 Monetary authority

The monetary authority sets the nominal interest rate according to a Taylor-like rule that takes into account the zero lower bound constraint

$$R_t = \begin{cases} R_t^{cb} & \text{if } R_t^{cb} > 1 \\ 1 & \text{if } R_t^{cb} \leq 1 \end{cases} \tag{22}$$

where

$$R_t^{cb} = R_{t-1}^{\gamma_R}\left[\tilde{R}_t^e\left(\frac{\pi_t}{\pi}\right)^{\gamma_\pi}\left(\frac{y_t/y_{t-1}}{\tilde{y}_t^e/\tilde{y}_{t-1}^e}\right)^{\gamma_y}\right]^{1-\gamma_R}\exp(\varepsilon_t^R) \tag{23}$$

where ε_t^R is a monetary policy shock while the coefficients $0 \leq \gamma_R < 1$, $\gamma_\pi > 1$ and $\gamma_y \geq 0$ control, respectively, the degree of interest rate smoothing, the response to deviations of inflation from the target, and the response to deviation of output growth from its perceived

potential. The variables $\tilde{R}_t^e$ and $\tilde{y}_t^e$ describe the central bank perceptions of the natural nominal interest rate $\tilde{R}_t \equiv \pi \tilde{r}_t$ and natural output $\tilde{y}_t$, respectively, where $\tilde{r}_t$ denotes the natural real interest rate. These natural quantities are defined as hypothetical values of the relevant variables that would be observed under fully flexible prices (i.e. $\theta = 0$) and absent stochastic shocks, but with demographic trends taken into account.

Unless indicated otherwise, the perceptions of the monetary authority are assumed to be consistent with current economic developments, i.e. $\tilde{R}_t^e = \tilde{R}_t$ and $\tilde{y}_t^e = \tilde{y}_t$. Alternatively, we assume that these perceptions are linked to the actual values with a constant gain learning process as in Evans and Honkapohja (2001)

$$\tilde{R}_t^e = \tilde{R}_{t-1}^e + \lambda(\pi \tilde{r}_{t-1} - \tilde{R}_{t-1}^e) \tag{24}$$

$$\tilde{y}_t^e = \tilde{y}_{t-1}^e + \lambda(\tilde{y}_{t-1} - \tilde{y}_{t-1}^e) \tag{25}$$

so that the central bank observes the true natural interest rate and output only with a lag, and updates their current guess with a fraction λ of the previous forecast error.[2] This way of formulating the feedback rule ensures the long-run consistency of the equilibrium with central bank targets, but also allows us to model imperfect knowledge of the monetary authority.

2.5   Market clearing conditions

The model is closed with a standard set of market clearing conditions. Equilibrium on the final goods market implies

$$y_t = c_t + i_t \tag{26}$$

The market clearing conditions for capital and labor can be written as

$$\frac{1}{N_t} \int_0^{N_t} k_t(i) = k_t \tag{27}$$

$$\frac{1}{N_t} \int_0^{N_t} h_t(i) = h_t \tag{28}$$

This allows us to write the aggregate production function as

$$y_t \Delta_t = \exp(\varepsilon_t^z) k_t^\alpha h_t^{1-\alpha} \tag{29}$$

where $\Delta_t \equiv \frac{1}{N_t} \int_0^{N_t} \left(\frac{P_t(i)}{P_t} \right)^{\mu_t/(1-\mu_t)} \mathrm{d}i$ measures the price dispersion across intermediate goods.

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[2]The resulting estimate of the unobserved variable is equal to an exponentially weighted average of all its past values.

Since bonds are traded only between (identical) investment funds, we have

$$B_t = 0 \tag{30}$$

Finally, without loss of generality, we rule out share consolidations and splits so that the number of shares issued by each intermediate goods is constant over time

$$D_t(i) = D(i) \tag{31}$$

2.6   Exogenous shocks

The model economy is driven by demographic processes that are characterized by the growth rate of initial young $n_{1,t}$ and age-specific mortality risk $\omega_{j,t}$ $(j = 1 : J)$. Both of them are treated as deterministic, i.e. known to all optimizing agents. Additionally, the economy is hit by stochastic shocks to productivity ε_t^z, household preferences ε_t^u, investment specific technology ε_t^i, monetary policy ε_t^R, and markups ε_t^μ.

3 Calibration

The model is calibrated for the euro area, using annual time frequency. We first explain how we parametrize the life-cycle characteristics, then present the assumptions regarding demographic trends, and finally describe the chosen values for other structural parameters as well as properties of stochastic shocks.

3.1 Life-cycle characteristics

Our model allows two exogenous household characteristics to vary with age, namely labor productivity z_j and weight on labor in utility ϕ_j, $j = 1, ..., JR - 1$. We calibrate the age profiles associated with these parameters by relying on calculations presented in Jablonowski (2018), who uses the second wave of the Household Finance and Consumption Survey (HFCS), conducted in 18 euro area countries between 2012 and 2014. The life-cycle characteristics are extracted at a household level, and the age of the household is determined by the age of the household head. We use data on labor income, including wage employment and self-employment, and hours worked, defined as time spent working at the main job. Since our model does not explicitly account for changes in the household composition or the family size, the extracted age profiles of labor income and hours worked are next divided by the square root of the number of household members, which is one of the equivalence scales used while working with household level data, see Fernandez-Villaverde and Krueger (2007) and OECD (2008). The thus obtained profiles are next smoothed using fourth-order polynomials, which are also used for extrapolation over the age groups 20-24. We approximate the life-cycle pattern of productivity z_j by dividing the age profile of income by that for hours worked. The age-specific weights on labor disutility ϕ_j are chosen such that in the initial steady state the model-implied hours worked match exactly the empirical age profile for this variable.

Figure 2 presents the matched age profiles for productivity and hours worked. The former follows the well-documented pattern, increasing up to late middle age of a household head, and then declining. As regards hours worked, they are almost flat, with some increase at young age and a drop close to retirement. In the figure we also show how our model matches two other important life-cycle profiles that we do not target directly, namely consumption and net assets. Again, as an empirical benchmark we use the HFCS data described above, with assets defined as net wealth excluding public and occupational pensions, and consumption approximated by spending on food (at home and outside) and utilities. As before, we use fourth-order polynomials to smooth the profiles and extrapolate them for age groups 20-24 and 67-70. Given the simple structure of our model, the profiles are matched remarkably well. In particular, we capture the timing of peaks in consumption and asset accumulation, the latter expressed relative to consumption, and the slopes are not very different from their

empirical counterparts as well, especially for assets. It might be interesting to note that our productivity and consumption profiles for the euro area peak later that those estimated for the US by Gourinchas and Parker (2002) or Fernandez-Villaverde and Krueger (2007). One of the reasons could be that we use more recent data, and hence our sample consists of households expecting longer lifetimes.[3]

3.2   Demographic trends

Our demographic scenarios use the past and projected estimates of the fertility and mortality rates for the European Union. We rely on the historical data provided by Eurostat and the Europop2013 projection, which encompasses years 2013-2080. Population data and age-specific death rates prior to 2015 come from the demo_pjan and demo_mlifetable series, respectively. For those years where mortality rates are not documented for the oldest cohorts, we employ exponential extrapolation. The future mortality rates are taken from the main projection scenario proj_13naasmr ,while population projections use the no-migration variant proj_13npzms for internal consistency.

This data is available for individual EU member states (28 countries), and we use the historical and projected cohort sizes to aggregate them over those countries for which the projected no-migration variant estimates are available. The historical mortality rates are available as of 2002. For years 1986-2001, we use the French estimates, as the EU-28 and France had similar mortality rates in the 2000s. As regards population data, we use directly the cohort sizes for years 2001-2014. For the period 1995-2000, we rescale proportionately the cohort sizes reported for EU-27 (Croatia is not included). Population prior to 1995 is constructed using French mortality rates. At the end of our projection horizon, we assume that mortality rates stabilize while the rate of change of the 20-year old cohort size stays at the level projected for 2080.

To ensure that the model predictions for the years that we focus on, i.e. 1980 and after, are not contaminated by frontloading effects, we start our deterministic simulations in the year 1900. To that end, we construct artificial population data for the years 1900-1994, and backcast the sizes of historical 20-year old cohorts while holding the historical mortality rates at the earliest available date (1986 for France) to accurately match the existing population structure from 1990 onwards. This is important as the consequences of the WW2 and post-war demographic booms are still visible in the age pyramids of European countries.

Finally, the mortality rates are smoothed using the Hodrick-Prescott filter with smoothing parameter 6.25, just to avoid jumps in the demographic input data produced by data revisions

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[3]The life-cycle consumption pattern in Gourinchas and Parker (2002), which is based on the CEX survey over the period 1980-1993, peaks at around the age of 45, while that estimated by Fernandez-Villaverde and Krueger (2007) using the CEX survey for years 1980-2001 peaks at around the age of 50. This already provides some evidence that, due to increasing longevity, the life-cycle consumption peak occurs later in life.

or by splicing historical data and projections. As for the growth rate of the size of the youngest cohort, we apply a much larger degree of smoothing (10,000) as we want it to capture secular trends in fertility rates rather than reflect the post-war baby booms and their echos. The resulting population pyramids for years 1995 (the first year in which we explicitly observe the population structure) and 2015 (the last year of available historical data) are depicted in Figure 3. It confirms that we are able to capture accurately the underlying demographic trends even after the applied smoothing.

