Business cycle implications of banking system heterogeneity and complexity

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Abstract

We investigate business cycle implications of banking system complexity and bank heterogeneity in individual leverage. We show that a more complex banking network generates higher individual bank leverage and increases economic volatility. Then, we build a general equilibrium business cycle model with three types of banks: deposit-taking, intermediary and lending. Keeping constant aggregate leverage in the banking system, we vary individual leverage and show that an increase in lending bank leverage increases costs of business cycle fluctuations. We argue that this can be mitigated by policymakers taxing the returns of lending banks.

Keywords: Financial intermediation; Network theory; Leverage; Welfare
*JEL Classifications*: E32, E44, E51

1   Introduction

The complexity of interbank linkages has gained deserved attention in recent years. It is now well-recognized that financial stability concerns may result from financial distress of banks that are only indirectly linked to each other through an intermediating bank. In particular, Cocco et al. (2009), Craig and von Peter (2014), Gabrieli and Georg (2014), Craig and Ma (2020) and Allen et al. (2020) document the core-periphery structure of the banking network in which banks can be broadly divided into three groups: deposit-taking banks (BD), intermediating banks (BI), and lending banks (BL).

Figure 1 from Craig and Ma (2020) illustrates this structure in the case of Germany. The right panel, in particular, shows that the banking sector can be well approximated by a linear stack, which is the structure that we will study.

Figure 1: German interbank market structure

(a) Unstructured   (b) Structured
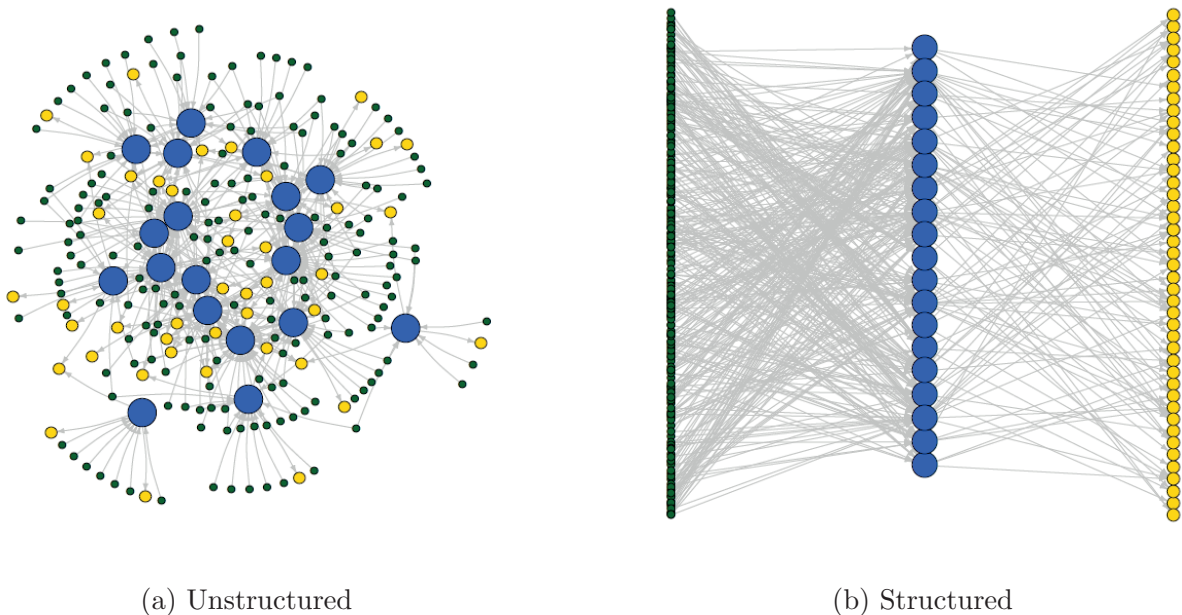
Source: Craig and Ma (2020). Note: Visual representation of the German over-the-counter interbank market as of end-2007 based on a simulated interbank network. Green = deposit-taking banks; Blue = intermediating banks; Yellow = Lending banks. The right panel depicts the same information but structures the network as a linear stack.

Notwithstanding empirical evidence, the banking sector is usually modelled parsimoniously in the macroeconomic literature, either consisting of a representative bank or two types of banks that trade with each other in the interbank market. In this way, existing theoretical

frameworks have tended to ignore the complexity of the banking system reported in empirical studies.[1] At the same time, the literature usually overlook the heterogeneity of financial intermediaries. The noticeable exception is the paper by Coimbra and Rey (2017) who point to a trade-off between stimulating the economy and financial stability when banks have heterogeneous leverage.

In this paper we account for a more realistic banking sector structure in an otherwise standard New Keynesian model in order to investigate the business cycle implications of the banking sector complexity and bank-leverage heterogeneity. To this end, we introduce financial frictions by modeling banks as facing a moral hazard problem as in Gertler and Kiyotaki (2010). We then study the consequences of various banking system networks and heterogeneity in banks' exposure to the financial friction.

We show that a more complex banking system generates higher individual bank leverage and makes the economy more volatile. Furthermore, we argue that an increase in lending-bank leverage substantially increases the welfare costs of business cycle fluctuations. We prove the result analytically in a 2-period partial-equilibrium framework and verify and quantify the result numerically in a richly-specified dynamic stochastic general equilibrium (DSGE) model. Finally, we show that appropriately chosen bank taxes can be used to lower lending-bank leverage and decrease the welfare costs of business cycles.

The article is related to a still growing financial friction literature that builds on seminal contributions by Kiyotaki and Moore (1997), Bernanke and Gertler (1989), Carlstrom and Fuerst (1997), Bernanke et al. (1999), Iacoviello (2005) and Christiano et al. (2014). The most closely related articles to our are Gertler and Kiyotaki (2010) and Gertler and Karadi (2011) as we follow their way of modelling the moral hazard faced by financial intermediaries. Our framework adds to the existing studies the realistic structure of the banking sector that allows us to investigate how its complexity and heterogeneity affect business cycles.

The paper is structured as follows. Section 2 investigates implications of the banking sector complexity. Section 3 shows how the heterogeneous allocation of leverage affects business cycle. Section 4 analyzes whether taxing some banks can improve welfare. Section 5 concludes.

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[1]An exception are agent-based models (ABMs) which can potentially capture rich interbank linkages (see for example Ozel et al. (2017). However, this type of model lacks general equilibrium mechanisms and so cannot be used for normative policy analysis.

2   Complexity of the banking system

In this section we investigate the role of banking sector complexity. Financial intermediaries in our framework form a stack which connects depositors (households) with borrowers (firms). Figure 2 illustrates four types of intermediation: from no banks (frictionless intermediation) to three types of bank. The first serves as a the frictionless benchmark. The second is the typical model of intermediation found in the literature. The latter corresponds to the core-periphery structure of the banking system reported in the empirical literature as the intermediating banks (BI) channeling funds from banks that collect deposits (deposit banks, BD) to banks that lend to firms (lending banks, BL). At the end of the section we generalize the structure of the banking sector to m banks.

Figure 2: Banking networks: Linear stacks

(a) Frictionless   (b) One bank   (c) 2-bank stack   (d) 3-bank stack
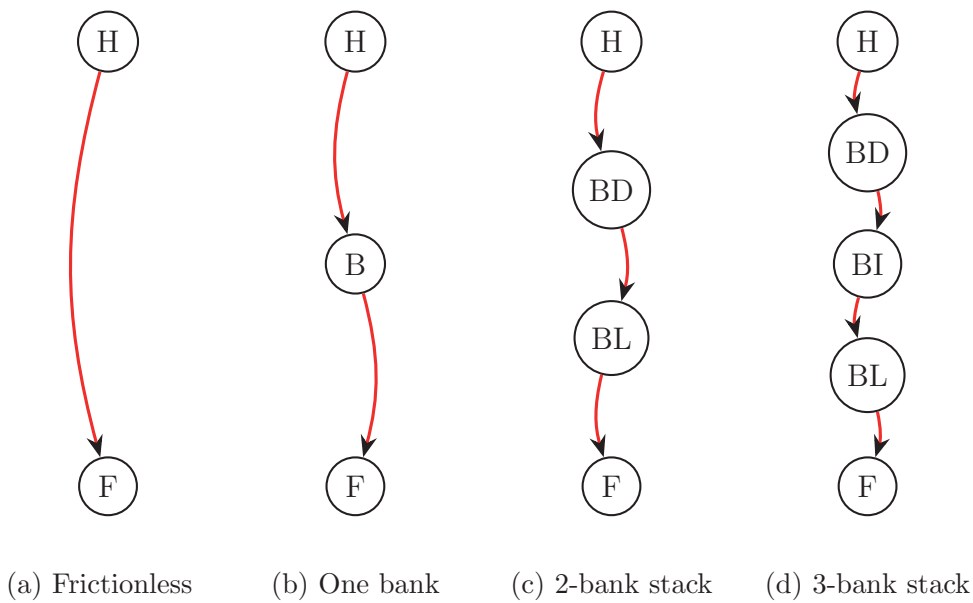
Note: H denotes households; B denotes banks; F denotes firms. Arrows indicate the flow of funds.

2.1   The set up

Consider, as way of introduction, the two-period $t \in \{0, 1\}$ models from figure 2(b) and 2(c). Households receive an endowment y (where $y = 1 - n$ and $n \in [0, 1]$) in period 0 and choose how to allocate consumption, c_t, between periods. Households transfer purchasing power to period 1 by supplying deposits, d. The deposit rate, R_d is set in equilibrium. Banks receive

exogenous net worth, n, in period 0. Banks are owned by households and households receive bank profits, π. Firms borrow in order to invest in assets, k, that earn exogenous return $R_k > 1$.

