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Contents

Abstract

We use a wide range of models (time series, semi-structural and structural) to estimate, project and explain the evolution of the natural rate of interest (NRI) in Poland (PL) and in the euro area (EA). Our findings are as follows: (i) the NRIs declined significantly in both PL and the EA over the last two decades, (ii) our most recent short-term estimates point towards a mildly negative NRI in PL and appreciably more negative NRI in the EA, (iii) our most recent medium-term estimates point towards a mildly positive NRI in PL and a slightly negative NRI in the EA, (iv) the NRI in PL remained consistently above the NRI in the EA by an average of 2-3 pp, (v) the main drivers of the declining NRIs were demographics and the worldwide productivity slowdown, (vi) the main driver of the higher NRI in PL than in the EA was the productivity catch-up in PL, (vii) we expect the NRI in PL to significantly converge to the EA level by the mid-2030s.

Keywords: natural rate of interest, monetary policy, Poland, euro area.

JEL codes: E43, E52, E58.

1. Introduction

The natural rate of interest (NRI) is a central concept in modern monetary economics. Introduced more than a hundred years ago by Knut Wicksell, it has evolved since and constitutes one of the cornerstones of contemporaneous central banking. What is the NRI and why do we think that knowing its level is important? What are its strengths and weaknesses? What do we find in this paper? What did other economists find out about the NRI and how does our paper contribute to the field? These are the questions that we answer in this and the next sections.

The natural rate of interest

Wicksell (1898) defined the natural rate as the real interest rate that stabilizes the price level. “There is a certain rate of interest on loans which is neutral in respect to commodity prices,” he wrote, and “tends neither to raise nor to lower them. […] It comes to much the same thing to describe it as the current value of the natural rate of interest on capital.”

Simultaneously, he presented his view on the consequences of not adjusting the market interest rate to the natural rate. Wicksell wrote about a “cumulative process”: should the market interest rate fall below the natural rate, investments would exceed savings. As a consequence, the economy would expand, prices rise and the real interest rate would fall, further stimulating the process.

Wicksell’s concept remained on the peripheries of economics for the next 100 years. It returned into the mainstream in the 1990s, with the widespread introduction of inflation targeting by central banks. Under this monetary policy strategy, interest rates are the central banks’ instruments, while inflation is the target. With a minor modification, the Wicksellian concept of the natural rate (redefined as the real interest rate that stabilizes inflation) takes a central stage in modern central banking. If known, it allows the central bank to stabilize, raise or lower inflation by setting the policy rate at, below or above the NRI, respectively. It proves particularly useful after periods of disinflation, when policy rates need to be returned to neutral in order to avoid deflation (if kept too high for too long) or a rebound of inflation (if lowered too much). As the central banks in Poland and in the euro area combat inflation at the time of writing this paper, we hope that our estimates can play a helpful role in restoring a balanced macroeconomic path.

Like any other economic concept, the NRI has strengths and weaknesses. Where do we see them? Let us start with the former. First, the concept organizes thinking about monetary policy, and in particular about its stance. Paraphrasing Robert Lucas Jr., once you start thinking about monetary policy through the lens of the NRI, it is hard to think about anything else. Second, in spite of the NRI being an unobservable variable, we have modelling techniques that allow for its estimation. We also have models that offer explanations for its evolution.

What are the weak sides of the NRI? We see two. First, since the variable is not directly observable, its estimation is subject to uncertainty. Our econometric estimates presented in Section 4 account for confidence bands and these can amount to more than 4 pp at the end of the sample. This is a lot and clearly reduces the applicability of the NRI in day-to-day policymaking.

Second, while so far, we have referred to a particular definition of the NRI (the real interest rate that stabilizes inflation), the practice is more nuanced. Several approaches to defining (and as a result calculating or estimating) the NRI coexist in the literature. For instance, in structural models the NRI is usually defined as the real interest rate that would prevail under flexible prices and wages. Other papers approach the NRI as the interest rate that keeps the output gap (itself another unobservable object without a unique definition) at zero and inflation stable. Statistical approaches to estimating the NRI often equalize it with a medium- or long-run trend in real rates.

The multitude of definitions and related estimation approaches are sometimes considered the main drawback of the NRI concept. We decided to turn this weakness into its strength. Instead of concentrating on one particular approach to estimating the NRI, we apply a wide range of economic and econometric models. This allows to check which features of the estimated NRI are robust.

We believe that our findings are interesting mainly if looked-upon jointly. Only then are we able to see the robust features of the NRI, but also understand where the differences stem from. In what follows, we offer a bird's eye view of our main findings, concentrating on the main common features of the estimated NRIs and explaining the differences encountered.

Overview of the results

We apply a wide range of models: time-series, semi-structural models and structural models, to estimate the NRI in Poland and in the euro area.

The most data-driven approach is based on time series decomposition into trend and stationary components in the spirit of Del Negro, Giannone, Giannoni and Tambalotti (2017), henceforth DGGT. We pick a representative open economy model of the interest rates term-structure to be presented in the section. The very nature of the decomposition is such that, in principle, the model ignores any structural macroeconomic relationships linking, for example, the NRI, the real interest rate and inflation. As a consequence, the estimated NRI likely reflects the financial markets' assessment of the natural rate.

Semi-structural approaches follow the papers by Holston, Laubach and Williams (2017), henceforth HLW, and by Brand and Mazelis (2019), henceforth BM. These models assume a backward-looking investment-savings (IS) curve and either accelerationist or partially forward-looking Phillips curves, as well as a relationship between the NRI and the potential growth rate of the economy. All relationships are loosely connected to standard micro-founded models, in particular the New Keynesian framework. The framework of Brand and Mazelis (2019) assumes additionally that the interest rate is driven by a Taylor-type monetary policy rule.

This assumption brings the model even closer to the New Keynesian framework. Importantly, this seemingly minor modification is far from innocent from a practical perspective. While the three relationships, described before, provide structural links between the NRI, the interest rate, output and inflation, the monetary policy rule adds the central banks' assessment on the likely level of the NRI to the econometricians' information set.

Our structural approach is based on a large quantitative life-cycle model of the euro area and Poland in the vein of Bielecki, Brzoza-Brzezina and Kolasa (2020), henceforth BBK. The model makes use of granular asset and demographic structures that are calibrated to reflect the main features of household balance sheets, fertility and longevity in the EA and PL. Moreover, we take into account past and expected future developments in technology, which in particular reflect the rapid catching-up process that Poland underwent during the last 30 years, but also its expected slowdown in the future as the convergence process slowly matures. The NRI calculated using this framework is driven by demographic and technology processes, and in contrast to the previous approaches, neither reflects assessments of financial markets and central banks, nor is affected by business cycle developments.

In what follows, we will briefly present our main findings in three dimensions. First, we show the estimates of the NRIs. Then, we discuss the divergence between the natural rates in Poland and in the euro area. Last but not least, we explain the main determinants which – in light of the structural model – underlie the trends of the NRIs. Details and more findings are presented in the next sections.