3.3   Other structural parameters

Our calibration of the remaining model parameters is based on the previous literature, complemented with econometric estimates performed outside of the model and a moment matching exercise. The chosen values of the structural parameters are reported in Table 1, while Table 2 reports the properties of shocks that we use in stochastic simulations.

The discount factor is calibrated at 1.001 to match the average real interest rate of 1.2% observed in the euro area over the years 1999-2008. This is the longest time span during which the Eurozone interest rates can be considered close to their equilibrium values. In this period inflation was relatively stable, and after 2008 the euro area faced a prolonged crisis that pushed the interest rates down for cyclical rather than structural reasons. Our calibration of the Frisch elasticity of labor supply at 0.25 is consistent with estimates from the microeconomic literature (see e.g. Peterman, 2016). Physical capital is assumed to depreciate at a standard rate of 10% annually. The capital elasticity of output is calibrated at 0.25, which ensures that the investment rate is close to the average values observed in the Eurozone. This parametrization, together with a markup of 4%, implies that the labor income share is about 72%. Since our model does not feature wage rigidities, we calibrate the Calvo probability for prices set by intermediate goods producers at a somewhat high value of 0.73, or 0.925 in quarterly frequency.

The parametrization of the monetary policy feedback rule, including the standard deviation of monetary shocks, is based on econometric estimation of a log-linearized version of the monetary policy feedback rule (23), using euro area data over the period 1980-2012 from the AWM database, converted to annual frequency. We cut the sample at 2012 as this was the last year during which the ECB monetary policy was not constrained by the zero lower bound. Prior to estimation, all series are detrended using the Hodrick-Prescott filter with the smoothing parameter set to 100, which is a conventional value used for annual data. Subtracting trends from the data is aimed to capture the time variation in the natural interest rate, potential output and the implicit inflation target.

Whenever we allow for imperfect information about the natural interest rate and potential output growth by the monetary authority, we consider two alternative parametrizations of the

speed of learning. The first one is based on the empirical literature documenting the observed speed of learning (Branch and Evans, 2006; Malmendier and Nagel, 2016; Milani, 2011), which suggests annual rates of approximately 8%. The second one is obtained by comparing two types of NRI estimates, i.e. smoothed and filtered, both based on the methodology of Holston et al. (2016).[4] The filtered estimates use only past data available at a given point in time to estimate the NRI, while the smoothed ones are based on the entire sample. As such, these two measures can be seen as proxies for the real-time and "true" estimates of the NRI, respectively.[5] Our alternative value of the speed of learning is obtained from a regression based on formula ((24)), which gives an estimate of 20%.

Apart from the demographic transition scenario, driven by purely deterministic evolution of the fertility and mortality rates, the model is also used in a stochastic context. The stochastic fluctuations are driven by five shocks, affecting productivity, intertemporal preferences, investment-specific technological progress, firms' monopolistic power and monetary policy. All of them follow first-order autoregressive processes, except for the monetary shock that we assume to be white noise, and whose volatility is estimated outside of the model together with the remaining parameters describing the monetary policy rule. The inertia and standard deviations of the remaining shocks, as well as the investment adjustment cost function curvature, are determined in a moment matching exercise, in which we use detrended data on euro area real GDP, real private consumption, real investment, HICP inflation and the short-term nominal interest rate. More specifically, we minimize the distance between the model-based moments (standard deviation, first-order autocorrelation and correlation with output) and their respective data counterparts. The former are calculated using the first-order approximation of the policy functions around the point defined as the mean of the state variables in our demographic transition scenario over the period 1999-2008, which is the period that we use to match the level of the real interest rate. We use the same weights for all matched moments, except for the volatility and inertia of the nominal interest rate, for which we assign much higher weights so that the fit is exact. Achieving a perfect match for this variable is important as one of our goals is to evaluate the impact of demographic transition on the probability of hitting the ZLB. As can be seen from Table 3, the achieved fit is very good also for other moments.

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[4]We used the codes made available by the authors.

[5]Clearly, this way of reasoning is not appropriate for the most recent part of the sample, as the smoothed estimates converge, by construction, to the filtered ones. For this reason, we decided to drop the last 10 years of observations and estimate the speed of learning on the sample 1980-2005.

4   Results of model simulations

In this section we seek to answer several important questions about the consequences of the demographic transition for monetary policy. As it is well know from the literature, population ageing can have a sizable impact on savings and the equilibrium interest rate. We start by documenting this effect. Then we move to analyzing the consequences for monetary policy: we compare the impact of the demographic change on inflation, under various assumptions about the speed with which the central bank notices the decline in the NRI and potential output growth. Finally, we asses how much the declining NRI raises the probability of hitting the zero lower bound on the nominal interest rates. This is done both for the case in which the central bank observes the declining NRI and potential output in real time, and when it learns about changes in these two latent variables only gradually.

We show our model predictions for the period 1980-2080. However, bearing in mind that long-term demographic projections are subject to high uncertainty, we formulate the main conclusions based on the outcomes predicted until 2030, which is not a very distant date given that agents enter our model at the age of 20, and the mortality rates can also be considered highly predictable over such a horizon.

4.1   Macroeconomic impact of the demographic transition

We begin with describing the impact that the demographic transition exerts on the main macroeconomic variables. To this end, we run a deterministic simulation, assuming that the demographic processes described in Section 3 are known to all agents. For now, we also shut down stochastic shocks and abstract away from imperfect learning by the monetary authority, i.e. we assume that the central bank observes the NRI and potential output in real time. This implies that the real interest rate is equal to NRI over the whole simulation horizon.

Figure 4 presents the main results. The upper-left panel shows that the economy faces a sharp increase in the dependency ratio, resulting from lower birth rates and higher life expectancy. These two forces significantly modify decisions made by households. First, due to a longer expected time in retirement, workers increase savings. Second, low birth rates result in a declining population, and thus result in higher per capita asset holdings. Both operate in the same direction – they drive the real interest rate down. The decline is substantial, though spread over time: the real interest rate goes down by more than 1.5 percentage points between 1980 and 2030. Figure 5 decomposes the impact of the two demographic processes, i.e. fertility and longevity, on the natural rate of interest. Clearly, both matter, with increased life expectancy being slightly more important and responsible for 50-60% of the NRI decline, depending on the considered time period.

Other macroeconomic developments depicted in Figure 4 that are worth mentioning include: a decline in labor supply, increasing real wage (as labor becomes scarce) and, as a consequence, a higher capital-labor ratio chosen by firms. This shift towards more intensive use of capital in the production process initially generates an investment boom, but as of 2020 capital per capita starts declining as the effect of shrinking working-age population relative to the number of retirees starts to prevail. The economy can produce less output per capita and the growth rate of potential output bottoms out in the 2030s about 0.6 percentage points below its 1980s level.

It might be interesting to compare our simulated path of the NRI to popular estimates of this latent macrovariable used in the literature. Figure 6 plots the outcomes of our simulation together with the smoothed and filtered estimates obtained with the Holston et al. (2016) method that we referred to in Section 3.3 (the available sample is 1972-2015). While (not surprisingly) the exact numbers differ, it is striking that our projected decline in the NRI is much in line with the downward trend in the econometric estimates. This is even more evident if we ignore the post-2008 part of the data, which is heavily affected by cyclical factors related to the global financial crisis. If we consider the period between 1972 and 2008, our simulated NRI declines by about 1 percentage point and the Holston et al. (2016) estimates are in the range of 1.1-1.5 percentage points. This comparison suggests that, over this pre-crisis period, demographic processes have been responsible for most of the downward trend in the Eurozone natural rate of interest.[6]

While motivating this paper we have stressed the importance of accounting for such life cycle features as age-dependent labor productivity and supply. The reason why this should matter is that, as the age distribution of households in the economy changes due to demographic forces, average productivity and labor input shift due to composition effects, which might have macroeconomic implications. To assess their quantitative impact, we rerun our demographic transition simulation under perfectly flat age profiles of labor productivity or labor supply.[7] The results for the real interest rate are presented in Figure 7. The constant productivity scenario generates almost the same decline in this variable as the baseline up to year 2015. After this date, the two lines diverge, and in 2030 the constant productivity variant implies the NRI which is approximately 0.1 percentage points higher than under the baseline. Assuming constant hours worked in the initial steady state has virtually no effect, which is not surprising given that the lifecycle profile of hours worked is relatively flat in the data that we match while calibrating the baseline variant of our model. Overall, we conclude that failing to account for life-cycle characteristics of labor productivity and supply leads to some underestimation of the scale of the fall in the real interest rate associated with population ageing.

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[6]Eggertsson et al. (2017) reach a similar conclusion for the US over the period 1970-2015.

[7]Since labor supply is endogenous in our model, we fix its profile in the initial steady state only, as we have done while fitting it to the HFCS data in our baseline calibration.

4.2 Consequences for monetary policy

We are now ready to move to the main focus of our paper, which is about the consequences of the demographic transition on monetary policy. We concentrate on two aspects.