Supply of loanable funds The household problem is given by

$$\max_{c_0, c_1} \frac{c_0^{1-\gamma}}{1-\gamma} + \beta \frac{c_1^{1-\gamma}}{1-\gamma},$$

$$\text{s.t.}$$

$$c_0 + d = y,$$

$$c_1 = R_d d + \pi,$$

where aggregate profits of the banking system are given by $\pi = R_k k - R_d d$. The aggregate (consolidated) balance sheet of the banking system is given by $k = d + n$. Thus, aggregate bank profits can be rewritten as $\pi = R_k (n + d) - R_d d$. Note that this expression for aggregate profits is independent of the structure of the banking system. Taking first-order conditions of the household problem and substituting in profits gives the supply curve of loanable funds:

$$d = \frac{(\beta R_d)^{1/\gamma} - \frac{n}{y} R_k}{(\beta R_d)^{1/\gamma} + R_k} y.$$

This is an upward sloping supply curve in (R_d, d) space. It is however more useful to work with the supply curve in (R_d, k), given by

$$k = \frac{(\beta R_d)^{1/\gamma}}{(\beta R_d)^{1/\gamma} + R_k}.$$

Note that the this supply curve is independent of n. Thus, exogenous changes in net worth in the banking system only affects equilibrium outcomes through shifts in the demand curve for loanable funds.

Demand for loanable funds: Frictionless In the frictionless case, $R_d = R_k$ and the demand curve is horizontal at R_k in (R_d, d) space.

Demand for loanable funds: One bank The bank problem is given by

$$\max_{d} R_k k - R_d d, \tag{1}$$

$$\text{s.t.}$$

$$k = n + d, \tag{2}$$

$$R_k k - R_d d \geq \theta k, \tag{3}$$

where (1) is bank profits, (2) is the bank's balance sheet, and (3) is the incentive compatibility constraint. Banks can run away with a fraction $\theta \in [0, 1]$ of total assets. Incentive compatibility requires that bank profits are weakly greater than the return from running away.

If the incentive constraint does not bind then $R_k = R_d$ and the demand curve is horizontal at R_k in (R_d, d) space. If the incentive constraint binds then $R_k k - R_d d = \theta k$ and $R_k > R_d$. In that case, the demand curve is given by

$$k = \frac{R_d}{R_d - (R_k - \theta)} n,$$

which is downward sloping in (R_d, k) space. The slope of the demand curve is given by

$$\frac{\partial R_d}{\partial k} = -\frac{(R_d - (R_k - \theta))^2}{(R_k - \theta)} \frac{1}{n} < 0,$$

and will be useful for the subsequent analysis.[2]

Demand for loanable funds: Two-bank stack In this case, deposit bank BD collects deposits and lends to lending bank BL and the latter lends to firms. Total net worth is $n = n_D + n_L$ with $n_D = \alpha n$ and $n_L = (1 - \alpha) n$. Total profits of the banking system is given by $\pi = \pi_D + \pi_L = R_k k - R_d d$. Thus, the profit condition is the same as in the single bank case and the household supply curve is unchanged.

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[2]We use the result that if $f(x) = \frac{x+a}{x+b}$ then $f'(x) < 0$ if $a > b$. In this case, $a = 0$ and $b < 0$.

The deposit bank problem is given by

$$\max_{d} R_b b - R_d d,$$

$$\text{s.t.}$$

$$b = \alpha n + d,$$

$$R_b b - R_d d \geq \theta_D b,$$

whereas the lending bank problem is given by

$$\max_{b} R_k k - R_b b,$$

$$\text{s.t.}$$

$$k = (1 - \alpha) n + b,$$

$$R_k k - R_b b \geq \theta_L k.$$

Assume (and verify ex-post) both incentive constraints bind. Then the aggregate demand curve is given by

$$k = \frac{R_d + (1 - \alpha) \theta_D}{R_d - (R_k - \theta_D) + \theta_L} n. \tag{4}$$

2.2   One- vs. two-bank models

Figure 3 presents demand and supply curves for the frictionless, one- and two-bank models. Adding a bank to the frictionless case setup introduces a positive wedge between the lending rate to firms (R_k) and deposit rate (R_d) when the incentive constraint binds. Given exogenous lending rate, it requires deposit rate to go down with higher assets, implying downward-sloping deposit demand. Similar logic applies for the model with two banks. When incentive constraint bind for two banks they introduce two wedges in interest rates: one between R_k and R_b and the second between R_b and R_d. If the constraint for the deposit bank does not bind then $R_b = R_d$ and effectively we return to one-bank case.

We will show that capital response to net worth shift is stronger in two-bank model than in a one-bank setup. In order to make such a statement, it would be useful to define leverage ratios.

Definition 1 *Aggregate consolidated leverage:* $\phi_{con} = k/n$

Figure 3: Demand-supply curves
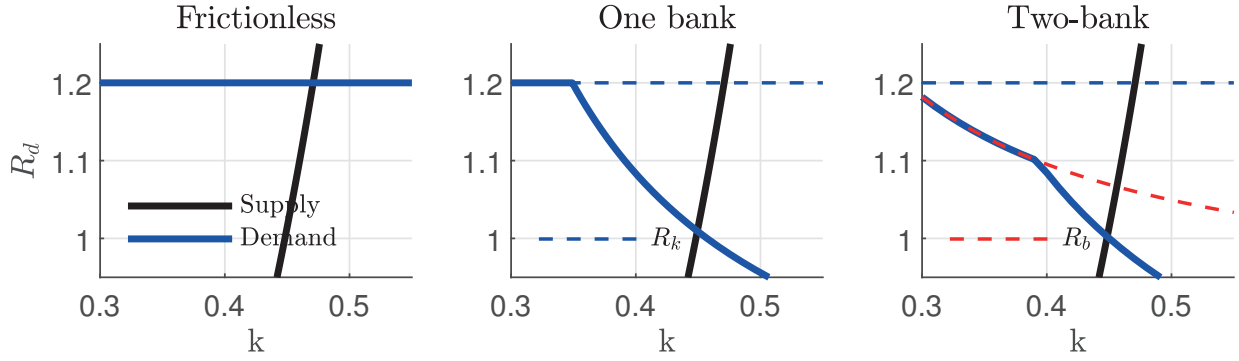
Note: Equilibrium in (R_d, k) is denoted by the intersection of the (black) supply curve and the (blue) demand curve. The $R_k - R_d$ spread is the vertical distance between the equilibrium point and the blue-dash line. Where the red-dash R_b line overlaps with the Demand curve, bank-1 is unconstrained. In the One-bank model, $\theta = 0.55$. In the Two-bank stack model, $\theta = 0.30186$.

Definition 2 *Bank-1 leverage:* $\phi_D = (k - (1 - \alpha) n) / \alpha n$

Definition 3 *Bank-2 leverage:* $\phi_L = k / (1 - \alpha) n$

From these definitions, we can show that individual bank leverage is lower than aggregate consolidated leverage. This result is provided in the following proposition:

Proposition 4 $\phi_{unc} > \phi_{con}$ *and* $\phi_1, \phi_2 > \phi_{con}$ *and* ϕ_{unc} *is the weighed average of* ϕ_1 *and* ϕ_2. *A necessary condition for* $\phi_1 < \phi_2$ *is* $\alpha \geq 0.5$.

Suppose we construct a scenario in which both the one-bank and two-bank economy has the same aggregate consolidated leverage (i.e. has the same k and R_d). Importantly, we set $\theta_D = \theta_L = \theta^*$ and $\alpha = 0.5$ (we will also allow for varying level of θ in the end of the section). This occurs when we set:

$$\theta = \theta^* \left(2 - \frac{1}{2\phi_{con}} \right). \tag{5}$$

Thus, in the above example, with $\theta = 0.55$ and $\phi_{con} = 2.81$, this requires $\theta^* = 0.302$.

The next proposition states that volatility is higher in the two-bank model (for a given consolidated leverage aggregate).

Proposition 5 *Suppose we choose θ and $\theta_1 = \theta_2$ such that $\phi_{con}^{one-bank} = \phi_{con}^{two-bank}$. In this scenario, the following result holds:*

$$\left.\frac{\partial k}{\partial n}\right|^{two-bank} > \left.\frac{\partial k}{\partial n}\right|^{one-bank}$$

Proof. Since we set θ^* consistent with (5), this means that the demand curve shifts by an equal amount. Thus, the steeper demand curve will generate more volatility from a shock to n. Solving for the slope of the demand curve, the condition for the two-bank demand curve to be steeper than the one-bank demand curve is: $\theta > \frac{3}{2}\theta^*$. This always holds since $\phi_{\mathbf{con}} \geq 1$. ∎

Let us relax the assumption that intermediation friction parameters of BL and BD are equal and show that the result still holds. Starting with the one-bank framework in which $\theta_L = 0.55$ we will gradually increase θ_D and lower θ_L to keep aggregate allocations constant. It is convenient to analyse this in the following expression that combines binding incentive constraints in the two-bank model:

$$k\left(\theta_L + \theta_D\right) = k\left(R_k - R_d\right) + R_d n + \theta_D\left(1 - \alpha\right) n. \tag{6}$$

In particular, by setting $\theta_D = 0$ this equation reduces to the binding incentive constraint of the lending bank only. [3] This equation also implies that in order to keep aggregate allocations constant θ_L must adjust to θ_D by less than one-to-one:

$$\theta_L = -\frac{B}{K}\theta_D \tag{7}$$

Both equations 6 and 7 suggest that the responsiveness of capital to changes in net worth should increase with θ_D, as the latter effectively increases the intermediation friction parameter (sum of θ_D and θ_L). This intuition is confirmed by model simulations. Notwithstanding the same steady state allocations in models with different values of intermediation friction parameters, the response of deposit rate (and also interest rate spread given exogenous lending rate) goes down while the reaction of capital to net worth shift rises with increasing θ_D and decreasing θ_L (figure 4, middle and right panels).