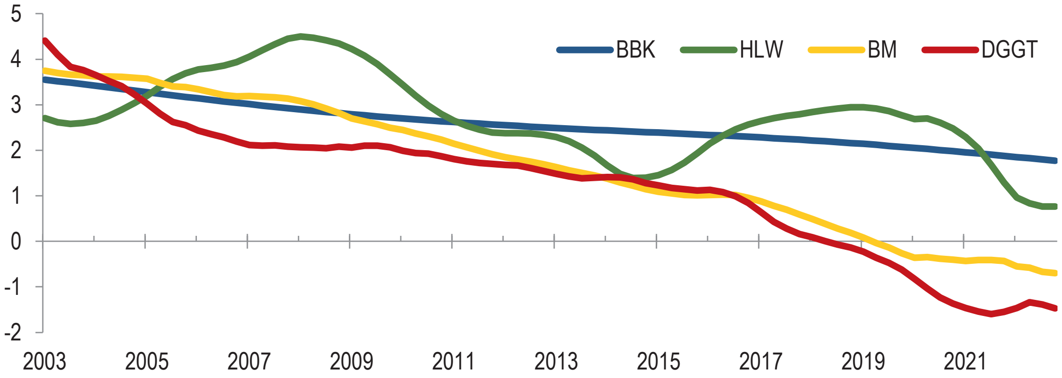
Figure 1 shows selected estimates of the NRI in PL in 2003Q1-2022Q4 (the common part of our samples). Two findings stand out. First, irrespective of the model, a clear, downward trend of the NRIs is visible. While in the beginning of our sample the natural rates vary between 2.7% and 4.5%, by the end of our sample these readings are much lower, between -1.5% and 1.8%. A similar tendency is clearly visible for our estimates in the EA (Figure 2). Our findings are consistent with much of the literature discussed below, which documents a substantial decline of natural rates both in advanced and emerging market economies.

Second, the estimates of downward trends in the NRIs are clustered into two groups. The NRIs from the HLW and BBK model tend to move together, with the HLW measure oscillating around the BBK which stems from the medium-term fluctuations in the GDP trend growth rate. The NRIs from the remaining two models, i.e. BM and DGGT, trend downwards at much faster pace, and diverge vividly from the first group. The DGGT measure persistently points toward comparatively lower levels of the NRI since 2005, and is joined since 2013 by the BM measure, coinciding with the period of stubbornly low inflation (and deflation) in Poland and the EA in the middle of the previous decade. The divergence between two groups of estimates becomes statistically significant in 2015 and grows wider in the last part of our sample.

It should be stressed that the BM and DGGT estimates take into account the assessment of the likely level of the NRI by the central bank and financial market participants, respectively, who additionally take into account the influence of economic forces that are largely outside of the information set of the NRI models based on medium- or long-run trends, i.e. HLW and BBK. These additional factors include, for example, the effects of the sovereign debt crisis for the weaker aggregate demand in the euro area, or the disturbances due to the Covid pandemic and the Russian aggression on Ukraine.

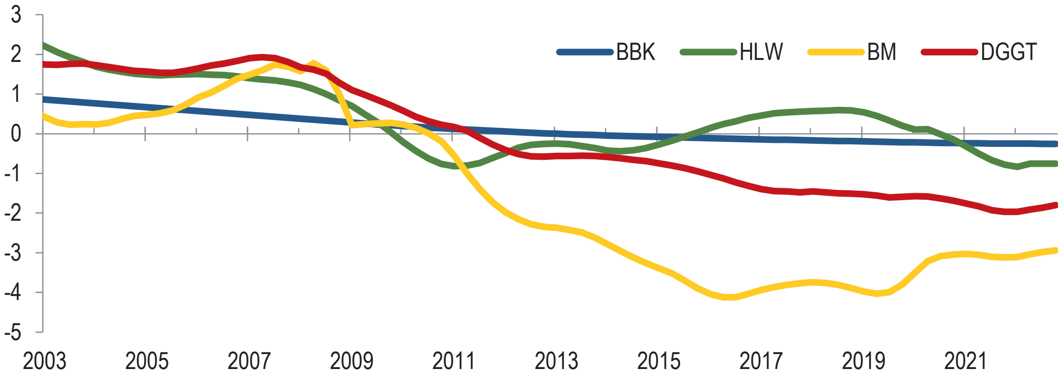
Our third conclusion results from comparing the NRIs in PL and in the EA. We plot the differences between them in Figure 3. It is clear that, in spite of much variability, the difference is one-sided. All models in all periods show a higher NRI in Poland. The average spread fluctuates between 2% and 3% and shows no trend throughout the sample period.

Figure 1 Estimates of the NRI in Poland (main results)



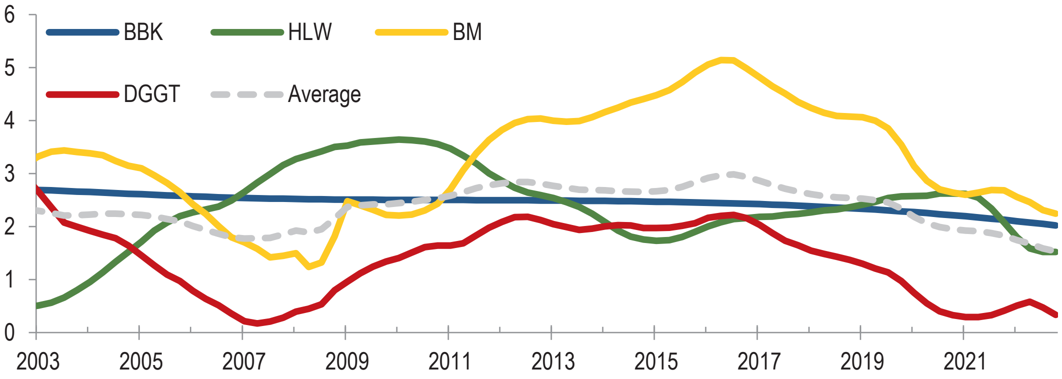
Source: Own calculations.

Figure 2 Estimates of the NRI in the euro area (main results)



Source: Own calculations.

Figure 3 Difference between NRI estimates in Poland and the euro area



Source: Own calculations.

What explains our main findings, in particular the downward trends in NRIs and the consistently higher NRI in Poland? Regarding the downward trend in natural rates, our structural BBK model points to two main groups of factors. The first is demographics. Trends in the euro area and in Poland are similar: fertility rates are low and life expectancy increases. These factors contribute to higher per capita asset holdings and, as a consequence, lower the equilibrium real rate.

The second factor that contributes to the downward trend is the worldwide slowdown in productivity growth. Here, the tendencies in Poland and in the euro area differ (on which below), nevertheless, as Poland is open to capital flows, the trend in the euro area contributes to lowering the NRI in Poland as well.

Differences in productivity play the leading role in explaining the consistently higher NRI in Poland. Over the last 30 years Poland has been catching up with the euro area. GDP per capita increased from 38% of the euro area average in 1995 to 77% in 2022. The fast pace of productivity improvements, instrumental in this catch-up process, raised the equilibrium interest rate in Poland by approximately 2 pp above the euro area level.

Last but not least, beyond explaining the past, we intend to say something meaningful about the future. From the policymakers' point of view, what matters mostly is the current and the future level of the NRI. We convey two messages. The first relates to the NRI in the near future that might be of interest to policymakers as they attempt to combat inflationary pressures. Our range of estimates at the end of the sample is quite wide, and clearly it matters whether, for example, in Poland the NRI is +1.8% (as suggested by the BBK model) or within the -3.4% to 0.3% interval (as suggested by the DGGT model). While it is difficult to judge unequivocally, it seems likely that the factors that have recently pushed the estimates from the BM and DGGT models below those from BBK and HLW have largely dissipated, and new factors (higher fiscal expenditures, potential AI revolution), which have not yet fully entered into the information set of our models, point in an upwards direction.[1]

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[1] See also a recent debate between Blanchard and Summers (2023).