First, we evaluate to what extent potential misperceptions of the NRI and potential output, resulting from the central bank's conviction that demographic trends are not important within the horizon that is relevant to monetary policy, could bias its stance. Indeed, in the past equilibrium interest rates have been frequently assumed constant or at least stationary. For instance, until around 2000 many economists and central bankers placed the NRI in the US, UK or euro area in the range of 2-3% (Laubach and Williams, 2003). With relatively rare exceptions, the monetary policy feedback rules that were calibrated or estimated for these (and many other developed) countries assumed a constant intercept, and hence implied a constant NRI (see e.g. Taylor, 1993; Smets and Wouters, 2003 and DSGE models developed in central banks around that time) or its stationarity (Orphanides and Williams, 2002; Trehan and Wu, 2007). In such an environment, possible misperceptions of the NRI could only result in short-run over- or under-restrictiveness of monetary policy.

Things change when the natural rate is in a declining trend for many periods. If the central bank observes the evolution of this latent variable in real time, then the impact of its secular decline on inflation and economic activity can be neutralized by appropriate adjustments in the policy rate, at least when it is not constrained by the zero lower bound. However, if the central bank notices the declining NRI only with a lag and fails to account for the fact that it is bound to continue a downward trend, the outcome will be a monetary policy stance that is unintentionally too deflationary and contractionary for an extended period of time. A similar problem applies to misperception of the potential output, though the consequences are in the opposite direction. If its declining growth rate is noticed only with a lag, the central bank will overestimate the amount of slack in the economy. As a consequence, the monetary policy stance would be too expansionary.

The second aspect that we analyze is the impact of a declining natural interest rate on the probability of hitting the zero lower bound. Clearly, even if the central bank tracks the NRI in real time, its secular decrease will bring the policy rate closer to the levels where the ZLB may be a concern. The possible misperception about the NRI and potential output, by affecting average inflation and monetary policy restrictiveness, will additionally influence the magnitude of this risk.

In the rest of this section, we use our model to quantitatively analyze these two potential consequences of the demographic transition on monetary policy in more detail. This will allow us to offer some normative implications for the monetary policy conduct.

4.2.1 Learning and inflation bias

As explained in Section 2, our monetary policy rule is designed to account for time variation in the NRI and potential output growth as it incorporates the central bank's perception of these two latent variables. In our baseline demographic transition scenario presented above, we assumed that the monetary authority observes both of them in real time. Absent stochastic shocks, this implied perfect stabilization of inflation at the target and output at its potential as the actual interest rate was adjusted one-for-one with changes in the NRI. As a result, the impact of demographic trends on the nominal side of the economy was neutralized and the economy evolved as under flexible prices. Now we compare these outcomes to the case when the central bank's perception about the NRI and potential output follows a learning process described by equations ((24)) and (25) so that every period the monetary authority's estimate of these two unobservable variables is updated for a fraction λ of the last period forecast error.

Before we show the outcomes of our model simulations, it is instructive to sketch how misperceptions about the NRI and potential output can influence inflation under flexible prices. To this end, let us abstract away for a moment from aggregate uncertainty and the ZLB constraint. If prices are fully flexible, output is always at its potential, i.e. $y_t = \tilde{y}_t$, and the Fischer equation implies $R_t = \tilde{r}_t \pi_{t+1}$. Then, using equations (22) and (23) that describe the monetary policy feedback rule, and assuming that the degree of misperception about the NRI and potential output growth is constant over time, the long-run deviation of inflation from the target is related to them via the following formula

$$
\frac{\pi_t}{\pi} = \left(\frac{\tilde{r}_t^e}{\tilde{r}_t} \right)^{\frac{1}{1-\gamma_\pi}} \left(\frac{\tilde{y}_t/\tilde{y}_{t-1}}{\tilde{y}_t^e/\tilde{y}_{t-1}^e} \right)^{\frac{\gamma_y}{1-\gamma_\pi}} \tag{32}
$$

Since $\gamma_\pi > 1$, overestimation of the natural interest rate generates a deflationary bias while too optimistic perceptions about potential output growth push inflation above the target, with the relative strength of these two effects dependent on γ_y, i.e. the degree to which the central bank responds to output growth. It is important to note that this bias exists even if monetary policy does not affect real allocations because of perfect flexibility of prices. If prices are sticky, misperception about quantities showing up in the monetary policy rule will have real effects, which may additionally feed into the reaction of inflation.

We now turn to a quantitative assessment of these forces by employing simulations with our model. Figure 8 documents the results by comparing selected macroeconomic variables under our two alternative parametrizations of the learning process to the baseline. The last two panels present the magnitude of misperception of the two latent variables showing up in the monetary policy feedback rule. Under the slower learning variant ($\lambda = 0.08$), the central bank overestimates the level of the NRI by an average of 0.32 percentage points over the

period 1980-2030, and the bias peaks at 0.4 around 2020. The difference between perceived and actual potential output growth is smaller, averaging to about 0.12 percentage points and reaching a maximum of 0.15. This misperception depresses real GDP, and especially investment, leading to slower capital accumulation. However, these real effects turn out to be very small. The effects on the nominal variables are much bigger. If the central bank fails to timely account for the demographic trends, it no longer stabilizes inflation at the target. Even though the NRI drops at an average rate of only 0.03 percentage points per year over the period 1980-2030, failing to account for it by the monetary authority has a substantial impact on inflation, which falls persistently below the target. The deflationary bias reaches as much as 0.35 percentage points around 2020 and averages to 0.28 over the period 1980-2030. If learning is faster ($\lambda = 0.2$), the degree of misperception, and hence deflationary bias, is smaller, but still non-negligible, amounting to nearly 0.15 percentage points at its peak and 0.13 on average. As the natural rate begins to stabilize after 2030, the degree of its misperception by the central bank starts shrinking and inflation slowly returns towards the target.

As already mentioned, two forces are at play to generate these outcomes – the declining NRI makes monetary policy overly restrictive and generates a deflationary bias, while the decelerating potential output growth acts in the opposite direction. Figure 9 shows the relative strength of these two opposing effects on inflation under the slow learning scenario ($\lambda = 0.08$). The misperception about potential output creates a positive inflation bias until the early 2040s, but its magnitude is dwarfed by the effect of overestimating the natural interest rate. Still, this decomposition illustrates that, somewhat paradoxically, if the central bank follows a dual mandate so that it cares both about inflation and output growth (as in our baseline calibration), slow learning about the impact of demographic trends on macroeconomic fundamentals can result in a lower deflationary bias than under strict inflation targeting regime.

Going back to Figure 8, it also shows that the deflationary bias resulting from imperfect observation of the economy's fundamentals translates into a significantly lower level of the nominal interest rate. This will have important effects when we move to the ZLB problem in the following section.

4.2.2 Probability of hitting the ZLB

During the recent decade, the interest rates in many countries have hit the zero lower bound (ZLB). Something that had looked like a textbook curiosity has become a part of central bank reality. While the affected banks managed to elaborate alternative tools that allowed (at least partially) to overcome the consequences posed by the constraint (e.g. quantitative easing), it seems that they still prefer to use the short-term interest rate as the main policy instrument.

Keeping this in mind, we now check whether the demographic trends substantially alter the probability of hitting the ZLB. If this is the case, central banks might want to consider increasing their inflation targets to compensate for the continued decrease in the NRI.

Ideally, we would like to run stochastic simulations using shocks calibrated as described in Section 3 together with the deterministic demographic transition scenario. Note, however, that the latter implies a permanent shift of the steady state, and hence standard local approximation methods cannot be applied. An alternative that we offer, which we claim still gives a good idea on how the ZLB probability is affected by population ageing, works as follows. We run a series of stochastic simulations for 100,000 periods in the viscinity of each point on our deterministic path driven by the demographic transition, using first-order Taylor approximation to the model equilibrium conditions.[8] In this sense, a given point on the deterministic path is treated as a quasi time-invariant steady state, around which the economy fluctuates. The simulations are done trice. First, we assume that the central bank knows the NRI and potential output. Second and third, we check what happens if the bank learns about these unobservable variables as described in Section 4.2.1, using two alternative parametrizations for the speed of learning. Since imperfect knowledge about the two latent variables results in a long period when the economy faces persistently low inflation and depressed nominal interest rates, we expect the probability of hitting the ZLB to be higher under learning. Then, we approximate the probability of hitting the ZLB by calculating the frequency of periods during which the nominal interest rate is constrained by this limit.

Our findings are summarized in Figure 10. According to the baseline scenario, the annual probability of hitting the ZLB in the 1980s has been relatively low (below 1%). However, as the equilibrium real rate was declining, the probability was increasing as well, reaching about 2% in 2010, and it is projected to exceed 4% by 2030. While the annual probabilities do not seem very high, one should note what they imply over longer horizons. For instance, the chance of hitting the ZLB during the whole 2020s decade increases to 30.6% from 3.7% in the 1980s. The results from the learning scenario are even more alarming. Not only does the annual probability increase to much higher levels, but even the numbers for the contemporaneous times are quite high, especially if the speed of learning is slow. In 2020 the probability of meeting the ZLB exceeds 4% under $\lambda = 0.2$ and rises above 5% if $\lambda = 0.08$.