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[3]Note that we keep $\alpha = 0.5$. One might consider changing it together with θ, however, then it turns out that its optimum value is always 1 implying one-bank model. We come back to pure shifts of net worth when we consider a three-bank case.

Figure 4: Dynamic responses in one- and two-bank models
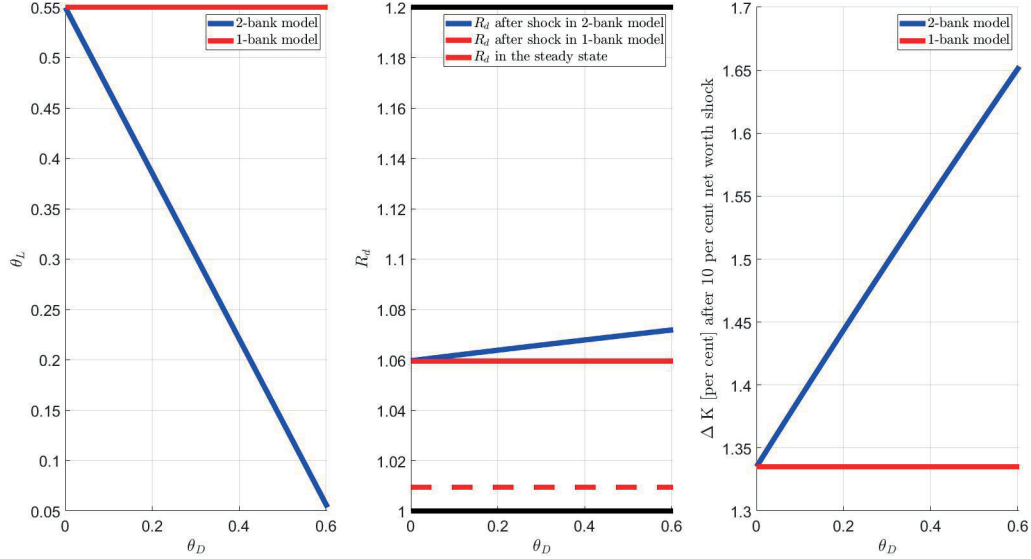
2.3 A m-Bank Stack

We next generalize the two-bank model to a m-bank stack to confirm that longer bank stacks are more volatile. We assume that θ_m is common across banks, that each bank has an equal fraction, $1/m$, of total net worth, and that the parametrization of the model is such that all incentive constraints bind at all times.[4] Figure 5 graphically presents the results of our simulations while below we derive them analytically.

The aggregate demand curve for capital is given implicitly by the following set of equations (we denote R_{ii-1} the interbank rate between bank i and the one below i in the stack):

$$\theta_m k = R_k k - R_{mm-1}\left(k - \frac{1}{m}n\right),$$

$$\theta_m\left(k - \frac{1}{m}n\right) = R_{mm-1}\left(k - \frac{1}{m}n\right) - R_{m-1m-2}\left(k - \frac{2}{m}n\right),$$

$$\vdots$$

$$\theta_m\left(k - \frac{m-1}{m}n\right) = R_{21}\left(k - \frac{m-1}{m}n\right) - R_d\left(k - n\right).$$

Thus, we have m equations and $m+1$ unknowns: $\{k, R_d, R_{mm-1}, R_{m-1m-2}, \ldots, R_{21}\}$. For

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[4]The role of heterogeneity, already touched upon in the previous section, will be further investigated in section 3.

Figure 5: Longer bank stacks are more volatile
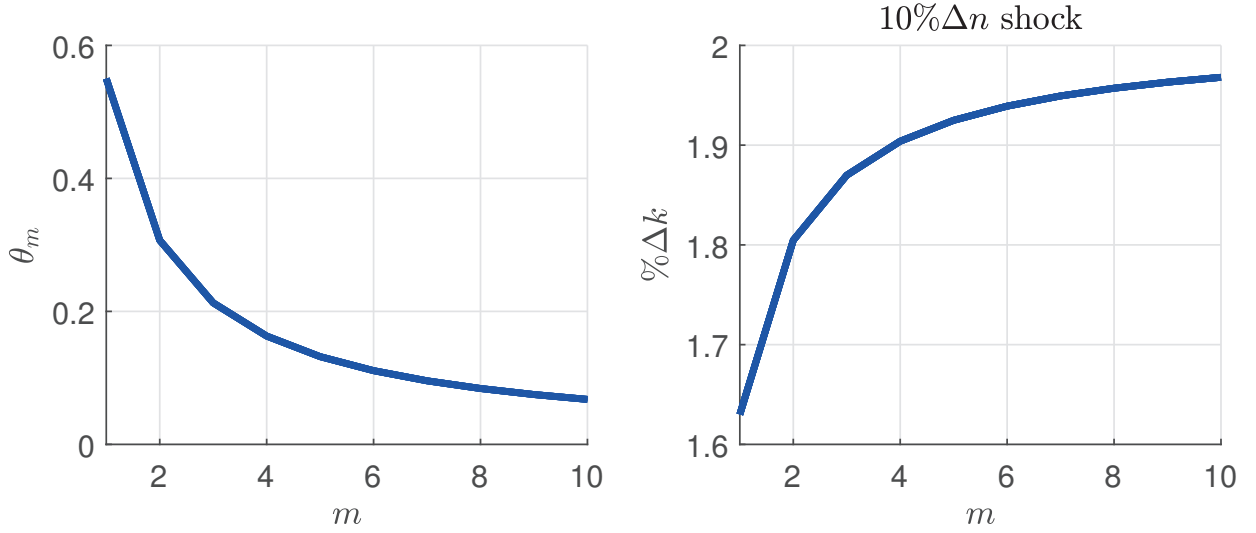
Note: $\theta_1 = 0.55$.

each economy, m, we find θ_m such that $\phi_{\mathsf{con}} = \phi_{\mathsf{con}}^m \quad \forall m$. This sequence of equations can be reduced to give

$$\theta\left(mk - \frac{n(m-1)}{2}\right) = R_k k - R_d(k-n). \tag{8}$$

Rearranging this gives the aggregate demand curve as

$$k = \frac{R_d + \theta_m \frac{m-1}{2}}{R_d - (R_k - \theta_m) + \theta_m(m-1)}n \tag{9}$$

Thus, the mapping from θ_1 to θ_m (for a given ϕ_{con}) is given by

$$\theta_1 = \theta_m\left(m - \frac{m-1}{2\phi_{\mathsf{con}}}\right). \tag{10}$$

The slope of the demand curve is given by

$$\frac{\partial R_d}{\partial k} = -\frac{n}{(k-n)^2}\left(R_k - \theta_m \frac{m+1}{2}\right). \tag{11}$$

Thus, the slope of the m-Bank economy is steeper than the slope of the 1-Bank economy if

$$\theta_1 > \theta_m \frac{m+1}{2}, \tag{12}$$

and the slope of the $(m + 1)$-Bank economy is steeper than the slope of the m-Bank economy if

$$\theta_m > \theta_{m+1} \frac{m + 2}{m + 1}. \tag{13}$$

We can write equation (10) in recursive form as follows[5]:

$$\theta_m = \theta_{m+1} \left(\frac{2\phi_{\text{con}} (m + 1) - m}{2\phi_{\text{con}} m - (m - 1)} \right) \tag{14}$$

Combining these expressions and simplifying leaves: $\phi_{\text{con}} > 1$. QED

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[5]Analogously to the case of one- and two-bank models from the previous section, it can be shown that starting with the m-bank stack, one may add another bank keeping aggregate allocations unchanged by increasing intermediation friction for the bank m+1 and lowering for m-banks. As in the case of one- and two-bank models, this would amplify reaction to net worth shock in the longer-stack model.

3 Leverage heterogeneity

In this section we investigate the importance of heterogeneity in leverage and show that more leverage concentration in lending banks deteriorates welfare. To obtain this result we build intuition using two-period models in several steps.

First, we show analytically and using simulations of a two-period two-bank model that lowering the leverage of lending bank (by shifting net worth from the deposit bank to the lending bank) improves welfare. Second, we argue that this result does not hold if we keep aggregate leverage in the banking sector and consumption allocations unchanged. We simulate net worth shifts in the two-period three-bank model and find that the impact of "pure" net worth shifts does not impact welfare. Third, we allow for heterogeneity in the intermediation friction keeping the individual bank leverage ratios unchanged. We show that welfare costs of business cycles rise with the degree of intermediation friction faced by lending banks.

Finally, we quantify effects of heterogeneity in bank leverage by simulating a medium scale DSGE model that accounts for 3 types of banks. Heterogeneity in leverage in this setup results from variations in intermediation friction faced by banks. Thus, the full model captures results that we obtained in steps one and three: the lower is the leverage of lending bank the higher welfare. We show that the effects of leverage heterogeneity are quantitatively substantial.

3.1 Net worth allocation

In this section we show that shifting net worth from the lending bank and making it more leveraged decreases capital and welfare.

Proposition 6 *The capital demand curve shifts outwards when net worth shifts from deposit bank to lending bank.*

$$\frac{\partial k^{demand}}{\partial \alpha} < 0. \tag{15}$$

This holds so long as both constraints bind. For an graphical example, see figure 6 and 7. When α is sufficiently low, $R_k = R_b$ and lending bank is unconstrained. A lowering α further increases the agency problem faced by deposit bank, pushing down on R_d and lowering k.