The second message is about future developments. Forecasting the natural rates 10 or 20 years ahead is very difficult but we believe that two relatively robust conclusions can be made. First, demographic forces (which are to a large extent predetermined over this horizon[2]) will continue to push the long-run trend of NRIs downwards, both in the euro area and in Poland. Second, we should observe a gradual narrowing of the NRI gap between PL and the EA as the productivity gap will slowly disappear. Our projection is that the NRI gap will close by the mid-2030s to a significant degree. This has profound implications for Polish monetary policy in the future. It means that there will be stronger downward pressure on the NRI in Poland, to which monetary policy will have to adjust. It also means that the probability of hitting the effective lower bound on interest rates will gradually increase in Poland.

Having discussed the main findings, let us now move to a brief overview of the relevant literature and a discussion of our contribution.

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[2] Bielecki et al. (2022) argue that the impact of migration flows on the NRIs in the near term is very limited.

2. Overview of the literature and our contribution

As already mentioned, the concept of the NRI rate came to prominence in the work of Knut Wicksell (1898), and it has been formalized in the context of modern macroeconomic theory by Michael Woodford (2003). The natural interest rate, also referred to as the neutral or equilibrium interest rate, is an unobservable variable and several approaches have been developed to estimate it.

Among them, the one proposed by Laubach and Williams (2003) belongs to the most commonly used. The model builds on equations for the IS curve and the Phillips curve. It assumes that the NRI is driven by the trend growth rate of output and other factors, such as households' rate of time preference. Laubach and Williams (2003) estimate jointly the NRI and potential output over 40 years of US data using the Kalman filter and find that the NRI varied significantly over 1961 and 2002 in the United States, with variation on the trend growth rate being an important determinant. In a follow-up paper, Holston et al. (2017) estimate a version of the Laubach-Williams (2003) model using data for the United States, Canada, the euro area and the United Kingdom and provide evidence of a downward trend in NRI over 1990-2016. Substantial co-movement of country-by-country estimates over time suggests an important role for global factors in determining the NRIs.

Several papers have used the Laubach-Williams' method and its modifications to estimate the NRI for various countries. In particular, Armelius et al. (2018) find that the NRI in Sweden fell from 3% at the end of 1995 to -1.8% at the beginning of 2017. The Swedish NRI was influenced in both the short- and long-run by fluctuations in the US NRI. In turn, Fujiwara et al. (2016) show that the NRI in Japan began to decline in the 1990s, and fell below zero during Japan's financial crisis of the late 1990s, and during the global financial crisis in 2008, and was about 0% from 2010 to 2016Q1, after the introduction of "Quantitative and Qualitative Monetary Easing (QQE) with a Negative Interest Rate" by the Bank of Japan. Arena et al. (2020) find that the NRIs declined in virtually all European countries between 2000 and 2019, with some modest recovery post-euro area sovereign debt crisis, starting in 2012. A significant part of that decline is accounted for by a fall in potential growth. Still, a wedge between the NRIs and potential growth rates remains, which can be partly attributed to external factors.

Another approach to estimate the NRIs is a time series decomposition. Del Negro et al. (2017) estimate a vector autoregression (VAR) with common trends for the US between 1960s and 2010s. In line with the ‘global saving glut’ hypothesis, they find a decline of the low-frequency component of the NRI in the United States since the late 1990s on account of a rising convenience yield for safe and liquid assets, and to a lesser extent, lower economic growth. In Del Negro et al. (2019), the analysis is extended to an open economy setup and applied to seven advanced economies dating back to 1870. It is documented that since the late 1970s the trend in the world real interest rate essentially coincides with that of the US. Cesa-Bianchi et al. (2022) use the approach developed by Del Negro et al. (2019) to obtain the estimates of the global equilibrium real interest rate in a panel of 31 countries from 1950 to 2015. In their simulation, the global equilibrium real interest rate rises from 1.25% in the mid-1950s to around 2.75% in the mid-1970s, declining steadily thereafter, reaching -0.25% in 2015.

Finally, the NRI can be calculated using a structural model. Krueger and Ludwig (2007) use a multi-country OLG model and predict a fall in the world interest rate due to demographic transition countries by approximately 1 pp between 2000 and 2060. Carvalho et al. (2016) calibrate the Blanchard-Yaari model to the average of several developed countries and simulate a more significant decline of the equilibrium interest rate in a shorter horizon (1.5 pp between 1990 and 2014). Bielecki et al. (2020) construct an open economy life-cycle model and investigate the impact of demographics on the NRI in the euro area. Demographic forces explain a decline of the NRI by about 2 pp between 1985 and 2030, of which about two-thirds can be attributed to population aging. In addition, the authors document that openness to international capital flows was not important in driving the real interest rate in the euro zone since the aging in the euro area was similar to that in the group of other advanced economies. Eggertsson et al. (2019) use a 56-period life cycle model calibrated to match the US economy in 2015 and calculate an NRI ranging from -1.5% to -2.2%. The main drivers of negative NRIs are an aging population, low fertility, and sluggish productivity growth. The main factor that has tended to counterbalance these forces is an increase in government debt. Finally, Platzer and Peruffo (2022) develop a heterogenous agent, overlapping generations model with non-homothetic preferences. The model can account for a 2.2 pp decline in NRI in the United States between 1975 and 2015. The decline in the NRI is mostly due to the slowdown in total factor productivity growth, demographic factors, namely a decline in fertility and a rise in life-expectancy, as well as rising income inequality. Growing public debt is, again, the major counteracting force.

Compared to the US or euro area, studies of the NRI for Poland are scarce. Brzoza-Brzezina (2006), using the Kalman filter and a structural VAR model, estimates the NRI for Poland between 1998 and 2003 and documents that the NRI averaged 4.6-5.0% over the analysed 6 years, showing relatively high variability. High productivity growth, high exchange rate risk and persistent budget deficits were the most probable causes of the high NRI in Poland at that time.

In turn, Stefański (2018) investigates the NRI in three Central and Eastern European economies: Poland, Czechia and Hungary, as well as the euro area between 1996 and 2017. He uses an open economy New Keynesian model of Galí and Monacelli (2005) as a base to derive an extended, open economy version of the Laubach-Williams framework. Stefański (2018) documents that the NRI fell after the financial crisis, but rebounded afterwards, still remaining lower than in the pre-crisis peak (in Poland and Czechia by 4-5 pp).

Last but not least, Bielecki et al. (2022) use an OLG model with nominal frictions to investigate how aging and migration affect the Polish economy, in particular the NRI. They show that the decline in the NRI due to demographic processes is substantial, amounting to around 1.5 pp, albeit spread over a period of 40 years. The impact of migration flows is relatively small and cannot significantly alleviate the downward pressure on the NRI induced by populating aging.

To generalize and conclude, different studies, with respect to methods and countries covered, find a decline in the NRIs over the recent decades. Estimates of the NRI tend to be imprecise and subject to real-time measurement error. Finally, changes in the NRI are driven by both macroeconomic factors, such as productivity growth, demographics, fiscal policy, and inequality, as well as by financial factors, such as international capital flows, supply of and demand for safe and liquid assets.