A natural question emerges to what extent these negative consequences can be mitigated by appropriate adjustments in the monetary policy conduct. One of the implications of the theoretical literature is that the presence of looming ZLB, the monetary policy should become more aggressive, see e.g. Adam and Billi (2006). The lower right panel of Figure 11 shows how the ZLB probability changes if the monetary authority becomes more hawkish, which we implement by setting the weight on output growth in the monetary policy feedback rule to zero. In the baseline model parametrization, i.e. when the central bank observes NRI

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[8]The simulations are run in Dynare OBC (Holden, 2016) in order to account for the zero lower bound.

and potential output in real time, switching to pure inflation targeting helps decrease the risk of hitting the ZLB, as the literature suggests. This is because more aggressive responses to deviations from the target decrease the volatility of the nominal interest rate (bottom left panel), without affecting its average level (upper right panel). A similar effect on interest rate volatility can be observed under learning. However, as we have discussed before, switching off the impact of potential output misperception on monetary policy also results in a deeper deflationary bias, which translates into a lower level of the nominal interest rate. This effect acts in the opposite direction, i.e. increases the risk of hitting the ZLB, and hence a more hawkish monetary policy mitigates this risk only moderately until the early 2030s.

Overall, our simulation results show that a slowly, but permanently declining equilibrium interest rate, especially if not properly accounted for, can result in a serious deterioration of monetary policy quality.[9] This finding stipulates our main conclusion for monetary policy. Even if the central bank is aware of the consequences presented in this paper, the demographic transition is an issue because it increases the chances of hitting the ZLB. However, if the bank fails to timely account for the declining NRI, monetary policy generates a deflationary bias that further increases the risk of ending up in a liquidity trap.

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[9]We do not discuss in detail the adverse consequences of hitting the ZLB, this has been done in many studies, see e.g. Gust et al. (2012); Ireland (2011); Neri and Notarpietro (2014).

5   Impact of introducing a pension system

To keep the structure of our model simple, while developing it we have abstracted away from the presence of a pension system. We now discuss how this simplification affects our main results. To this end, we augment our baseline model with a streamlined pay-as-you-go pension system. Participation in the system is mandatory so that all working households have to pay social security contribution that is levied on their labor income at rate τ_t^p. Retired households receive pension benefits pen_t, independent of their age, and calculated as the product of the replacement rate ϱ_t and the economy-wide net wage. For simplicity, the pension system is assumed to be balanced at all times so that the following condition is satisfied

$$\tau_t^p w_t \sum_{j=1}^{JR-1} N_{j,t} z_j h_{j,t} = pen_t \sum_{j=JR}^{J} N_{j,t} \tag{33}$$

where

$$pen_t = \varrho_t (1 - \tau_t^p) w_t \tag{34}$$

The budget constraint of the households needs to be modified as follows

$$P_t c_{j,t} + P_t a_{j+1,t+1} = (1 - \tau_t^p) W_t z_j h_{j,t} + R_t^a P_{t-1} a_{j,t} + P_t beq_t + \mathbf{1}_{j \geq JR} P_t pen_t \tag{35}$$

while the rest of the model remains unchanged.

In the presence of dynamically changing demographics, the pension system can remain balanced only if either the contribution rate or the replacement rate (or both) appropriately adjust over time. In this respect, the exact institutional arrangements differ across individual euro area economies, and some of them might be subject to significant modifications in response to increasing fiscal burden associated with a growing dependency ratio. Therefore, we opt for considering two extreme illustrative scenarios as these could be considered as defining the boundaries for the range of solutions that will be implemented by the governments of the ageing societies in reality. In the first one, the contribution rate is held constant at $\tau^p = 0.2$, while the replacement rate is adjusted accordingly. As a second extreme, we keep the replacement rate unchanged at $\varrho = 0.2$ and it is the contribution rate that needs to adjust. Both scenarios imply the effective net pension replacement rate (ratio of net pension to pre-retirement net labor income) of approximately 80% in 1980.

The consequences of those two scenarios for the selected macroeconomic variables are depicted in Figure 12. As in the baseline, in the two alternatives we abstract away from imperfect knowledge by the monetary policy so that the real interest rate and output are equal to their natural (potential) levels. It is clear that, given the past demographic trends and their projections of the next decades, keeping the pension system balanced requires either a significant increase in the contribution rate or a massive decrease in the replacement

rate. The choice of the method of preserving the pension system stability has important implications for the response of the key macrovariables, including the natural interest rate. Under constant contribution rate we see faster accumulation of assets as households expecting low pensions in the future increase their private saving. This exerts additional downward pressure on the NRI, which decreases at a higher rate and in 2030 falls short of its 1980 level by nearly 2 percentage points. In contrast, the constant replacement rate scenario initially generates a similar decrease in the NRI as in the baseline, but then this variable starts decelerating and around 2030 levels off about 1.4 percentage points below its level observed in 1980. This different behavior can be traced back to the adjustment in the contribution rate, whose rise decreases disposable income, discourages work and, as a result, leads to slower accumulation of assets.

Overall, the two extreme variants of the pension system lead to qualitatively similar predictions about the evolution of the natural interest rate, but their implications about the magnitude differ by about 0.5 percentage points on a cumulative basis. Importantly, however, our baseline scenario, in which we abstract away from any mandatory pension system, generates a very similar path of the NRI as the constant replacement variant until about 2015, and most of the universal pension systems in Europe (Pillar 1) over this period could be classified as defined benefits schemes (Eichhorst et al., 2011). Over the projection horizon, during which some decrease in the replacement rate seems unavoidable given the growing dependency ratio, the baseline variant rests firmly between the two extreme scenarios and in this sense can be seen as a reasonable guess on the possible future developments.

6   Conclusions

How does the demographic transition, resulting from lower fertility rates and higher life expectancy, affect monetary policy? While the question has already been tackled in the literature, and there seems to be an agreement that the equilibrium interest rate will be affected, many issues remain unclear. First, whether the impact is quantitatively large or small. Second, what happens if the central bank learns about the declining equilibrium interest rate only with a lag. Third, to what extent the demographic transition affects the probability of hitting the zero lower bound on the nominal interest rates.

We believe that our modeling approach, based on a fully-fledged OLG framework, carefully calibrated to closely match the age profiles of key decisions made at a household level, and using projected birth and mortality rates, is able to deliver more precise simulations than the earlier studies. We show that the effects can be substantial. In particular, between 1980 and 2030, the equilibrium interest rate in the euro area declines by more than 1.5 percenage points and the growth rate of potential output falls by about 0.6 percentage points. In principle, this decline should not pose a problem for monetary authorities – they should simply adjust the interest rates to follow the declining natural rate and potential output. Two issues emerge, however. First, both variables are not directly observable, and it seems possible that monetary policy learns about their decline only with a lag. Second, a lower natural rate implies, ceteris paribus, a higher probability of hitting the zero lower bound.

We show that both problems are acute. Learning about the declining natural rate and potential output growth rate can result in a prolonged period of below-target inflation. While the two mismeasurements work in the opposite direction, the NRI effect proves much stronger. Our simulations show a deflationary bias during the whole transition process of up to 0.35 percentage points. As expected, the annual probability of hitting the ZLB also increases, from a benign 0-1% in the 1980s to over 4% in 2030. Taking additionally learning into account makes the problem much more pronounced. In this case, the probability of hitting the ZLB reaches 4-6% already in 2020.

All in all, our main conclusion is that, in spite of being spread over a long period of time, the demographic transition can have a significant impact on the conduct of monetary policy and should be taken into account by the interest rate setters in a timely manner. Otherwise, the outcome will be a deflationary bias and overly restrictive monetary policy stance, resulting in a dramatic increase in the probability of hitting the zero lower bound – already elevated even if the monetary policy is perfectly aware of the impact that ageing exerts on the economy.