When α is sufficiently high, $R_b = R_d$ and deposit bank is unconstrained. Increasing α decreases the net worth of the constrained bank, increasing the pushing down on R_b and R_d and again lowering capital. The non-obvious result is when both banks are constrained. The explanation is as follows: Suppose R_d and d are unchanged. A one-unit transfer of net worth from deposit bank to lending bank requires, that for the lending bank incentive constraint to remain unchanged that R_b to rise by $R_b / (k - (1 - \alpha) n)$. In contrast, for deposit bank, the incentive constraint remains unchanged if R_b rises by a smaller amount: $(R_b - \theta_1) / (k - (1 - \alpha) n)$. Thus, to restore equilibrium, k and b need to increase, requiring a rise in R_d and d.

Figure 6: Shift in net worth from bank-1 to bank-2 (ceteris paribus)
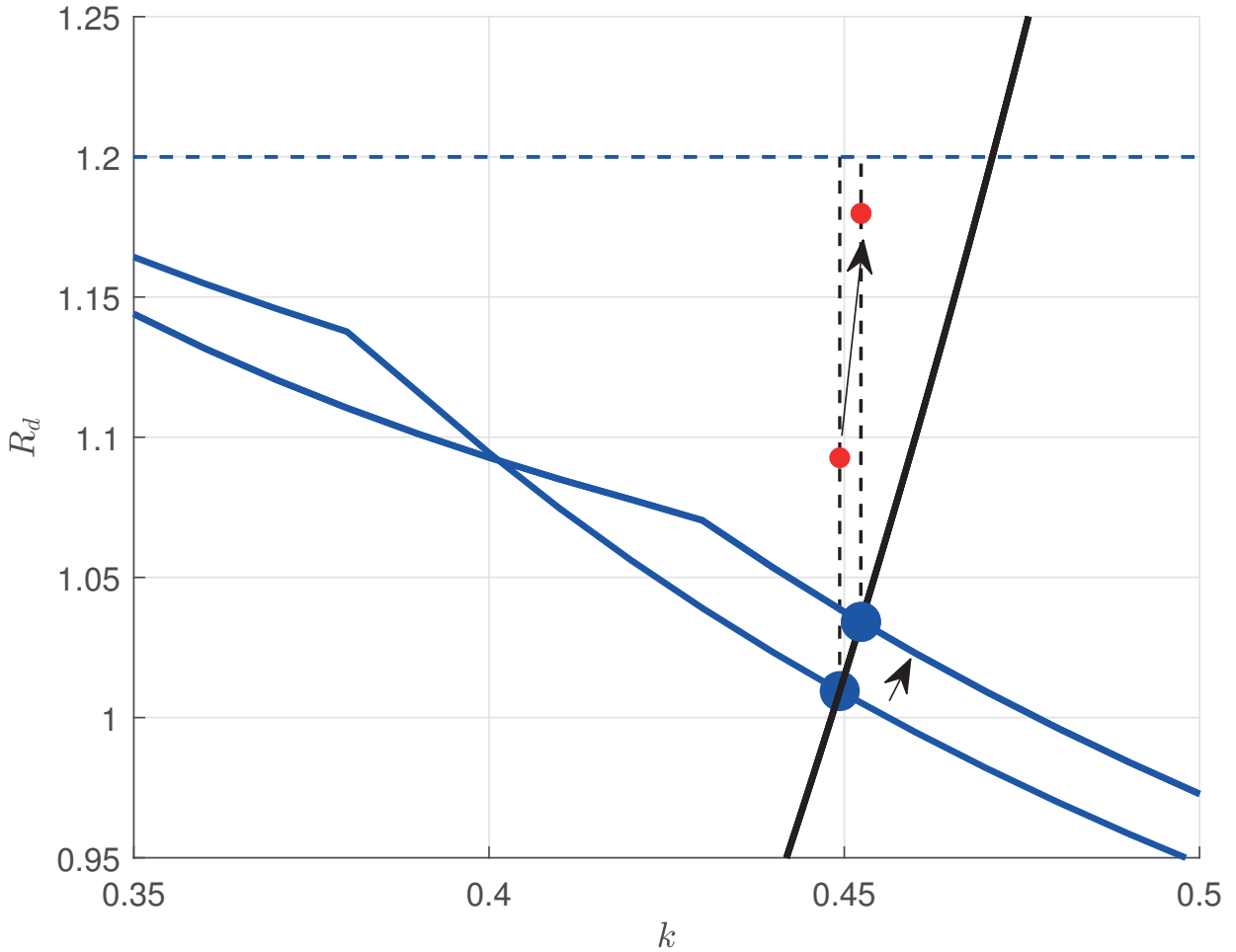
Note: In the baseline $\theta = 0.30186$ and $\alpha = 0.5$. The shock is $\alpha' = \alpha \times 0.65$. Thus, net worth falls in bank-1 and rises in bank-2. The bank-1 spread rises and the bank-2 spread falls.

Figure 7: Shift in net worth from bank-1 to bank-2 (ceteris paribus)
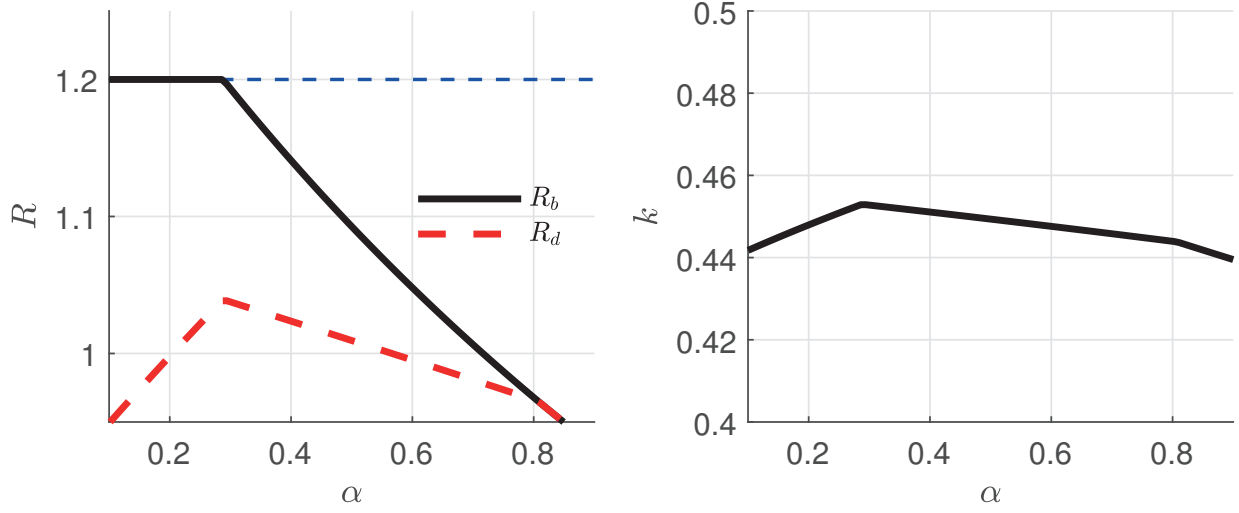
Note: $\theta = 0.30186$.

Having established the relation between α and capital we formulate the next proposition that relates α to welfare and considers it as a policy instrument.

Proposition 7 *Optimal policy A social planner that wants to maximize welfare will maximize k and therefore set α^* such that the lending banking is just at its incentive constraint.*

In the above example this is when $\alpha = 0.2879$. In this situation, bank-1 is highly leveraged while bank-2 is less leveraged. The equilibrium is reported in Table 1.

Now, let us consider a three-bank model. This framework will be useful because in the next section we will consider shifting net worth between banks while keeping the aggregate leverage constant. Such exercise is feasible in 3-bank model whereas in 2-bank it is not. In the 3-bank 2-period model the demand curve for capital is given by[6]

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[6]In general, the left-hand side term is given by

$$mk - (m-1)n + \sum_{j=1}^{m-1} (m-j)\alpha_j n.$$

Rearranging, the demand curve is given by

$$k = \frac{R_d + \theta\left(m - 1 - \sum_{j=1}^{m-1}(m-j)\alpha_j\right)}{R_d - (R_k - \theta m)}n.$$

$$\theta \left(3k - \left(2 \left(1 - \alpha_1\right) - \alpha_2\right) n\right) = R_k k - R_d \left(k - n\right).$$

Proposition 8 *The optimal weight on net worth on lower banks receives greater weighting that higher banks. Thus, one wants to load the net worth on the higher banks until they are unconstrained.*

A numerical example is given in Table 1. In this example, the optimal outcome is for bank-1 to be highly leveraged, and for bank-2 and bank-3 to be just unconstrained.

Table 1: Equilibrium outcomes

Model	θ	$\alpha_{1,2}$	S_{kd}	S_{k2}	S_{21}	S_{1d}	$\phi_{1,2,3}$	k	n
Optimal distribution of net worth									
Three-bank	0.208	0.173,0.374	10.18	0.00	0.00	10.18	11.2,6.1,4.0	0.4598	0.16

Note: $S_{ij} = 100 \left(R_i - R_j\right)$.

3.2 Net worth shifts keeping aggregate leverage constant

In the previous section we analyzed how changes in net worth and resulting from them heterogeneous leverage levels affect equilibrium outcomes. In this section we disentangle the pure effects of shifting net worth from their impact on aggregate capital and consumption allocation (see Table 2). To this end we analyze the effects of shifting net worth while keeping aggregate leverage constant. In order to get basic intuition we analyze a two-period economy with three banks: deposit-taking, intermediary and lending. Parameters α_1 and α_2 denote shares of net worth allocated respectively to the deposit and the intermediary banks. The total net worth is exogenous and is equal to N. We shock the model with 1 percent increase in net worth for all banks and analyze its impact on welfare under different values of α's[7].