How is our paper related to the existing studies and where do we see our contribution? The models we use are not novel and have been used before. However, as already mentioned, the original models have often been developed for closed economies. Our paper extends them consistently, so that they reflect the open economy feature of Poland.

In the time series model setup as in Del Negro et al. (2017), we consistently estimate the NRI in Poland and in the euro area from a bunch of different interest rates (home and foreign interest rates, including policy rates, short- and long-term government bonds). Following Del Negro et al. (2019), we use an assumption of uncovered interest rate parity to compare the investment in Treasury securities of Poland and euro area, both denominated in euro. On a quarterly basis we are able to decompose the changes in the nominal interest rate into the NRI, the cyclical part of the interest rate and long-run trends in the inflation rate. At the same time, we also provide valuable information on current fluctuations in convenience yield on government bonds, and time premia.

In, both time series and semi-structural approaches to NRI measurement, we use the most up-to-date estimates of policy interest rates for the periods of quantitative easing. Thus, we follow the current research trend of incorporating into the analysis the shadow rates when the economy is within its zero lower bound (ZLB).

Additionally, we propose a tractable and relatively prior-free method of dealing with the very unusual sequence of shocks affecting the economies starting from early 2020, which make estimating the standard HLW-like models quite tricky at the end of the current sample.

In our structural life-cycle model we treat Poland as a small open economy interacting with the large closed economy of the euro area. A novel feature employed in this paper is to allow both regions to systematically differ in their productivity levels, while the small open economy gradually catches-up and slowly closes the historical productivity gap. This catch-up process turns out to be crucial in explaining the systematic differences between NRIs in Poland and in the euro area over our sample period.

Moreover, existing studies usually apply a single method to estimate the NRI. In contrast, we use a wide spectrum of methods, thus allowing to draw conclusions that are robust to such a test. Last but not least, as evidenced above, the NRI has been only scarcely covered for Poland. Our study provides policymakers in Poland and in the euro area with up-to-date, robust knowledge about one of the most important objects in monetary economics.

3. Models

In this section we present the main features of our modelling frameworks: the time series models, the semi-structural models and the structural model. A more detailed description of the models and the data used can be found in the Appendix.

Time series models

In the time series approach we use non-structural models to disentangle common trends from a number of financial market time series. Hence, we decompose each of n quarterly times series, collected in a row vector y_t, into their long-run trend, $\bar{y}_t$, and transitory components, $\tilde{y}_t$. As in the model of Del Negro et al. (2017), each of the latent variables is generated from either a multivariate random walk or a stationary VAR and has a state space representation:

$$y_t = \Lambda\bar{y}_t + \tilde{y}_t, \qquad \text{measurement equation} \qquad (1)$$

$$\bar{y}_t = \bar{y}_{t-1} + e_t, \qquad \text{transition equation for trend components} \qquad (2)$$

$$\tilde{y}_t = \Phi^{-1}(L)\varepsilon_t, \qquad \text{transition equation for transitory components} \qquad (3)$$

where y_t is an $n \times 1$ vector of observables, both $\bar{y}_t$ and $\tilde{y}_t$ are row vectors of latent variables with q and n elements, respectively, Λ is a $n \times q$ selection matrix with all elements restricted to be zero or one, and $\Phi(L) = I - \sum_{l=1}^{p} \Phi_l L^l$ is an inversible lag polynomial of order p. In this framework, $q \leq n$, the rank of Λ determines the number of common trends in observable variables, while $n - q$ is the number of cointegrating relationships in the system. The $q + n$ shocks are identically normally distributed:

$$\begin{bmatrix} e_t \\ \varepsilon_t \end{bmatrix} \sim N\left(\begin{bmatrix} 0_q \\ 0_n \end{bmatrix}, \begin{bmatrix} \Sigma_e & 0 \\ 0 & \Sigma_\varepsilon \end{bmatrix}\right), \qquad \text{NIID shocks} \qquad (4)$$

where $N(0,\cdot)$ denotes the multivariate Gaussian distribution, with zero mean and positive definite covariance matrices Σ_e and Σ_ε.

The model (1-4) is an independent trend-cycle decomposition which resembles the multivariate decomposition of Stock and Watson (1988) with shocks to trend being orthogonal to shocks to transitory components. In the model we also account for missing observations in y_t, and initial values in $\bar{y}_t$ and $\tilde{y}_t$. For the brevity of exposition, we omit technical details of Bayesian inference and postpone information on hyperparameters to the Appendix. We start with a closed-economy model and then enhance it to an open economy setup.

Closed-economy model

A model with inflation, policy-, short- and long-term interest rates in a closed economy setup for the Polish economy consists of the following measurement equations:

$$\pi_t = \bar{\pi}_t + \tilde{\pi}_t, \tag{5}$$

$$\pi_t^e = \bar{\pi}_t + \tilde{\pi}_t^e, \tag{6}$$

$$R_t = \bar{r}_t + \bar{\pi}_t + \tilde{R}_t, \tag{7}$$

$$NBP_t = \bar{\pi}_t + \bar{r}_t - \overline{cy}_t + \widetilde{NBP}_t, \tag{8}$$

$$I_t = \bar{\pi}_t + \bar{r}_t - \overline{cy}_t + \overline{tp}_t + \tilde{I}_t, \tag{9}$$

where π_t denotes headline inflation (CPI), π_t^e inflation expectations or (alternative version) the inflation target, R_t the short-term interbank interest rate (WIBOR), except for the ZLB period when it is substituted with shadow rate estimates based on the Krippner (2013) method, NBP_t is the policy interest rate (with missing values during the ZLB period), and I_t is the 5-year (baseline version) or 10-year (alternative version) Treasury bond interest rate.

We allow for divergence between the risk-free (Treasury or policy) and risk-bearing (interbank) interest rates. The difference, labelled convenience yield[3], $\overline{cy}_t$, lowers the

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[3] Our interpretation of the convenience yield is not in line with Del Negro et al. (2017). Still, the convenience of purchasing government bonds versus investment in the interbank market has its origin in safety and liquidity features as in Del Negro et al. (2017).

interest rate of government bonds compared to interbank interest rates of similar characteristics. The trend component of the real interest rate, $\bar{r}_t$, common to all interest rates is our measure of the NRI and one of the three stochastic trends that govern the dynamics of the model in the long run, next to the inflation trend, $\bar{\pi}_t$, and the time premium for longer maturities trend, $\overline{tp}_t$. Cyclical components, $\tilde{y}_t$, describe the short-run deviations of observed variables from their respective long-run trends.

Open-economy model

To enhance the model with open economy features we take into account two sources of spillovers from the EA, to interest rates and to the exchange rate. We utilize the no-arbitrage (uncovered interest rate parity) condition from Del Negro et al. (2019) to introduce a long-run relationship between the yield on Poland's government bonds, I_t^{EUR}, and euro-area government bonds, I_t^{EA}, both denominated in euro:

$$I_t^{EUR} = (\bar{r}_t + \bar{\pi}_t) + \overline{tp}_t - \overline{cy}_t^{EUR} + \overline{cy}_t - \bar{x}_t + \tilde{I}_t^{EUR}, \tag{10}$$

In the equation above $\overline{cy}_t^{EUR}$ is the convenience yield on bonds denominated in EUR, and $\bar{x}_t$ is the long-run return on the nominal bilateral exchange rate (PLNEUR).

Hence, equation (10) states that the long-run interest rate on Polish bonds denominated in the euro increases when either the domestic natural real interest rate, long-run inflation, or time premium[4] increases, the convenience yield on the euro denominated bonds decreases or the exchange rate appreciates.