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Tables and figures

Table 1: Calibrated structural parameters

Parameter	Value	Description
β	1.001	Discount factor
φ^{-1}	0.25	Frisch elasticity of labor supply
δ	0.1	Capital depreciation rate
α	0.25	Capital share in output
S_1	4.39	Investment adjustment cost curvature
μ	1.04	Product markup
θ	0.73	Calvo probability
π_{ss}	1.02	Inflation target
γ_R	0.41	Interest rate smoothing
γ_π	1.97	Reaction to inflation
γ_y	0.42	Reaction to GDP growth
λ	0.2; 0.08	Learning parameter values

Table 2: Calibrated stochastic shocks

Parameter	Value	Description
ρ_z	0.00	Inertia of productivity shocks
ρ_u	0.75	Inertia of preference shocks
ρ_i	0.73	Inertia of investment specific shocks
ρ_p	0.82	Inertia of price markup shocks
σ_z	0.005	Standard dev. of innovations to productivity shocks
σ_u	0.029	Standard dev. of innovations to preference shocks
σ_i	0.033	Standard dev. of innovations to investment specific shocks
σ_p	0.019	Standard dev. of innovations to price markup shocks
σ_R	0.009	Standard dev. of monetary shocks

Table 3: Matched data moments

| Variable	Standard dev.		Autocorrelation		Corr. with GDP	
	Model	Data	Model	Data	Model	Data
GDP	1.56	1.65	0.79	0.59	1.00	1.00
Consumption	1.62	1.56	0.83	0.77	0.73	0.89
Investment	4.48	4.47	0.95	0.69	0.56	0.97
Inflation	0.98	0.76	0.42	0.63	0.40	0.47
Interest rate	1.49	1.49	0.64	0.64	0.22	0.60

Figure 1: Demographics
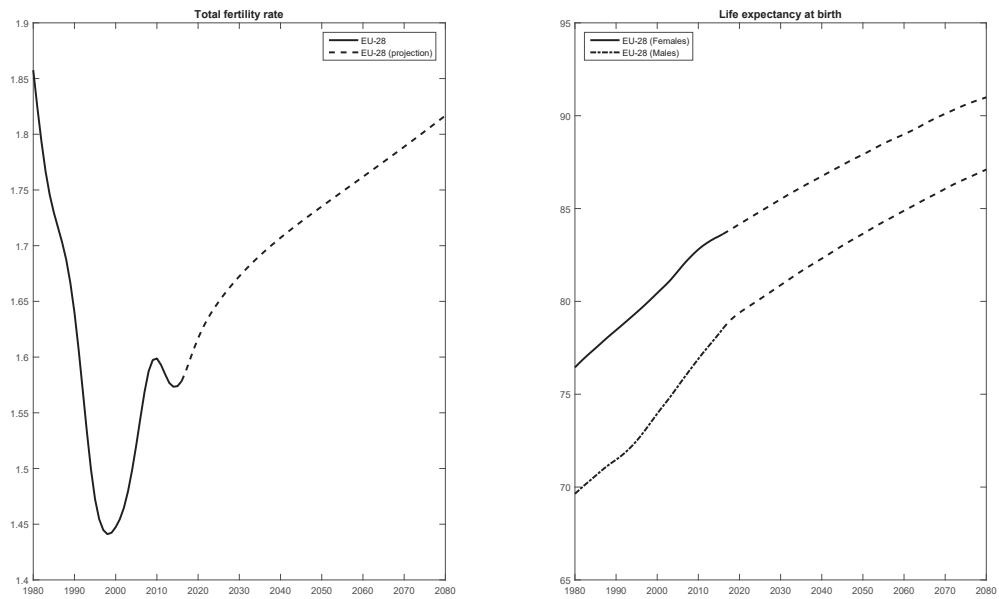
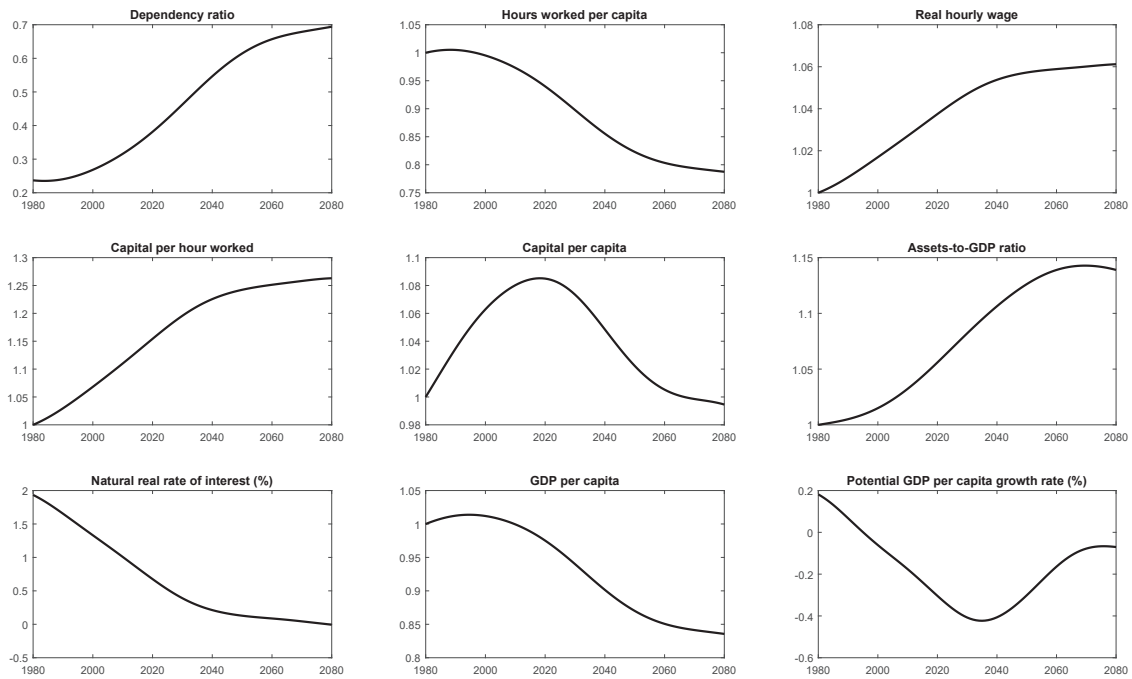Source: Eurostat and Europop2013 projections.

Figure 2: Lifecycle profiles
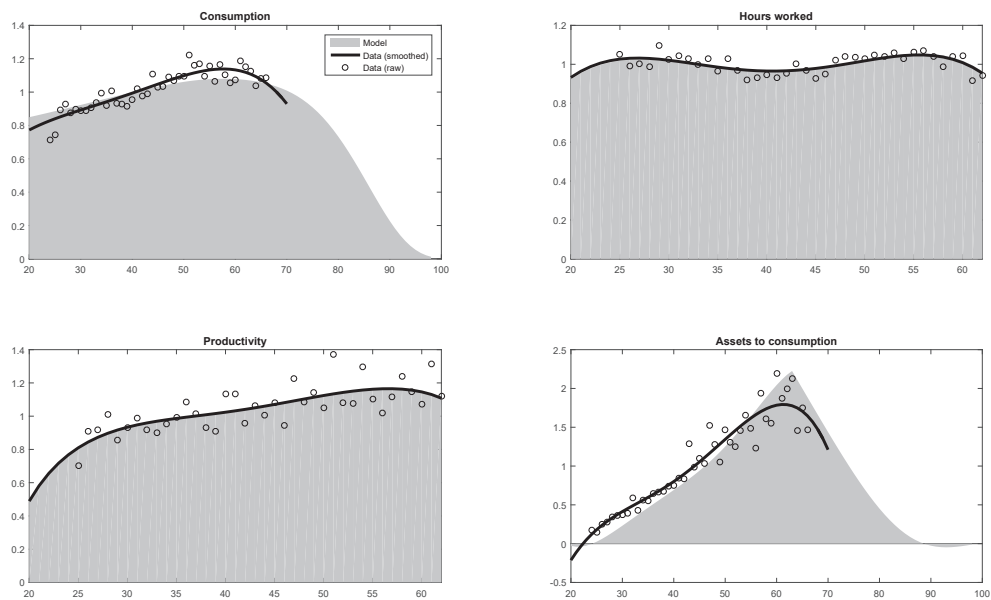
Source: HFCS data for the euro area and model simulations.

Figure 3: Population pyramids

Population pyramid in 1995 | **Population pyramid in 2015**
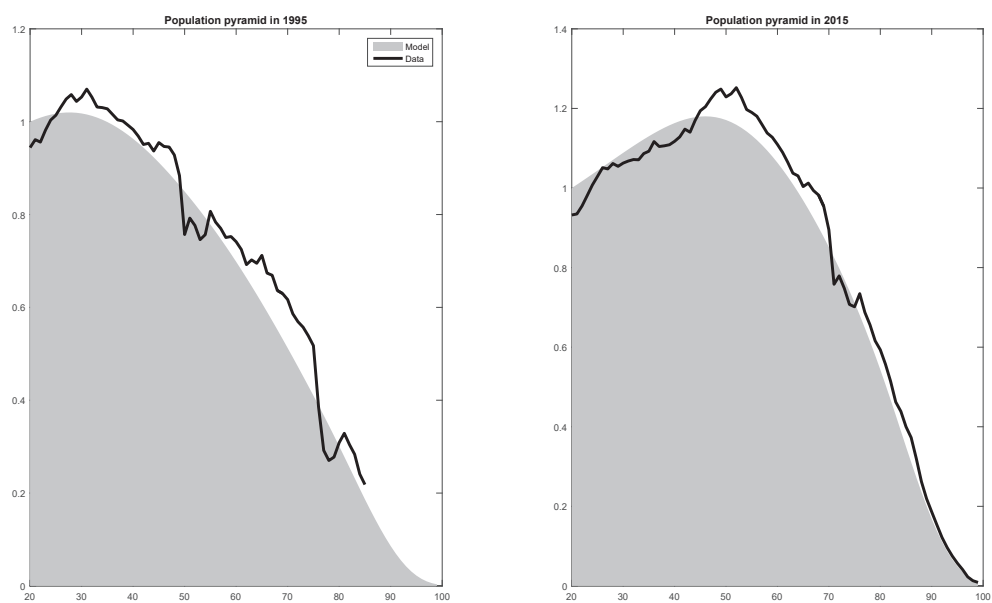
Source: Eurostat and model simulations.