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[7]Note that we need at least three banks and two α parameters to adjust so that the aggregate leverage is constant

Table 2: Equilibrium outcomes

Model	θ_D	θ_I	θ_L	α_1	α_2	ϕ_{con}	ϕ_D	ϕ_I	ϕ_L	k	n
Baseline											
Frictionless	·	·	0.00	·	·	·	·	·	·	0.471	·
One bank	0.550	·	·	·	·	2.81	2.81	·	·	0.449	0.16
Two-bank	0.302	·	0.302	0.500	·	2.81	4.62	.	5.62	0.449	0.16
Three-bank	0.208	0.208	0.208	0.333	0.333	2.81	6.43	7.43	8.43	0.449	0.16
DSGE 3-b	0.208	0.208	0.208	0.267	0.329	2.15	5.32	5.32	5.32	5.99	2.78
Shift in net worth from deposit bank											
Two-bank	0.302	.	0.302	0.288	.	2.83	7.36	.	3.98	0.453	0.16
Three-bank	0.208	0.208	0.208	0.288	0.424	2.81	7.28	5.95	9.75	0.449	0.16
Shift in intermediation friction parameter (θ_D goes down)											
Two-bank	0.257	.	0.339	0.500	.	2.81	4.62	.	5.62	0.449	0.16
Three-bank	0.198	0.212	0.212	0.333	0.333	2.81	6.43	7.43	8.43	0.449	0.16
DSGE 3-b	0.198	0.213	0.213	0.257	0.332	2.15	5.49	5.24	5.24	5.98	2.78

Figure 8 presents the results of simulations. The left panel shows how net worth allocation changes between banks. With more net worth going to the deposit bank, less is allocated to the intermediary bank to keep the total leverage constant. The net worth of the lending bank is equal to $1 - \alpha_1 - \alpha_2$ and must be increasing with rising α_1. The right panel presents the welfare consequences of business cycle (1% net worth shock in this case) by finding the portion of consumption in the steady state that households would need to sacrifice to be indifferent between the world with and without shocks, where the former is given by:

$$\mathbb{E}\left(W\right) = \frac{1}{2}\left(\frac{c_{0,n_L}^{1-\gamma}}{1-\gamma} + \beta\frac{c_{1,n_L}^{1-\gamma}}{1-\gamma}\right) + \frac{1}{2}\left(\frac{c_{0,n_H}^{1-\gamma}}{1-\gamma} + \beta\frac{c_{1,n_H}^{1-\gamma}}{1-\gamma}\right),$$

Figure 8: Impact of the net worth shock on welfare when α's shift (3-bank, 2-period model)
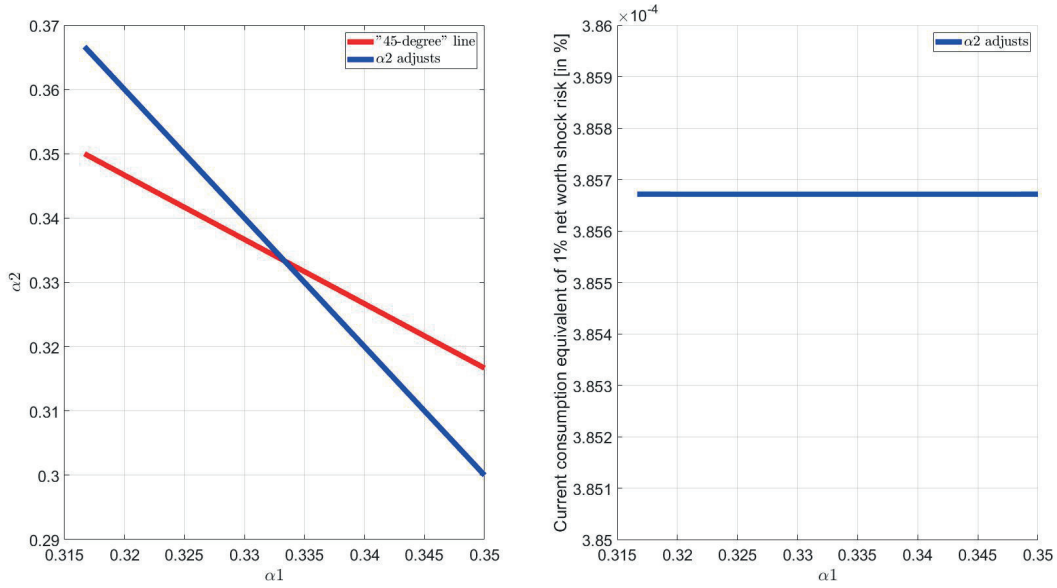
while the latter is defined as:

$$\mathbb{E}\left(W\right) = \frac{\left(c_{0,\bar{n}}\left(1 - \lambda\right)\right)^{1-\gamma}}{1 - \gamma} + \beta \frac{c_{1,\bar{n}}^{1-\gamma}}{1 - \gamma},$$

where $n_L = \bar{n} - \epsilon$ and $n_H = \bar{n} + \epsilon$. Equating these two formulas we find the consumption equivalent cost of business cycle, λ. It is constant for all $\alpha 1$ considered as the right panel shows. It means that shifting net worth between banks is unimportant for the dynamic outcomes. Thus, negative impact of net worth allocation found in the previous section does not result from heterogeneity in leverage but rather from changes in aggregate allocations.

3.3 Heterogeneity in the degree of intermediation friction in the 2-period model

Instead of shifting net worth to change the leverage, in this section we investigate welfare consequences of varying intermediation friction parameter. This is interesting because in the full general equilibrium model with endogenous net worth allocation (as opposed to the 2-period model in which it is exogenous) leverage heterogeneity may result from variations in these parameters.

As we show in section 2.2, gradually increasing (θ_D) we obtain stronger response of capital to net worth shock. Figure 9 shows that the relationship between changes in θ's and business

cycle fluctuations is monotonic. This is reflected in the welfare costs of the shock that rise with θ_D as presented in the right panel. After net worth shock, the deposit rate increases more when deposit banks are exposed more to the financial friction since households are more sensitive to the behaviour of riskier banks they lend to. Given fixed lending rate, this squeezes the interest rate spread after the shock making the economy more volatile. The centre panel illustrates, in turn, that leverage ratios in both banks stay constant when run-away parameters change. This results from two assumptions: 1) exogenous net worth level and 2) constant aggregate leverage across simulations which implies capital level to be constant as well.

Figure 9: Impact of the net worth shock on welfare in the 2-bank 2-period model when only θ's shift
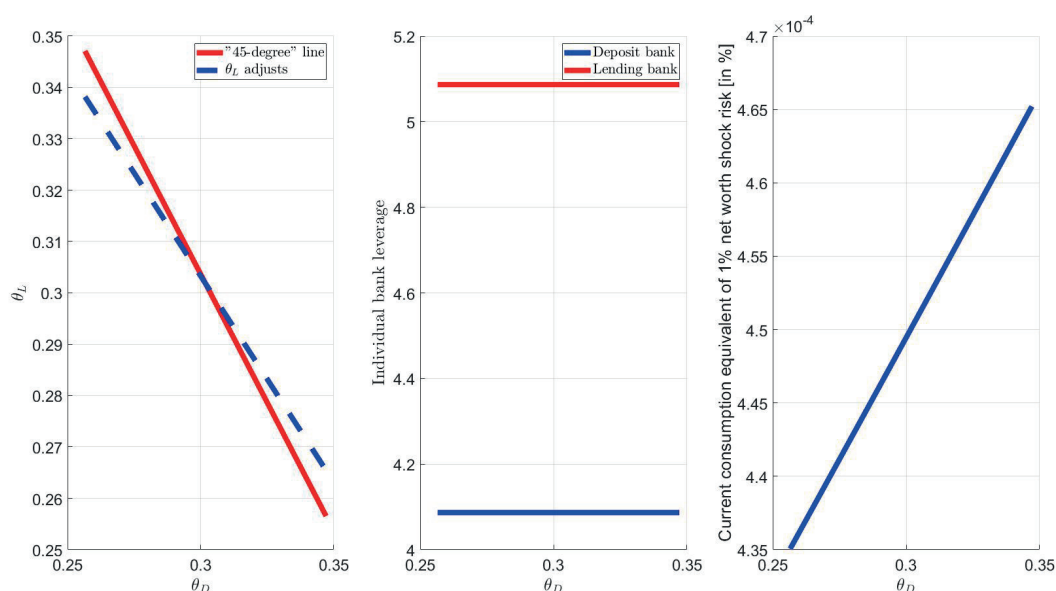
3.4 Importance of the vertical heterogeneity

We now turn to investigate how results reported in the previous section translate into a setup with three banks and whether heterogeneity in financial intermediation friction does influence the macroeconomic dynamics in contrast to heterogeneity in leverage stemming from net worth allocation that does not have such impact as presented in section 3.1 (recall Table 2 for the steady state allocations). To this end we analyze net worth shocks in the 2-period economy with three banks: deposit-taking, intermediary and lending. Net worth is allocated equally among banks. We simulate an increase in θ_D parameter that is compensated

(to keep the steady state allocation of main variables the same) either by adjusting θ_C and θ_L or just by θ_L. In this way we check whether concentration of financial friction across the banks matters. As previously, if the intermediation friction parameter of the deposit bank decreases, costs of the business cycle go down. The new insight is that this impact is stronger if intermediation friction of lending bank, θ_L, is the only one to adjust (figure 10).