Symmetrically, we assume a common trend in headline inflation (HICP) and the inflation target in the EA analogously to equations (5) and (6), and we model the interest rate for the euro area in a similar vein as in equations (8) and (9):

$$ECB_t = \bar{r}_t^{EA} + \bar{\pi}_t^{EA} - \overline{cy}_t^{EUR} + \widetilde{ECB}_t, \tag{11}$$

$$I_t^{EA} = \bar{r}_t^{EA} + \bar{\pi}_t^{EA} + \overline{tp}_t - \overline{cy}_t^{EUR} + \tilde{I}_t^{EA}, \tag{12}$$

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[4] For a matter of parsimony, we also assume that respective time premia in financial markets for government bonds denominated in EUR and PLN are close to each other.

where ECB_t is a an approximation of the ECB policy interest rate obtained with the Krippner (2013) method, I_t^{EA} is the long-term interest rate on EA bonds, and $\bar{r}_t^{EA}$ and $\bar{\pi}_t^{EA}$ are trends of the interest rate and inflation in the EA, respectively.

The second source of external shocks are exchange rate fluctuations. In the baseline version of the model (with 5-year interest rates), like for other observables, we decompose the annual dynamics of the exchange rate into a long-run trend and a transitory component:

$$x_t = \bar{x}_t + \tilde{x}_t, \tag{13}$$

where $\bar{x}_t = \bar{x}_{t-1} + e_t^x$.

In an alternative version of the model (5Y+FEER) we additionally relate the trend in the exchange rate, $\bar{x}_t$, to the fundamental equilibrium exchange rate, FEER, calculated externally, in the spirit of Kuziemska-Pawlak and Mućk (2022):

$$FEER_t = \bar{x}_t + \widetilde{FEER}_t. \tag{14}$$

To this end, we keep the prior variance of $\widetilde{FEER}_t$ close to zero to shrink a common exchange rate trend, $\bar{x}_t$, in (13) and (14) towards the dynamics of FEER.

We estimate the open economy DGGT models with a lag order in VAR, $p = 4$. The details on the priors and the initial values for latent variables can be found in the Appendix.

Semi-structural models

The vantage point of the semi-structural models is the baseline New Keynesian framework with sticky prices. The non-policy equations, log-linearized around a steady state with inflation target $\bar{\pi}$, can be expressed in the following manner:

$$\tilde{y}_t = \tilde{y}^e_{t+1} - a_r(i_t - \pi^e_{t+1} - \bar{r}_t) + \epsilon^{\tilde{y}}_t, \qquad \text{New Keynesian IS curve} \qquad (15)$$

$$\pi_t = b_1\pi^e_{t+1} + (1 - b_1)\bar{\pi} + b_y\tilde{y}_t + \epsilon^{\pi}_t, \qquad \text{New Keynesian Phillips curve} \qquad (16)$$

where in this model $\tilde{y}_t$ and i_t denote the output gap and the nominal interest rate, respectively. According to equation (15), the current output gap is a function of its future expected value, the difference between the ex-ante expected real interest rate and the NRI, as well as a demand-type shock $\epsilon^{\tilde{y}}_t$. In equation (16) current inflation is a function of inflation expectations, the central bank's target, the current output gap, and a cost-push-type shock ϵ^{π}_t. Parameters a_r, b_1 and b_y can be in principle derived from fundamentals like household preferences, production function parameters or the degree of price stickiness, see e.g. Galí (2015).

The NRI of the baseline New Keynesian model is defined by:[5]

$$\bar{r}_t = \rho + \Delta\bar{y}^e_{t+1}, \qquad \text{New Keynesian NRI} \qquad (17)$$

where ρ is households' discount factor and $\Delta\bar{y}^e_{t+1}$ is the expected rate of growth of flexible price output.

The popular semi-structural approach of Laubach and Williams (later extended in HLW) recasts the above forward-looking system into its backward-looking analogue. Flexible price output is replaced by the GDP trend, and in both IS and Phillips curve relationships the expected terms are replaced with the (averages of) past values of the variables in question.

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[5] For simplicity of exposition and following common practice it is assumed that the elasticity of intertemporal substitution equals 1.

The block of GDP trend $\bar{y}_t$, output gap and the IS curve in the HLW model can be expressed using our notation as:

$$y_t = \bar{y}_t + \tilde{y}_t, \quad \text{Trend-cycle decomposition} \quad (18)$$

$$\bar{y}_t = \bar{y}_{t-1} + g_{t-1} + \epsilon_t^{\bar{y}}, \quad \text{GDP trend evolution} \quad (19)$$

$$g_t = g_{t-1} + \epsilon_t^{g}, \quad \text{GDP trend growth rate evolution} \quad (20)$$

$$\tilde{y}_t = a_1 \tilde{y}_{t-1} + a_2 \tilde{y}_{t-2} - \frac{a_r}{2}(r_{t-1} - \bar{r}_{t-1} + r_{t-2} - \bar{r}_{t-2}) + \epsilon_t^{\tilde{y}}, \quad \text{HLW IS curve} \quad (21)$$

where the rate of growth of GDP trend g_t follows a random walk, the GDP trend itself is also potentially affected by random level shifts $\epsilon_t^{\bar{y}}$, while the current output gap is a function of its past two realizations and past two real interest rate gaps.

Inflation in the HLW model follows an accelerationist version of the Phillips curve, and is a function of its lagged values, past output gap and a cost-push shock:

$$\pi_t = b_1 \pi_{t-1} + \frac{(1-b_1)}{3}(\pi_{t-2} + \pi_{t-3} + \pi_{t-4}) + b_y \tilde{y}_{t-1} + \epsilon_t^{\pi}. \quad \text{HLW Phillips curve} \quad (22)$$

The real interest rate used in equation (21) is constructed by subtracting from the current nominal interest rate the average inflation rate over the course of the past year:

$$r_t = i_t - \frac{1}{4}(\pi_t + \pi_{t-1} + \pi_{t-2} + \pi_{t-3}). \quad \text{HLW real interest rate} \quad (23)$$

The natural real interest rate is obtained as a sum of the GDP trend growth rate g_t and a catch-all component z_t, reflecting, for example, shifts in ρ, assumed to follow a random walk as well:

$$\bar{r}_t = g_t + z_{t,} \quad \text{HLW NRI} \quad (24)$$

$$z_t = z_{t-1} + \epsilon_t^{z}. \quad \text{Evolution of } z \quad (25)$$

One shortcoming of the HLW model is its assumption of the nominal interest rate being a fully random variable. As a result, identification of the NRI level is typically quite weak, with large error bands, as the model relies ultimately on information obtained from the IS curve, where the slope parameter is subject to the pile-up problem, see, for example, Stock and Watson (1988). Therefore, while the HLW procedure can recover the medium-run level of the NRI, it cannot be sufficiently informative about the movements in the NRI at high frequency.