Figure 4: Macroeconomic effects of the demographic transition

Note: Real hourly wages, capital per hour worked, capital per capita, assets-to-GDP ratio and GDP per capita are normalized to unity in 1980.

Figure 5: Decomposition of demographic forces affecting the NRI
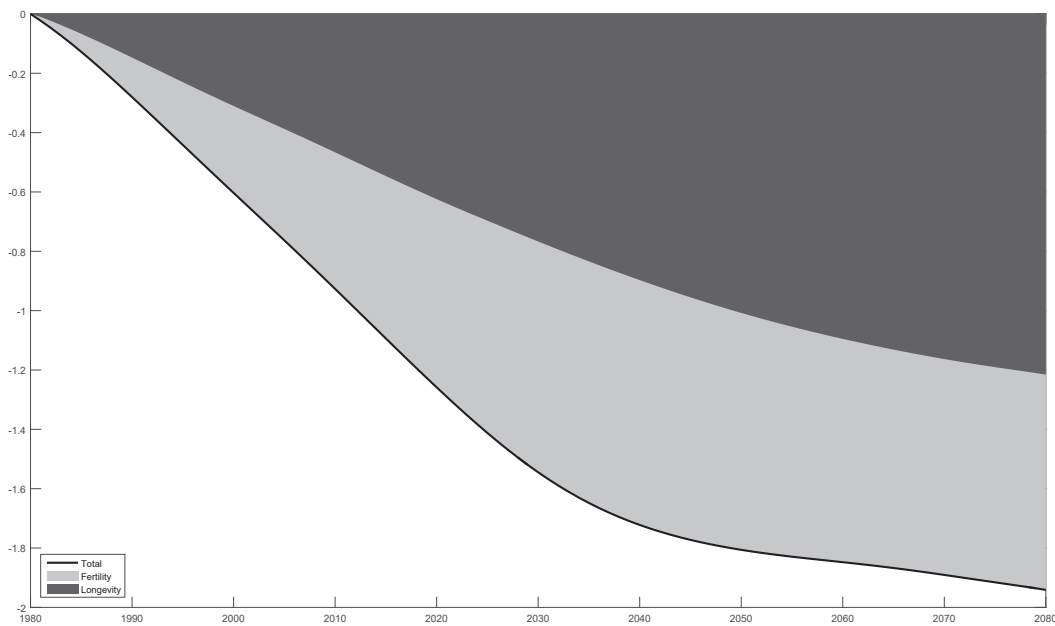
Figure 6: The natural rate of interest: comparison with Holston et al. (2016) estimates
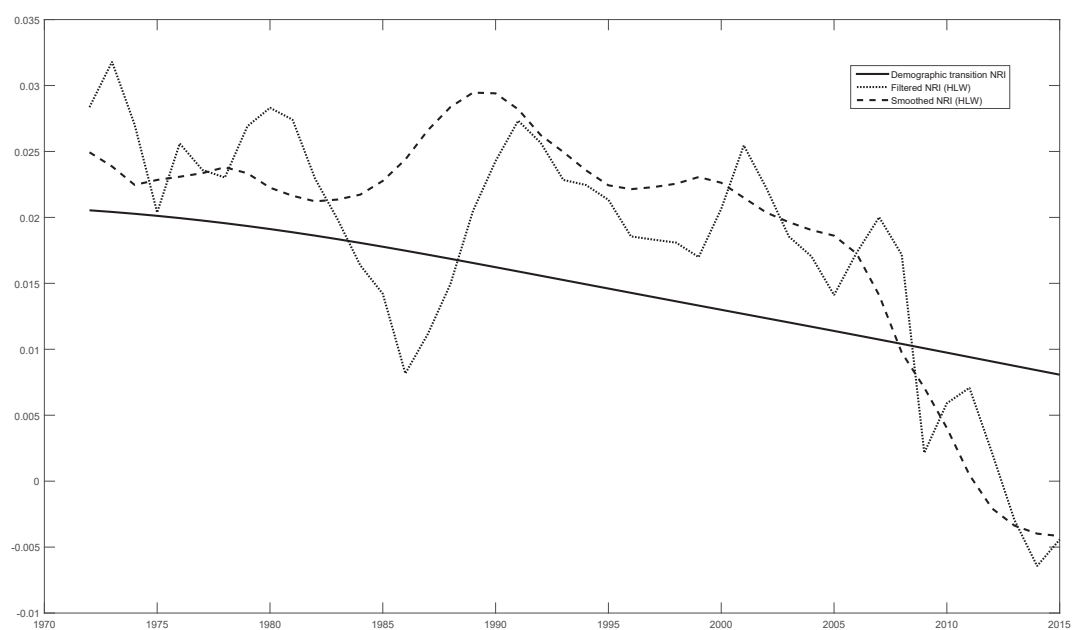
Figure 7: Robustness: alternative assumptions regarding hours worked and productivity profiles
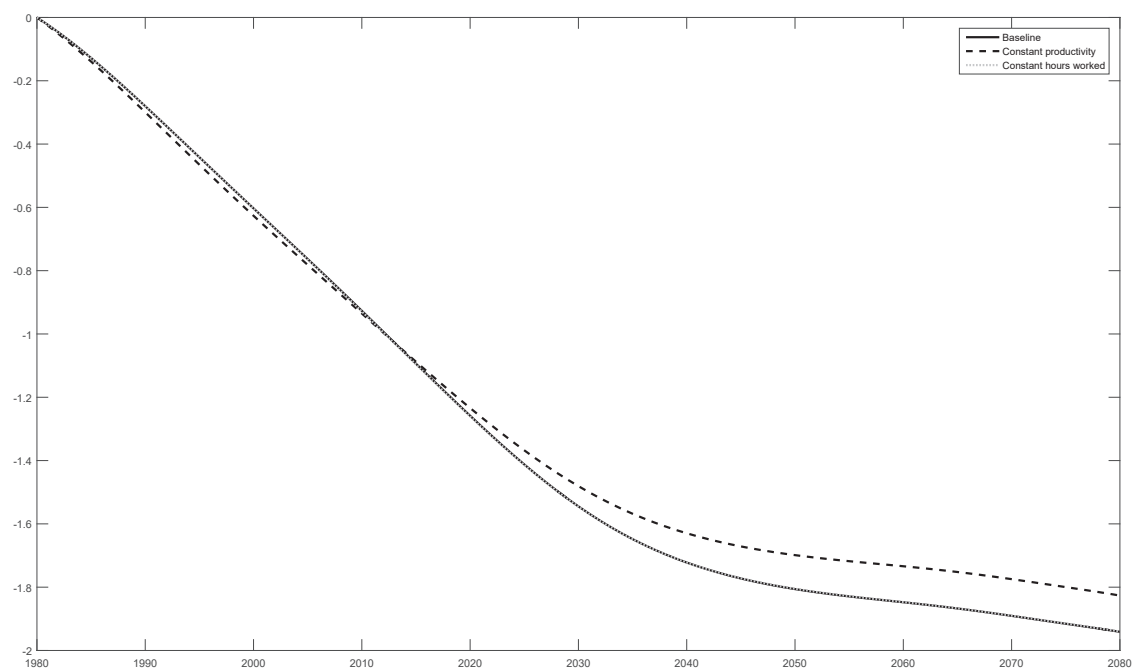
Figure 8: Macroeconomic effects of the demographic transition under learning
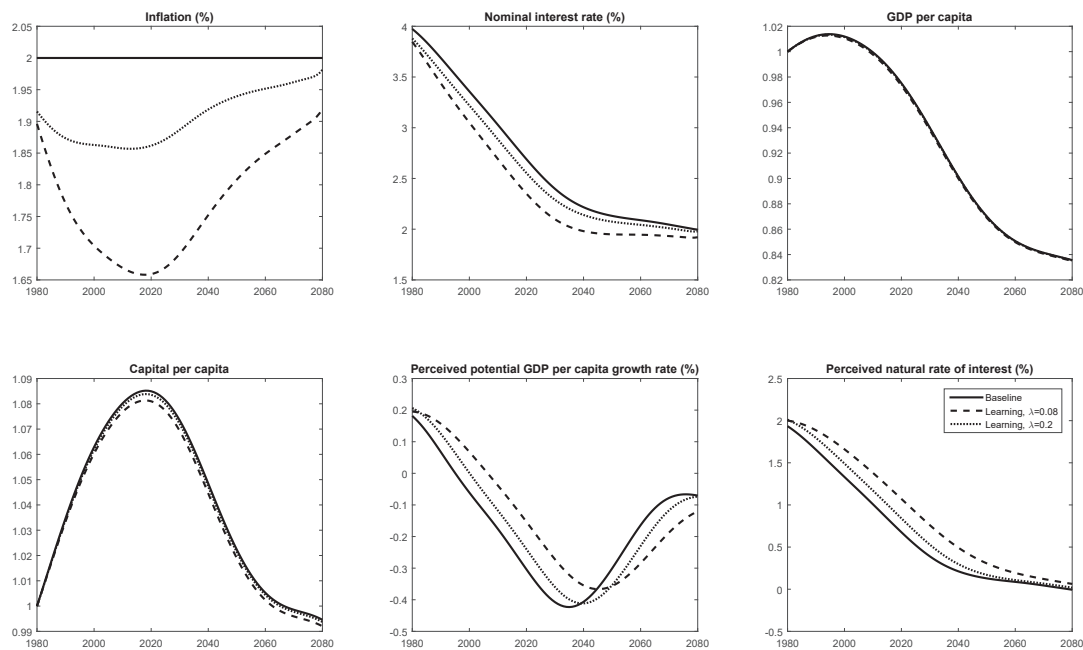
Figure 9: Decomposition of inflation bias under learning ($\lambda = 0.08$)
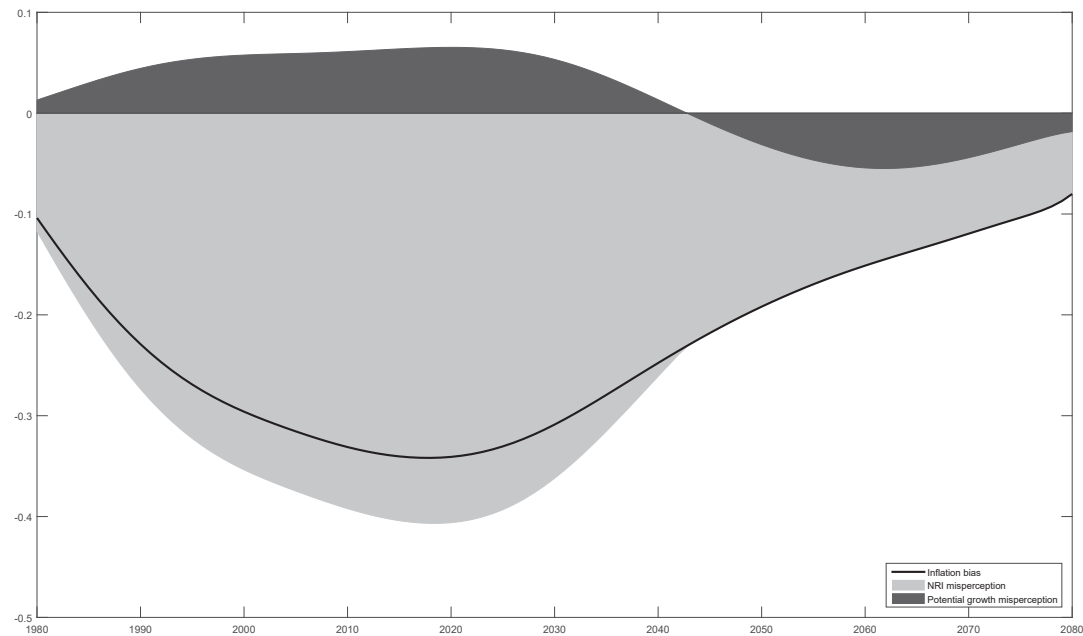
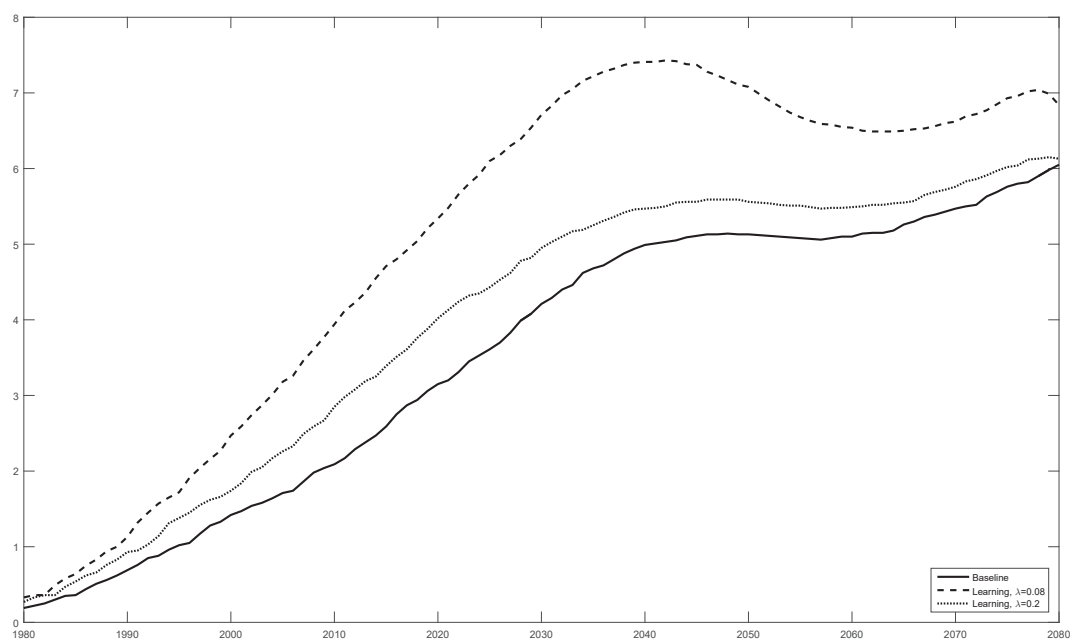Figure 10: Annual probability of hitting the ZLB