Figure 10: Impact of the net worth shock on welfare when only θ's shift (2-period, 3-bank model)
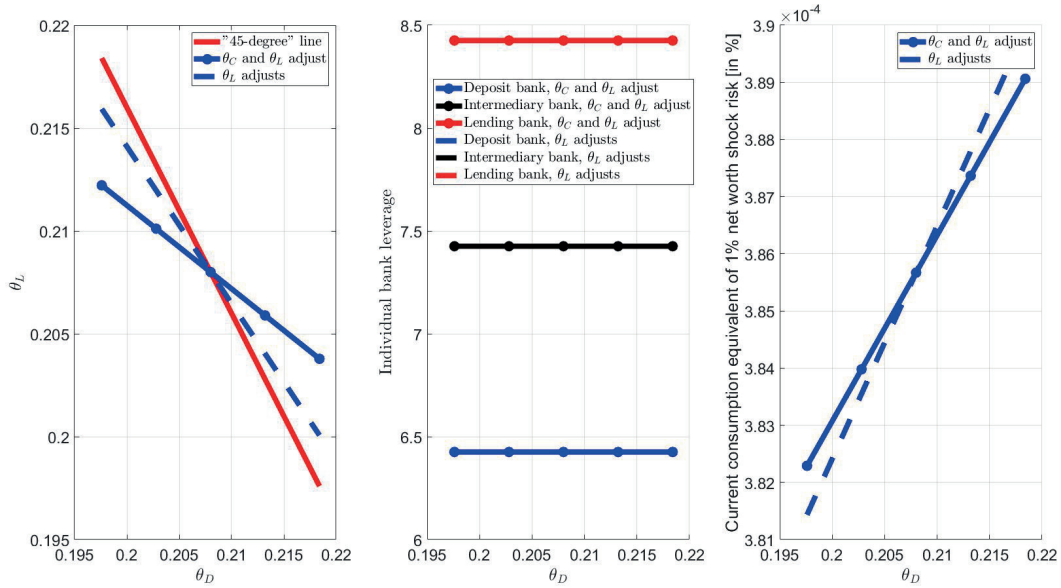
In order to understand these results better it is useful to consider the demand function for the 3-bank model analogous to the eq. 4:

$$k = \frac{R_d + (1 - \alpha_1)\,\theta_D + (1 - \alpha_1 - \alpha_2)\,\theta_C}{\theta_D + \theta_C + \theta_L - (R_k - R_d)} n. \tag{16}$$

As in the two-bank model, run-away parameters do not enter symmetrically into the function. On top of that, the formula for capital in the 3-bank model reveals that if θ_L is to adjust alone, it has to change more than in the case when both θ_L and θ_C shift (figure 10, left panel). The right panel of figure 10 reveals that when only θ_L adjusts welfare costs of fluctuation react more to shifts in θ_D. Since in this scenario financial friction of lending bank is stronger, it shows that the more banks are constrained earlier in the stack, the stronger dynamic amplification of the net worth shock.

3.5 Quantifying the vertical heterogeneity in leverage

In previous sections the distinction between the leverage and intermediation friction was clear-cut. This was possible since in the 2-period model, net worth was assumed to be exogenous and hence the leverage was not affected by the degree of the friction for a given bank. In order to make more realistic case in which the leverage depends on the degree of banks' exposure to financial frictions, we utilize the infinite horizon three-bank DSGE model as presented in Appendix A. In this framework the leverage depends on intermediation friction: the less constrained a bank is (the lower its θ), the more it can lend and the higher leverage it has. We introduce in the model three types of shocks: capital quality, interest rate and productivity calibrated to the shocks to their values as in Gertler and Karadi (2011). Using second-order model approximation around the steady state we calculate the one-period consumption cost of the business cycle, λ defined as:

$$\mathbb{E}W\left(C, L\right) = W\left(\left(1 - \lambda\right)\bar{C}, \bar{L}\right)$$

As previously, we find parameter pairs that leave the aggregated leverage unaffected. Left panel of figure 11 presents the trade-off between run-away parameters. It turns out, that in the general equilibrium framework θ_L and θ_D change one to one to keep the consolidated leverage constant. To understand why these parameters are more substitutable than in 2-period model recall eq. 16 and note that in the full model net worth allocations (depicted by α's in the formula) are endogenous. Thus, for concurrent adjustment in parameters there are additional degrees of freedom that may ensure the constant aggregate leverage ratio across simulations.

Comparing the left and middle panels of figure 11 reveals, as expected, that the lower the run-away parameter for a given bank is, the higher is the leverage of this bank. Right panel, in turn, shows that in the full model simulated with all shocks, costs of business cycle fluctuations turn out to be significant. Furthermore, it shows that the intuition we built in the previous sections still holds: the more constrained are the deposit banks, the more costly are business cycle fluctuations. This is straightforward from comparing figure 11 with its counterpart for two-period case (figure 10).

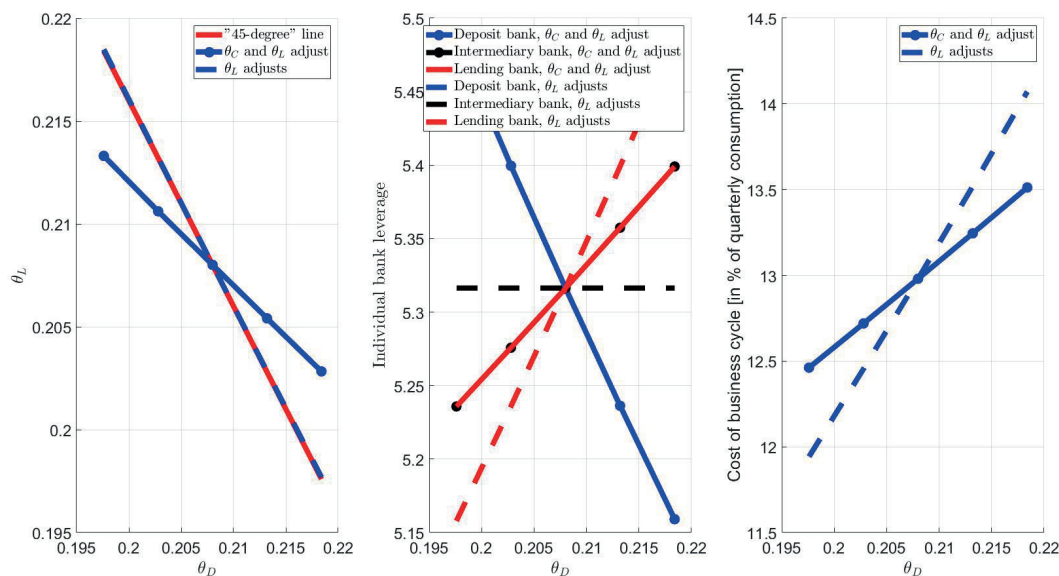Figure 11: Leverage heterogeneity in the infinite horizon three-bank DSGE model

Table 3: Intermediation friction and welfare

	$\theta_D = 0.208$ $\theta_C = 0.208$ $\theta_L = 0.208$	$\theta_D = 0.198$ $\theta_C = 0.213$ $\theta_L = 0.213$	$\theta_D = 0.198$ $\theta_C = 0.208$ $\theta_L = 0.219$
productivity	0.142	0.139	0.136
capital quality	12.76	12.24	11.73
monetary policy	0.163	0.159	0.155
all shocks	12.98	12.46	11.94

Note: all values denote consumption equivalent cost of business cycles

In order to show that the result presented above does not hinge on any specific shock calibration, table 3 presents welfare calculated for individual shocks under three parameter combinations. For all of them our results hold, i.e. increasing bank leverage further in the stack strengthens the banking sector amplification of structural shocks.

4 Policy implications

Since leverage heterogeneity may impact the costs of the business cycles, we investigate whether government may impact it and lower macroeconomic volatility by considering tax levied on a given type of banks. First, we build an intuition with a one-bank two-period model. Then, we simulate the general equilibrium model from the section under different tax parametrizations to assess its potential benefits.

4.1 Economic intuition

We consider an economy as in section 2.1 assuming additionally that the government may impose a proportional tax τ. Then, the bank problem is given by:

$$\max_{d} R_k k - R_d d,$$

$$\text{s.t.}$$

$$k = n + d,$$

$$(R_k k - R_d d)(1 - \tau) \geq \theta k.$$

We assume that the incentive constraint binds so we have that $(R_k k - R_d d)(1 - \tau) = \theta k$ and $R_k > R_d$. In that case, the demand curve is given by

$$k = \frac{R_d (1 - \tau)}{(1 - \tau)(R_k - R_d) - \theta} n,$$

If we substitute $\theta^* = \frac{\theta}{1-\tau}$ we see that by increasing the tax rate the policymaker effectively increases bank's run-away parameter. Thus, the policymaker controls the degree of intermediation friction and θ^* can be treated as a policy tool with capital demand given by:

$$k = \frac{R_d}{(R_k - R_d) - \theta^*} n,$$

Taxing bank's profits may be interpreted in broader terms - introducing more restrictive policy towards a given type of banks. Higher capital requirements, minimum LtV ratio or reserve requirements - just to name few - will have effectively the same impact as they lower bank's profitability.

4.2   Model simulations

Quantitative importance of banks' taxation is illustrated in figure 12. We levy a fiscal-neutral tax(subsidy) on lending and deposit banks according to the following rules:

$$\tau_t^L = \gamma_\tau \left(\Phi_{L,t} - \Phi_{L,t-1} \right)$$
$$\tau_t^D = -\tau_t^L \frac{K \left(R_{t+1}^k - R_t^{bcl} \right)}{B_t^{dc} \left(R_t^{dc} - R_t \right)};$$

Tax (subsidy) for the lending bank is compensated by subsidy(tax) for the deposit bank. The costs of the business cycle rise when tax policy shifts resources from the deposit bank to the lending one (figure 12).