To counteract this deficiency, Brand and Mazelis (2019) propose a setting where the nominal interest rate level results from conscious decisions made by an inflation-targeting central bank. The HLW framework is amended as follows. First, the Phillips curve relationship assumes that inflation is anchored at the inflation target:

$$\pi_t = (1 - b_1)\bar{\pi} + \tfrac{b_1}{2}(\pi_{t-1} + \pi_{t-2}) + b_y \tilde{y}_{t-1} + \epsilon_t^{\pi}. \qquad \text{BM Phillips curve} \qquad (26)$$

Second, the central bank follows a Taylor-type rule when setting the nominal interest rate:

$$i_t = \rho_1 i_{t-1} + (1 - \rho_1)\big(\bar{r}_t + \bar{\pi} + \rho_\pi(\pi_t - \bar{\pi}) + \rho_y \tilde{y}_t\big) + \epsilon_t^{i}. \qquad \text{BM Taylor rule} \qquad (27)$$

Importantly, this policy rule specification assumes that the average nominal interest rate level carries valuable information regarding the evolution of the NRI. An inflation-targeting central bank, after, for example, setting interest rates too low relative to the NRI, observes above-target inflation and is able to correct the course by hiking the policy interest rate. As a result, the identification of the NRI strongly relies on central bank actions, and this is the primary source of the difference in BM results compared to HLW.

Finally, since now inflation expectations can be defined within the model, the real interest rate entering the IS curve is the nominal interest rate less model-consistent expected inflation:

$$r_t = i_t - E_t \pi_{t+1}. \qquad \text{BM real interest rate} \quad (28)$$

Structural model

According to standard neoclassical single-sector growth models, the NRI depends predominantly on household preferences, as well as on variables exogenous from the point of view of the model, like the rate of technological progress and the population growth rate. For example, in a variant of the Ramsey model the long-run NRI can be expressed as:[6]

$$\bar{r} = \rho + n + x, \qquad \text{Long-run NRI} \qquad (29)$$

where x is the rate of technological progress. While it is not possible to characterize the NRI in a single, closed-form equation when households are heterogeneous, the same intuition regarding the NRI drivers still applies. In our BBK framework, the household sector is modelled as an aggregate of 80 cohorts, aged from 20 to 99, that are subject to time- and age-dependent mortality risk. As their lives are divided into periods of labour market activity and retirement, worker-consumers, in order to smooth their consumption over time, accumulate assets until they become retired, and later draw them down to support their old-age consumption. When life expectancy grows over time, it acts as lowering of the effective discount rate ρ and leads households to accumulate more assets.

Other important drivers of the observed (and projected) drop in the NRI encompass changes in pension system generosity that exacerbate the life-cycle savings motive due to the declines in pension to labour income ratio, sub-replacement fertility rate resulting in declining (and even negative) adult population growth rate n, and last but not least the slowdown in rate of technological progress x.

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[6] This equation results from the households' Euler equation when welfare of subsequent cohorts is not weighed by their population (a so-called "average utilitarian" perspective).
A similar but less elegant expression can also be obtained in the standard two-period OLG model, where $\bar{r} = [\alpha/(1-\alpha)](2+\rho)(1+n)(1+x) - 1$, which can be approximated as $\bar{r} \approx \rho/2 + n + x$ for $\alpha \approx 1/3$.

The full description of the BBK model can be found in Bielecki et al. (2020), and the complete set of model equations is reproduced in the Appendix. Compared to the original BBK model, we introduce two modifications to better reflect the historical experience of Poland.

First, while the TFP level of the EA follows productivity improvements at the world technology frontier, Polish economy initially faces a relatively low level of TFP. In line with historical data, we assume that since the 1990s Poland experiences a rapid catch-up. Second, to capture the effects of the gradual opening of the Polish economy to international trade and financial market integration before accession to the European Union, we assume a time-varying country risk-premium coefficient.

4. Results

Time series models

We use Bayesian methods to approximate the low-frequency component of the NRI by a common trend in interest rates, $\bar{r}_t$, from the DGGT model. The time series models use quarterly financial data[7] from Poland and the EA for the period 2001Q1-2022Q4, which brings the financial markets perspective on the NRI. The general outcomes are relatively robust to reasonable changes in the model specification (see two alternative versions in the Appendix).

Statistical inference in the baseline version of the DGGT model indicates that there is a downward trend in the NRI in Poland over the last 20 years (Figure 4). The median estimate of $\bar{r}_t$ decreased by 5.9 pp from 4.4% in 2003Q1 to -1.5% in 2022Q4. This decrease is statistically significant at the 5% level, as the 95% highest posterior density (HPD) interval of the change in $\bar{r}_t$ in the analysed period is strictly negative (i.e. from -8.2 to -3.7). However, it is not certain whether the NRI from the DGGT model fell significantly below zero in any of the last 12 quarters as the zero line stays within the 95% HPD interval. The 95% HPD intervals for the NRI level are twice as wide and at the end of the sample (above 3 pp) than in the middle which is, to a certain extent, a feature of the estimation method (filtering and smoothing step in the Gibbs sampler).

At the same time, the results confirm a downward NRI trend in the EA (Figure 5). Between 2003 and 2022 the NRI corresponding to the real ECB policy rate decreased by 3.6 pp. The 95% HPD interval of this change ranges from -4.9 pp to -2.2 pp, and it

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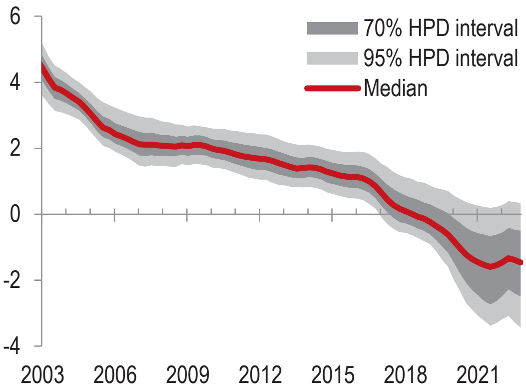
[7] In the baseline version of the DGGT model these are: 3-month WIBOR (complemented with the shadow interest rate), NBP policy rate (with the zero lower-bound 6-quarter period treated as missing data), interest rates on 5-year government bonds, annual inflation (CPI), and its NBP target, and their equivalents for the euro area (the shadow policy EBC rate, 5-year euro area government bonds, HICP inflation and its ECB target), additionally interest rates on 5-year Polish Treasury bonds denominated in euro, and the bilateral exchange rate of the EURPLN.

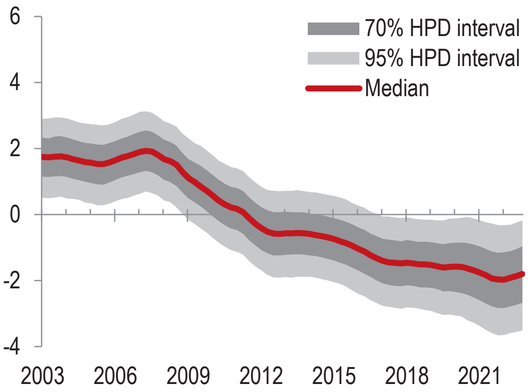
is highly probable that the NRI in the EA has been negative at least since the first quarter of 2017.

Figure 4 Estimates of the NRI in PL, baseline DGGT model



Source: Own calculations.