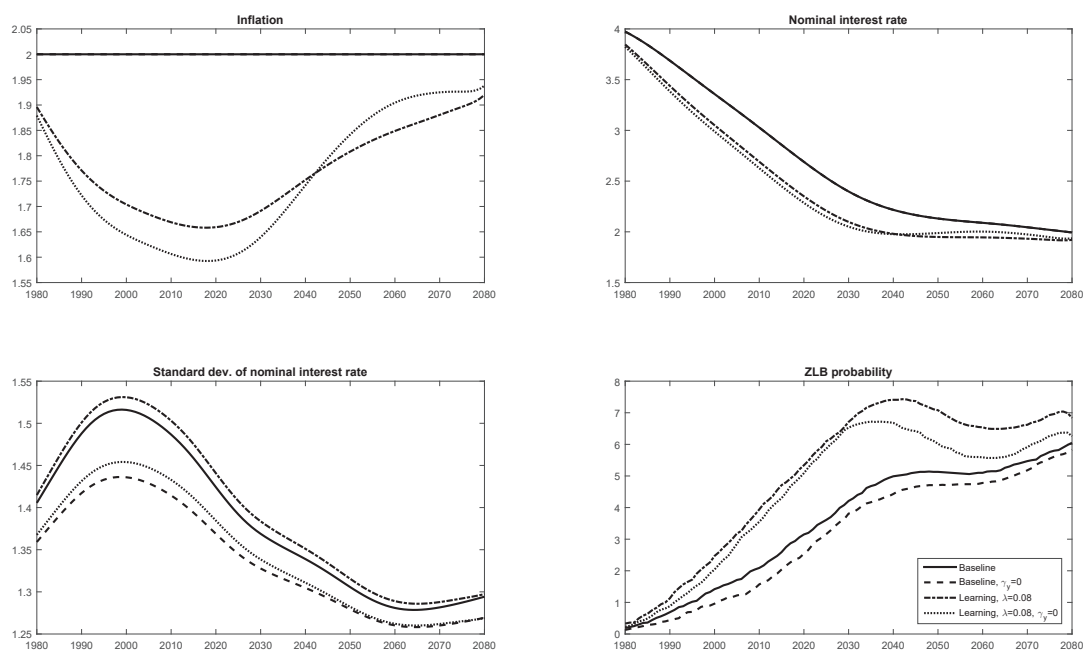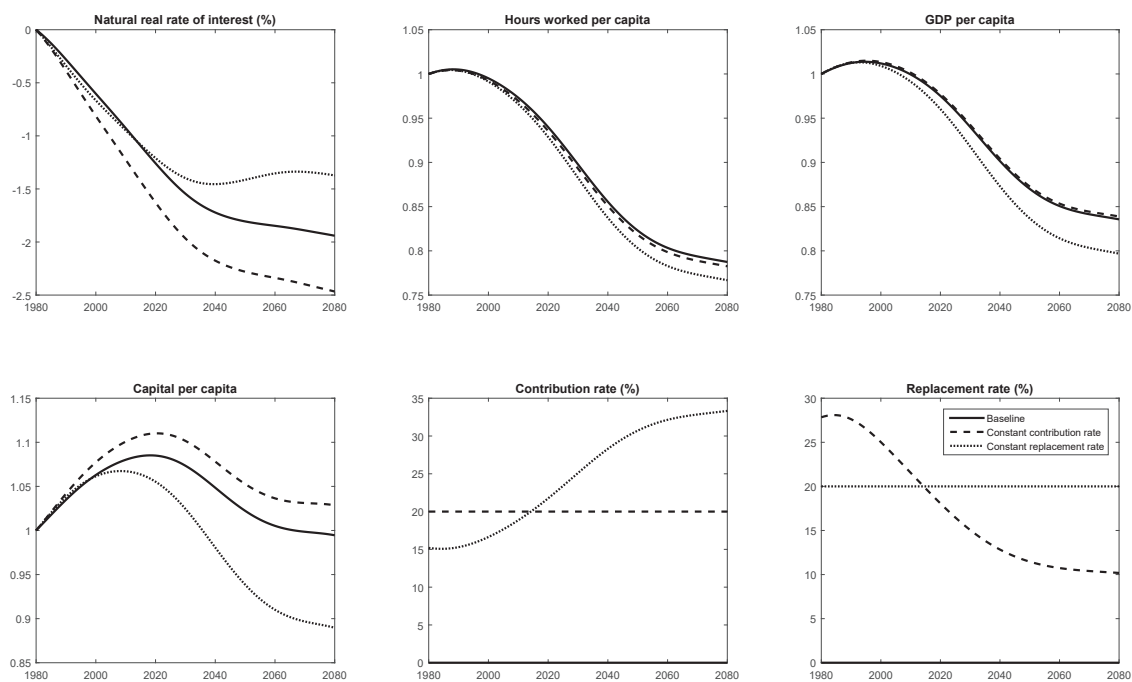Figure 11: Hawkish monetary policy and ZLB probability

Figure 12: Demographic transition under alternative pension systems

Note: For each variant, the real interest rate is normalized to zero in 1980 while hours worked per capita, GDP per capita and capital per capita are normalized to unity in 1980.