Figure 12: Tax on banks and its impact on welfare in the three-bank DSGE model
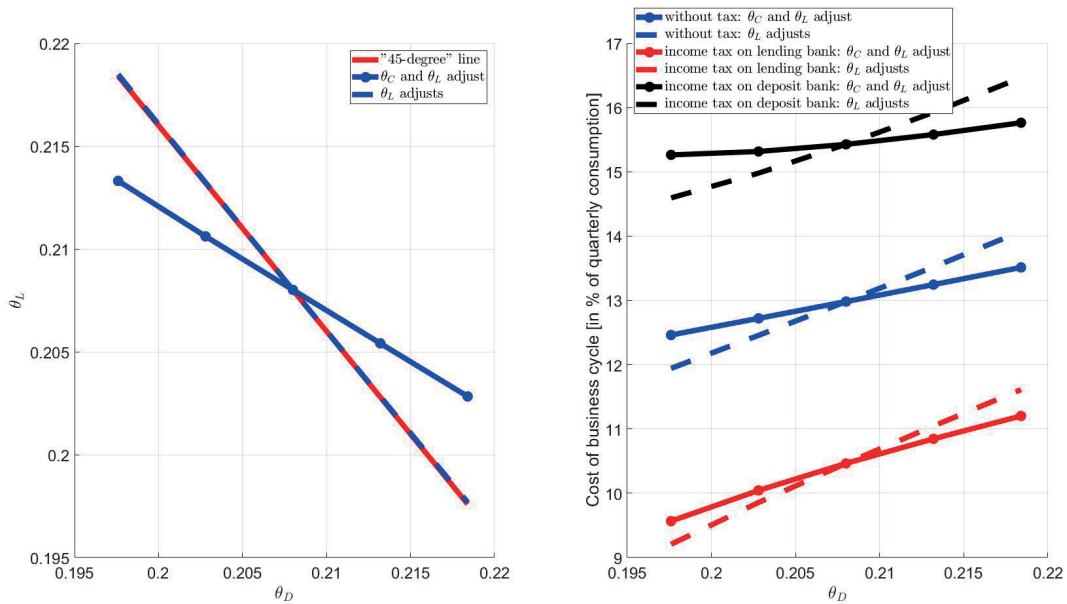
This is the case because positive tax rate on the deposit bank effectively increases its intermediation friction parameter as we showed in the previous section. At the same time the parameter for the lending bank is lowered. This is consistent with our previous results when we shifted financial intermediation friction from lending bank to deposit one obtaining higher costs of business cycles.

5   Conclusions

This paper argues that the complexity of banking system and bank heterogeneity in leverage have important business cycle implications. We utilize a general equilibrium framework with heterogeneous financial intermediaries and establish a number of novel results.

First, in the two-period partial equilibrium macroeconomic model we introduce a banking system that consists of many intermediaries that lend to each other (a bank stack). In this setup we show that individual bank leverage and macroeconomic volatility increase with the length of the linear bank stack.

Next, we analyze the impact of heterogeneity in bank leverage assuming empirically relevant banking system in which there are three types of intermediaries: deposit-taking, intermediary and lending. Keeping constant steady-state aggregate leverage in the banking system, we vary the tightness of the intermediation friction between individual banks both in a stylized two-period framework and a medium-size DSGE model. In both cases rising tightness for the deposit bank results in increasing macroeconomic volatility. In the DSGE model this outcome is reflected in the positive relationship between lending bank leverage and business cycle costs. Finally, we show that policymakers may mitigate this impact by taxing lending bank returns.

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Appendix

A DSGE model

This appendix presents the model applied in the quantitative analysis in section 3.5. It is a medium-scale New Keynsian Dynamic Stochastic General Equilibrium (DSGE) model with three types of financial intermediaries: deposit, intermediary and lending banks. These banks are linked with each other as presented in Figure 2. Otherwise the modelling framework follows Gertler and Karadi (2011). Below we present economic problems faced by agents and the calibration of parameters.

A.1 Households

Households consume, supply labor and save maximizing their lifetime utility:

$$\max_{\{C_t, L_t\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u\left(C_t, L_t\right),$$

where the period-utility function is given by

$$u\left(C_t, L_t\right) = \ln(C_t - hC_{t-1}) - \frac{\chi}{1+\varphi} L_t^{1+\varphi},$$

The maximisation problem is subject to the period-budget constraint

$$C_t + D_t \leq W_t L_t + \Pi_t + R_{t-1}^d D_{t-1}. \tag{17}$$

where D_t denotes deposits and R_t^d - the corresponding deposit rate set by the central bank. First order conditions are given by

$$\frac{\partial \mathcal{L}_t}{\partial C_t} = 0 \quad \Longleftrightarrow \quad \mu_t = \frac{1}{C_t - hC_{t-1}} - \beta h \frac{1}{C_{t+1} - hC_t},$$

$$\frac{\partial \mathcal{L}_t}{\partial L_t} = 0 \quad \Longleftrightarrow \quad \chi L_t^{\varphi} = \mu_t W_t,$$

$$\frac{\partial \mathcal{L}_t}{\partial D_t} = 0 \quad \Longleftrightarrow \quad \mu_t = \beta \mu_{t+1} R_t^d.$$

where mu_t denotes the Lagrange multiplier depicting the marginal utility of consumption.

A.2   Financial intermediaries

There are three types of financial intermediaries in the model: deposit, intermediary and lending banks. Below, we describe them.

Deposit banks   The deposit banks collects deposits from households and lends to the intermediary bank. Thus, its balance sheet is given by:

$$B_{DI,t} = N_{D,t} + D_t.$$

The bank faces an agency problem. It may divert a fraction θ_D of its assets if its higher than the continuation value. In order to guarantee its creditors that it will not do so, the bank has to satisfy the following incentive compatibility constraint

$$V_{D,t} \geq \theta_D B_{DI,t}$$

The bank's value function maximizes its expected terminal net worth. As the bank survives into the next period with probability σ, the value function is given by

$$\max_{B_{DI,t},D_t} V_{D,t} = \mathbb{E}_t \sum_{j=0}^{\infty} (1-\sigma)\sigma^{j-1}\Lambda_{t,t+j}N_{t+j}$$

If a bank survives to the next period, his net worth evolves as follows

$$N_t = R_{BI,t-1}B_{BI,t-1} - R_{D,t-1}D_{t-1}$$

The value function may be rewritten recursively:

$$V_{D,t-1} = \mathbb{E}_{t-1}\Lambda_{t-1,t}[(1-\sigma)N_t + \sigma \max_{B_{DI,t},D_t} D, t],$$

We guess its form:

$$V_t(B_{DI,t}, D_t) = V_{D,B_{DI,t}} B_{DI,t} + V_{D,D_t} N_t$$

where:

$$V_{I,B_{DI,t}} = \mathbb{E}_t \Lambda_{t,t+1} \Omega_{D,t+1} \left(R_{DI,t} - R_{D,t} \right),$$
$$V_{D,D_t} = \mathbb{E}_t \Lambda_{t,t+1} \Omega_{D,t+1} R_{D,t}.$$

and

$$\Omega_{D,t} \equiv (1 - \sigma) + \sigma \left(V_{B_{DI,t}} B_{DI,t} + V_{D_t} N_t \right),$$

The net worth of deposit banks follows:

$$N_{D,t} = \sigma(R_{BI,t} B_{BI,t-1} - R_{D,t-1} D_{t-1}) \varepsilon_t^N + \omega_{BI,t-1}$$

Intermediary banks Intermediary banks borrow from deposit banks and lend to lending banks. Their problems are analogous to deposit banks. Their balance sheet is given by:

$$B_{IL,t} = N_{I,t} + B_{DI,t}.$$

The incentive compatibility constraint:

$$V_{I,t} \geq \theta_I B_{IL,t}$$

The value function:

$$\max_{B_{IL,t}, B_{DI,t}} V_{I,t} = \mathbb{E}_t \sum_{j=0}^{\infty} (1 - \sigma) \sigma^{j-1} \Lambda_{t,t+j} N_{I,t+j}$$

If a bank survives to the next period, his net worth evolves as follows

$$N_{I,t} = R_{IL,t-1} B_{IL,t-1} - R_{BI,t-1} B_{BI,t-1}$$

The value function may be rewritten recursively:

$$V_{I,t-1} = \mathbb{E}_{t-1}\Lambda_{t-1,t}[(1-\sigma)N_{I,t} + \sigma \max_{B_{IL,t},B_{DI,t}} V_{I,t}],$$

We guess its form:

$$V_{I,t}(B_{IL,t}, B_{DI,t}) = V_{I,B_{IL,t}}B_{IL,t} + V_{I,B_{DI,t}}N_{I,t}$$

where:

$$V_{I,B_{IL,t}} = \mathbb{E}_t\Lambda_{t,t+1}\Omega_{I,t+1}\left(R_{IL,t} - R_{DI,t}\right),$$
$$V_{I,B_{DI,t}} = \mathbb{E}_t\Lambda_{t,t+1}\Omega_{I,t+1}R_{DI,t}.$$

and

$$\Omega_{I,t} \equiv (1-\sigma) + \sigma\left(V_{I,B_{IL,t}}B_{IL,t} + V_{I,B_{DI,t}}N_{I,t}\right),$$

The net worth of intermediary banks follows:

$$N_{I,t} = \sigma(R_{IL,t-1}B_{IL,t-1} - R_{BI,t-1}B_{BI,t-1})\varepsilon_t^N + \omega_{B_{IL,t-1}}$$

Lending banks   Lending banks borrow from intermediary banks and lend to firms. Their problems are analogous to other banks. Their balance sheet is given by:

$$Q_t K_t = N_{I,t} + B_{IL,t}.$$

The incentive compatibility constraint:

$$V_{L,t} \geq \theta_L Q_t K_t$$

The value function:

$$\max_{K_t B_{IL,t}} V_{L,t} = \mathbb{E}_t \sum_{j=0}^{\infty} (1-\sigma)\sigma^{j-1}\Lambda_{t,t+j}N_{I,t+j}$$

If a bank survives to the next period, his net worth evolves as follows

$$N_{L,t} = R_{IL,t}Q_tK_t - R_{IL,t-1}B_{BI,t-1}$$

The value function may be rewritten recursively:

$$V_{L,t-1} = \mathbb{E}_{t-1}\Lambda_{t-1,t}[(1-\sigma)N_{L,t} + \sigma \max_{K_tB_{IL,t}} V_{L,t}],$$

We guess its form:

$$V_{L,t}(K_tB_{IL,t}) = V_{L,K_t}Q_tK_t + V_{L,B_{IL,t}}N_{L,t}$$

where:

$$V_{L,K_t} = \mathbb{E}_t\Lambda_{t,t+1}\Omega_{L,t+1}\left(R_{K,t+1} - R_{IL,t}\right),$$
$$V_{L,B_{IL,t}} = \mathbb{E}_t\Lambda_{t,t+1}\Omega_{L,t+1}R_{IL,t}.$$

and

$$\Omega_{L,t} \equiv (1-\sigma) + \sigma\left(V_{L,K_t}Q_tK_t + V_{L,B_{IL,t}}N_{L,t}\right),$$

The net worth of intermediary banks follows:

$$N_{L,t} = \sigma(R_{IL,t}Q_tK_t - R_{IL,t-1}B_{BI,t-1})\varepsilon_t^N + \omega_{Q_tK_t}$$

A.3 Firms

We consider two stages of the prodcution process. Competitive firms produce intermediate goods using labor and capital and sell it to retail firms that mark goods and sell them in monopolistically competitive market.