Figure 5 Estimates of the NRI in the EA, baseline DGGT model

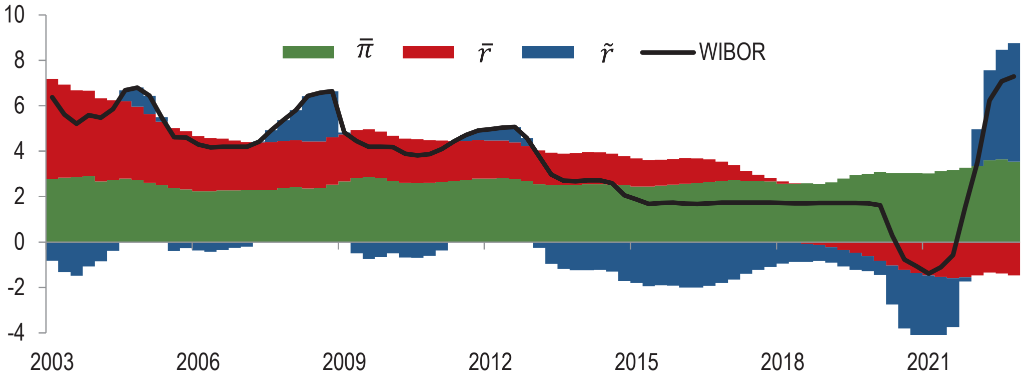


Source: Own calculations.

Time series models do not allow for an economic interpretation of the NRI movements. Nevertheless, we see two areas where some conclusions can be drawn from looking at the estimated latent trend components (Figure 34 in the Appendix) and interest rate decomposition (Figure 6 and Figure 7). The first observation is that in contrast to the NRIs, these components are relatively stable, both in PL and in the EA. The biggest fluctuations can be observed for the inflation trend in PL, which (not surprisingly) declines during the final disinflation period 2001-2002, then stabilizes close to the NBP inflation target (2.5%) and rises in the final four years of our sample. In contrast, the inflation component in the EA is remarkably stable at approximately 2% throughout the sample. Both the time premium and the convenience yields for Polish bonds decline somewhat in the run-up to the Global Financial Crisis and increase afterwards.

Figure 6 decomposes the Polish interbank rate (WIBOR) into the NRI, trend inflation and cyclical components. This exercise allows to draw further conclusions. First, the bulk of decline of WIBOR between 2003 and 2021 can be attributed to the financial markets' perception of the declining equilibrium real interest rate. Second, the recent sharply increasing interest rate is primarily perceived as a cyclical monetary policy tightening. In contrast, between 2013 and 2021 the interest rates were perceived as being below the equilibrium level (i.e. expansionary). Third, in the last part of our sample the inflation trend is above the NBP inflation target (3.6% in 2022Q4). This means that the nominal equilibrium rate from the DGGT model (approximately 1.8% in 2022Q4) is actually higher than a simple correction of the real NRI for the inflation target would suggest.

Figure 6 Decomposition of WIBOR (baseline DGGT model, pp)



Source: Own calculations.

Figure 7 shows an equivalent\ decomposition for the EA shadow interest rate[8]. Two things stand out. First, as in Poland, the long-term downward force driving the trend of the nominal rate is the steadily pushing down the NRI. Second, the assesment of policy stance is roughly similar to that for Poland. In particular, monetary policy was perceived as expansionary between 2012 and 2021 and became contractionary in 2022. Last but not least, however, it should be noted that likewise for NRIs, there is also a systematic divergence of nominal equilibrium rates, with the one in PL being substantially above the one in the EA. The most recent estimate of the nominal equilibrium rate in the EA hovers around zero. This points to a higher probability of

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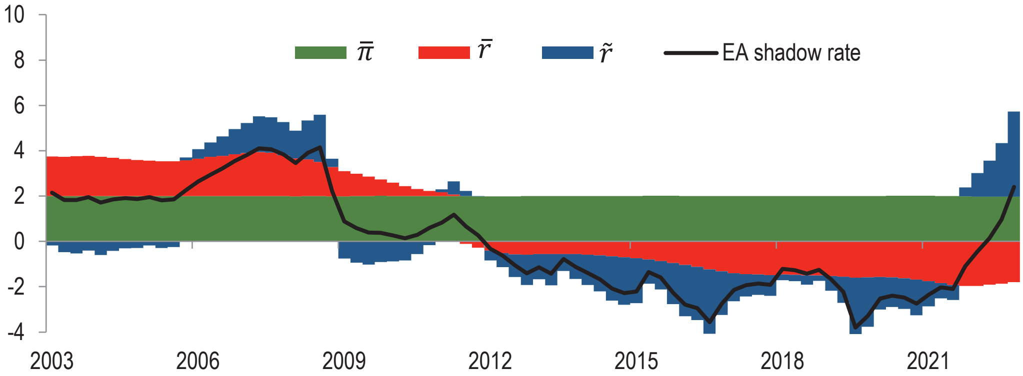
[8] For brevity of exposition, we omit the contribution of the euro convenience yield, which is mostly stable throughout the sample and stays at the level of 1.5 pp.

the EA facing the effective lower bound problem once the current inflationary episode is over.

Figure 7 Decomposition of the EA shadow interest rate (baseline DGGT model, pp)



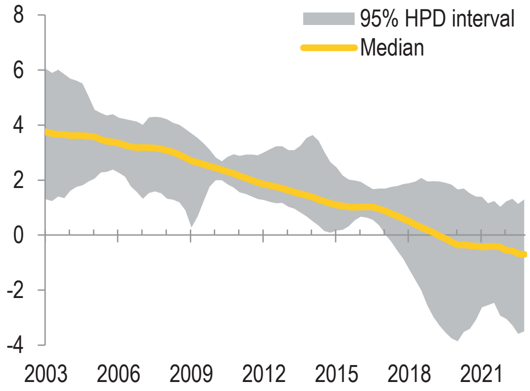
Source: Own calculations.

Semi-structural models

Both semi-structural models are estimated using Bayesian methods on the 1999Q1-2022Q4 sample for PL and the EA. The estimation procedure uses data on real GDP, core inflation and the interbank nominal interest rate (in the periods of binding ZLB we use Krippner (2013) estimates). Due to the unusual sequence of shocks experienced by both economies starting in 2020, we first estimate the output gaps using the Hamilton (2017) filter outside of the model and then provide them as additional observables with a relatively wide measurement error (see Appendix for details on the procedure and assumed priors).

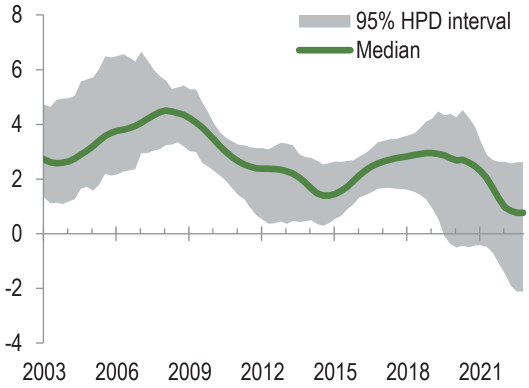
Similarly to the DGGT, the BM model (Figure 8) finds a downward trend of the Polish NRI, declining from the median estimated value of 3.7% in 2003Q1 to -0.7% in 2022Q4, hence an overall drop of around 4.4 pp. Admittedly, the 95% highest posterior density (HPD) intervals are wide, especially at the beginning and end of our sample. A big part of the uncertainty in the NRI level is the consequence of the estimated Taylor rule parameter uncertainty.

Figure 8 Estimates of the NRI in PL, BM model



Source: Own calculations.