A   Appendix

A.1   Complete list of model equations

This appendix presents the full set of model equilibrium conditions that jointly determine the evolution of real per capita allocations and real prices, for given intitial conditions and for given realizations of exogenous deterministic demographic variables ($n_{1,t}$ and $\omega_{j,t}$, $j = 1,...,J-1$) and stochastic shocks (ε_t^z, ε_t^u, ε_t^i, ε_t^R, μ_t). Real prices are obtained by dividing their nominal values by the aggregate price level P_t. We also use the following definitions of aggregate profits and average stock prices: $f_t \equiv \frac{1}{N_t} \int_0^{N_t} f_t(i)D(i)\mathrm{d}i$ and $p_t^d \equiv \frac{1}{N_t} \int_0^{N_{t+1}} p_t^d(i)D(i)\mathrm{d}i$.

Households

$$c_{j,t} + a_{j+1,t+1} = w_t z_j h_{j,t} + \frac{R_t^a}{\pi_t} a_{j,t} + beq_t \tag{A.1}$$

$$a_{0,t} = 0 \tag{A.2}$$

$$a_{J,t} = 0 \tag{A.3}$$

$$1 = \beta(1-\omega_{j,t})\mathbb{E}_t \left\{ \frac{\exp(\varepsilon_{t+1}^u)}{\exp(\varepsilon_t^u)} \frac{c_{j,t}}{c_{j+1,t+1}} \frac{R_{t+1}^a}{\pi_{t+1}} \right\} \tag{A.4}$$

$$h_{j,t} = \left(\frac{w_t z_j}{\phi_j c_{j,t}} \right)^{1/\varphi} \qquad z_{j \geq JR} = 0 \tag{A.5}$$

Demography

$$N_{1,t}^{rel} = 1 \tag{A.6}$$

$$N_{j+1,t+1}^{rel} = \frac{(1-\omega_{j,t})N_{j,t}^{rel}}{1+n_{1,t+1}} \tag{A.7}$$

$$N_t^{rel} = \sum_{j=1}^{J} N_{j,t}^{rel} \tag{A.8}$$

$$1 + n_{t+1} = \frac{N_{t+1}^{rel}}{N_t^{rel}}(1+n_{1,t+1}) \tag{A.9}$$

Aggregation

$$c_t = \frac{\sum_{j=1}^{J} N_{j,t}^{rel} c_{j,t}}{N_t^{rel}} \tag{A.10}$$

$$h_t = \frac{\sum_{j=1}^{J} N_{j,t}^{rel} z_j h_{j,t}}{N_t^{rel}} \tag{A.11}$$

$$a_{t+1} = \frac{\sum_{j=1}^{J} N_{j,t}^{rel} a_{j+1,t+1}}{N_{t+1}^{rel}(1 + n_{1,t+1})} \tag{A.12}$$

$$beq_t = \frac{\sum_{j=1}^{J} \left[N_{j-1,t-1}^{rel} - N_{j,t}^{rel}(1 + n_{1,t}) \right] a_{j,t}}{N_t^{rel}(1 + n_{1,t})} \frac{R_t^a}{\pi_t} \tag{A.13}$$

Financial intermediary

$$(1 + n_{t+1})k_{t+1} = (1 - \delta) k_t + \exp(\varepsilon_t^i) \left[1 - \frac{S_1}{2}(1 + n_t)^2 \left(\frac{i_t}{i_{t-1}} - 1 \right)^2 \right] i_t \tag{A.14}$$

$$(1 + n_{t+1})a_{t+1} = q_t(1 - \delta)k_t + i_t + p_t^d \tag{A.15}$$

$$\frac{R_t^a}{\pi_t} a_t = [r_t^k + (1 - \delta)q_t]k_t + p_t^d + f_t \tag{A.16}$$

$$R_t q_t = \mathbb{E}_t \left\{ [r_{t+1}^k + (1 - \delta)q_{t+1}]\pi_{t+1} \right\} \tag{A.17}$$

$$R_t p_t^d = \mathbb{E}_t \left\{ (1 + n_{t+1})(p_{t+1}^d + f_{t+1})\pi_{t+1} \right\} \tag{A.18}$$

$$1 = q_t \exp(\varepsilon_t^i) \left[1 - \frac{S_1}{2}(1 + n_t)^2 \left(\frac{i_t}{i_{t-1}} - 1 \right)^2 - S_1(1 + n_t)^2 \left(\frac{i_t}{i_{t-1}} - 1 \right) \frac{i_t}{i_{t-1}} \right] \tag{A.19}$$

$$+ \mathbb{E}_t \left\{ \frac{\pi_{t+1}}{R_t} q_{t+1} \exp(\varepsilon_{t+1}^i) S_1(1 + n_{t+1})^3 \left(\frac{i_{t+1}}{i_t} - 1 \right) \left(\frac{i_{t+1}}{i_t} \right)^2 \right\}$$

Intermediate goods producers

$$\frac{r_t^k}{w_t} = \frac{\alpha}{1-\alpha}\frac{h_t}{k_t} \tag{A.20}$$

$$mc_t = \frac{1}{\exp(\varepsilon_t^z)}\left(\frac{r_t^k}{\alpha}\right)^{\alpha}\left(\frac{w_t}{1-\alpha}\right)^{1-\alpha} \tag{A.21}$$

$$\tilde{p}_t = \mu_t\frac{\Omega_t}{\Upsilon_t} \tag{A.22}$$

$$\Omega_t = mc_ty_t + \theta\mathbb{E}_t\left\{\frac{\pi_{t+1}}{R_t}\left(\frac{\pi}{\pi_{t+1}}\right)^{\frac{\mu_t}{1-\mu_t}}\Omega_{t+1}\right\} \tag{A.23}$$

$$\Upsilon_t = y_t + \theta\mathbb{E}_t\left\{\frac{\pi_{t+1}}{R_t}\left(\frac{\pi}{\pi_{t+1}}\right)^{\frac{1}{1-\mu_t}}\Upsilon_{t+1}\right\} \tag{A.24}$$

Inflation and price dispersion dynamics

$$1 = \theta\left(\frac{\pi}{\pi_t}\right)^{\frac{1}{1-\mu_t}} + (1-\theta)\left(p_t^{new}\right)^{\frac{1}{1-\mu_t}} \tag{A.25}$$

$$\Delta_t = (1-\theta)\left(p_t^{new}\right)^{\frac{\mu_t}{1-\mu_t}} + \theta\Delta_{t-1}\left(\frac{\pi}{\pi_t}\right)^{\frac{\mu_t}{1-\mu_t}} \tag{A.26}$$

Monetary policy

$$R_t = \begin{cases} R_t^{cb} & \text{if } R_t^{cb} > 1 \\ 1 & \text{if } R_t^{cb} \leq 1 \end{cases} \tag{A.27}$$

$$R_t^{cb} = R_{t-1}^{\gamma_R}\left[\tilde{R}_t^e\left(\frac{\pi_t}{\pi}\right)^{\gamma_\pi}\left(\frac{y_t/y_{t-1}}{\tilde{y}_t^e/\tilde{y}_{t-1}^e}\right)^{\gamma_y}\right]^{1-\gamma_R}\exp(\varepsilon_t^R) \tag{A.28}$$

Alternative assumptions about perceived latent variables

$$\tilde{R}_t^e = \tilde{R}_t \qquad \text{or} \qquad \tilde{R}_t^e = \tilde{R}_{t-1}^e + \lambda(\pi\tilde{r}_{t-1} - \tilde{R}_{t-1}^e) \tag{A.29}$$

$$\tilde{y}_t^e = \tilde{y}_t \qquad \text{or} \qquad \tilde{y}_t^e = \tilde{y}_{t-1}^e + \lambda(\tilde{y}_{t-1} - \tilde{y}_{t-1}^e) \tag{A.30}$$

Market clearing

$$y_t\Delta_t = \exp(\varepsilon_t^z)k_t^{\alpha}h_t^{1-\alpha} \tag{A.31}$$

$$y_t = c_t + i_t \tag{A.32}$$

$$f_t = y_t - w_th_t - r_t^kk_t \tag{A.33}$$

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