Production firms Production firms borrow capital from lending banks and combine it with labor to produce intermediate goods according to the Cobb-Douglas production function:

$$Y_{m,t} = A_t\left(U_t\xi_t^KK_t\right)^\alpha\left(L_t\right)^{1-\alpha}$$

where A_t denotes productivity, U_t - capital utilization, ξ_t^K - capital quality shift that allows to obtain in the model parsimonious exogenous volatility in the value of capital. Firms borrow capital at the end of a period t, produce, and sell undepreciated capital at the beginning of period $t+1$. Subject to the production function, firms maximize profits given by the following formula:

$$\Pi_{m,t} = Y_{m,t} P_{m,t} + Q_{t+1} K_t \xi_t^K - W_t L_t - R_t^K Q_{t-1} K_t - K_t \delta(U_t) \xi_t^K$$

where $P_{m,t}$ denotes the price of intermediate goods.

They act in a perfectly competitive market choosing capital used in the period $t+1$, utilization rate U_t and labor demand according to first order conditions:

$$w_t = P_{m,t}(1-\alpha)\frac{Y_t}{N_t}\xi_t^K K_t$$

$$P_{m,t}\alpha\frac{Y_t}{U_t} = \delta'(U_t)$$

$$R_{t+1}^K = \frac{\left[P_{m,t+1}\alpha\frac{Y_{t+1}}{\xi_{t+1}^K K_{t+1}} + Q_{t+1} - \delta(U_{t+1})\right]\xi_{t+1}^K}{Q_t}$$

Capital producers   Capital producers buy capital from final goods producers at price 1, invest it and sell newly created at price Q_t. Profits are transferred to households. Thus, capital producers maximize:

$$\max_{\{I_{n,t}\}_t^\infty} \mathbb{E}_t \sum_{j=0}^\infty \beta^{t+j}\Lambda_{t,t+j}\left[(Q_t-1)I_{n,t} - f\left(\frac{I_{n,t}+I_{ss}}{I_{n,t-1}+I_{ss}}\right)(I_{n,t}+I_{ss})\right],$$

where: f is a function such that: $f(1) = f'(1) = 0$, $f''(1) > 1$, while I_t denotes investments in period t, while $I_{n,t}$ - net investments. It follows that:

$$I_{n,t} = I_t - \delta(U_t)\xi_t^K K_t$$

where capital is accumulated as follows:

$$K_{t+1} = I_{n,t} = K_t \xi_t^K + I_{n,t}$$

The first order condition is given by:

$$Q_t = 1 + f\left(\frac{I_{n,t}+I_{ss}}{I_{n,t-1}+I_{ss}}\right) + \frac{\partial f\left(\frac{I_{n,t}+I_{ss}}{I_{n,t-1}+I_{ss}}\right)}{\partial I_{n,t}}\left(\frac{I_{n,t}+I_{ss}}{I_{n,t-1}+I_{ss}}\right) - \mathbb{E}_t\beta\lambda_{t,t+j}\frac{\partial f\left(\frac{I_{n,t+1}+I_{ss}}{I_{n,t}+I_{ss}}\right)}{\partial I_{n,t}}\left(\frac{I_{n,t+1}+I_{ss}}{I_{n,t}+I_{ss}}\right)^2$$

Retailers Retailers buy intermediate products from production firms, differentiate them and set their price $\tilde{P}_{F,t}(i)$ in a monopolistically competitive market in order to maximize:

$$\max_{\tilde{P}_t,\{y_{F,t}\}_{s=0}^{\infty}} E_t\sum_s (\beta\theta_F)^s \Lambda_{t,t+s}\left(\frac{\tilde{P}_{F,t}\pi^{\gamma}_{t,t+s}}{P_{t+s}} - P_{m,t+s}\right)Y_{F,t+s}$$

subject to the demand from users of final output:

$$Y_{F,t} = \left(\frac{\tilde{P}_{F,t}}{P_t}\right)^{\frac{\mu}{1-\mu}}Y_t$$

where $\pi^{\gamma}_{t,t+s} = \pi^{\gamma}_{t+1}\cdot\ldots\cdot\pi^{\gamma}_{t+s}$. Solving the maximization problem we get the following condition:

$$\tilde{P}_{F,t} = \mu\frac{\Omega_t}{\Upsilon_t}$$

where:

$$\Omega_{F,t} = \Lambda_t P_{m,t}p_{H,t}^{\frac{\mu}{\mu-1}}Y_{F,t} + \beta\theta_F E_t\left(\frac{\pi^{\gamma}_{t+1}}{\pi_{t+1}}\right)^{\frac{\mu}{1-\mu}}\Omega_{F,t+1}$$

and

$$\Upsilon_{F,t} = \Lambda_t P_{F,t}^{\frac{\mu}{\mu-1}}Y_{F,t} + \beta\theta_H E_t\left(\frac{\pi^{\gamma}_{t+1}}{\pi_{t+1}}\right)^{\frac{1}{1-\mu}}\Upsilon_{F,t+1}$$

Finally, given that fraction $1 - \theta_F$ firms reset their prices, price aggregator is given by:

$$P_t^{\frac{1}{1-\mu}} = \theta_F\left(P_{F,t-1}\pi_t^{\gamma}\right)^{\frac{1}{1-\mu}} + (1-\theta_F)\left(\tilde{P}_{F,t}\right)^{\frac{1}{1-\mu}}$$

A.4 Goods market clearing

Integrating output over firms gives:

$$Y_{F,t}\Delta_{F,t} = Y_{m,t}$$

where $\Delta_{F,t}$ measures the price dispersion:

$$\Delta_{F,t} = \int_0^1 \left(\frac{P_{F,t}(i)}{P_{F,t}}\right)^{\frac{\mu}{1-\mu}} di = \left(\frac{P_{F,t}}{P_{F,t-1}}\right)^{\frac{\mu}{\mu-1}} \theta_F \Delta_{F,t-1} \left(\frac{\pi_t^{\gamma}}{\pi_t}\right)^{\frac{-\mu}{\mu-1}} + (1-\theta_F)\left(\frac{\tilde{P}_{F,t}}{P_{F,t}}\right)^{\frac{-\mu}{\mu-1}}$$

Finally, monetary policy is given by a simple Taylor rule:

$$i_t = i_{t-1}^{\gamma_r} \left(\pi_t^{\gamma_\pi}\left(\frac{Y_t}{Y}\right)^{\gamma_y}\right)^{(1-\gamma_r)} \exp \varepsilon_t^r$$

and from Fisher equation we have:

$$i_t = \mathbb{E}_t\left[R_{t+1}\pi_{t+1}\right]$$

A.5 Exogenous processes

As explained in the main text, there are four shocks in the model: productivity ε_t^A, capital quality ε_t^K, net worth ε_t^N and interest rate ε_t^r. The latter two are assumed white noise, while remaining two enter the autoregressive processes:

$$A_t = A_{t-1}^{\rho_A} \exp \varepsilon_t^A$$

$$\xi_t^K = \xi_{t-1}^{K}{}^{\rho_K} \exp \varepsilon_t^K$$

A.6 Calibration

Table 4 presents calibrated parameters in the DSGE model that to large extent follow Gertler and Karadi (2011). The only parameters that we modify are associated with banking sector. First, we set run-away parameters $(\theta_L, \theta_I, \theta_D)$ at the same level as in the two-period model,

i.e. 0.208 (see also table 1). Second, we set the proportion of transfer to entering bankers to bank assets, ω, at 0.003 to reduce the difference in moments between first order approximation around the steady state of our model and Gertler and Karadi (2011).

Table 4: Calibrated parameters

Households

| β | 0.99 | h | 0.81 | χ | 3.41 | φ | 0.28 | | |

Financial intermediaries

| θ_L | 0.21 | θ_I | 0.21 | θ_D | 0.21 | σ | 0.97 | ω | 0.003 |

Firms

| α | 0.33 | $\delta(U)$ | 2.50% | μ | 1.32 | θ_F | 0.78 | γ | 0.24 |

Government

| $\frac{G}{Y}$ | 0.2 | γ_π | 1.50 | γ_r | 0 | γ_y | -0.12 | σ_r | 0.01 |

Shocks

| ρ_k | 0.66 | σ_k | 0.05 | ρ_A | 0.95 | σ_A | 0.01 | σ_N | 0.01 |

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