Figure 9 Estimates of the NRI in PL, HLW model

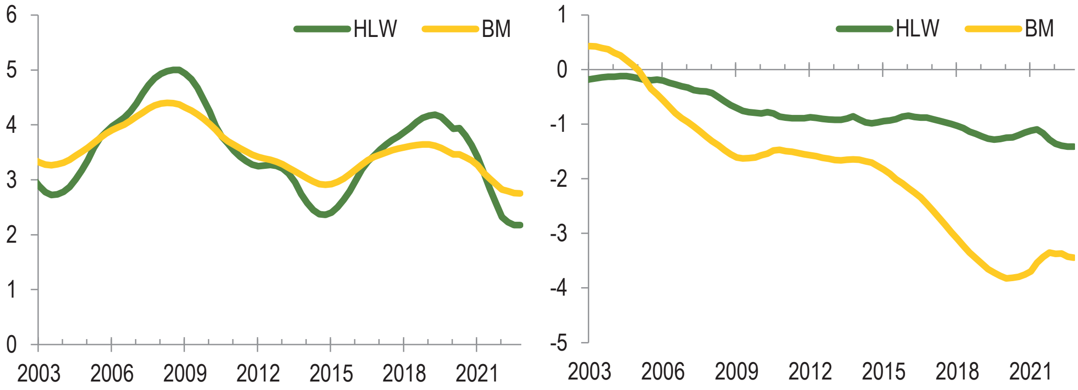


Source: Own calculations.

In comparison, the HLW model (Figure 9) suggests a rather modest decline in the NRI, which also occurs at a non-uniform pace, reflecting medium-frequency fluctuations in the underlying growth rate of the GDP trend. The median estimate decreases from 2.7% in 2003Q1 (close to its cyclical low) to 0.8% in 2022Q4, constituting an overall drop of less than 2 pp.

Given these contrasting outcomes of related frameworks, a discussion of the sources of the discrepancy is in order. As seen in the Appendix, the estimated parameters common to both models are largely in line, and the extracted processes of output gap or inflation expectations are also alike. The two models also recover a similar growth rate of the GDP trend, which is only somewhat more volatile in the HLW case. The two approaches markedly differ in their interpretation of the random-walk catch-all component z_t. While the HLW model generates a gradual decline of less than 1.5 pp since 2003Q1, in the case of the BM model the drop is much more pronounced, reaching almost 4 pp.

Figure 10 Estimates of the drivers of NRI in PL, BM and HLW models



GDP trend growth rate g

Remaining factors z

Source: Own calculations.

Source: Own calculations.

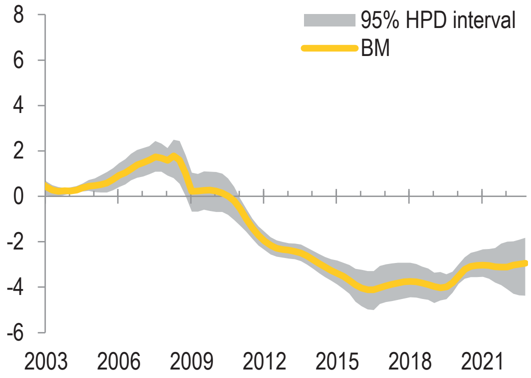
What accounts for this discrepancy is that the BM model, by taking into account the policy rule, adds the policymakers perspective to our information set. This may carry valuable (though most probably short-term) information about the natural level of both nominal and real interest rates.

The HLW model lacks the ability to reliably pick up NRI short-run movements and therefore is more suitable for determining the medium-run evolution of the NRI. Both the DGGT and BM models can additionally uncover short-run movements of the NRI (assessed by financial market participants and the central bank), although these movements can in principle also arise due to the misinterpretation of the "true" processes or the influence of additional shocks.

In line with the above interpretation, the BM method, when supplied with the shadow rate estimate of the EA money market conditions, finds a very pronounced drop of the NRI after the financial crisis and over the entire decade of the 2010s, which was first plagued by the sovereign debt crisis, and later suffered from aggregate demand deficiency. As a result, the obtained NRI estimate typically moves below the real interest rate implied by the shadow rate over the 2010s period, and reaches its low point of less than -4% in 2016. Notably, this estimate picks up at the onset of 2020 and starts converging toward the DGGT and HLW estimates. The HLW estimate again follows quite closely the fluctuations in the growth rate of the GDP trend, dropping around the financial crisis, recovering in the second half of the 2010s decade, and falling again in the pandemic episode.

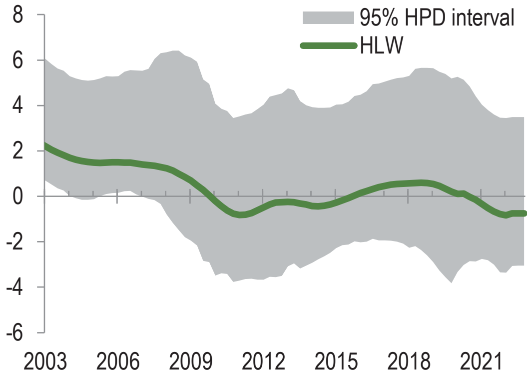
Over the last 20 years, for the euro area the median NRI estimate according to the BM method declined from 0.4% in 2003Q1 to -2.9% in 2022Q4, with an overall drop of around 3.4 pp. The HLW method finds a decline from 2.2% in 2003Q1 to -0.8% in 2022Q4, an overall drop of around 3 pp.

Figure 11 Estimates of the NRI in the EA, BM model



Source: Own calculations.

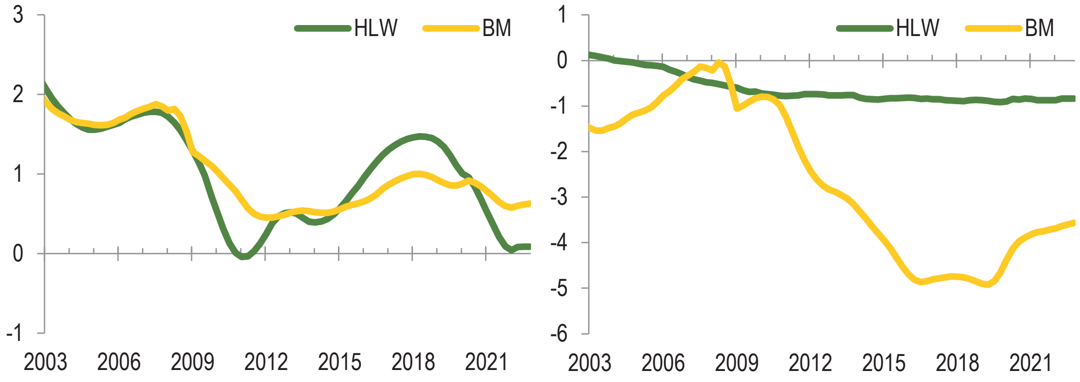
Figure 12 Estimates of the NRI in the EA, HLW model



Source: Own calculations.

Again, while the HLW and BM approaches do not differ substantially regarding the GDP trend growth rates, they recover staggeringly different levels of the catch-all z_t process. According to the BM model, the latter is subject to a dynamic decline starting at the onset of the financial crisis, becoming even more negative during the sovereign bond crisis in the EA, and only starting to stabilize in 2016 and recover at the onset of the pandemic. Such a sequence of events aligns quite well with our interpretation of the short-run NRI movements driven by various shocks whose influence dissipates over the medium-run.

Figure 13 Estimates of the proximate drivers of NRI in the EA, BM and HLW models



GDP trend growth rate g

Remaining factors z

Source: Own calculations.

Source: Own calculations.

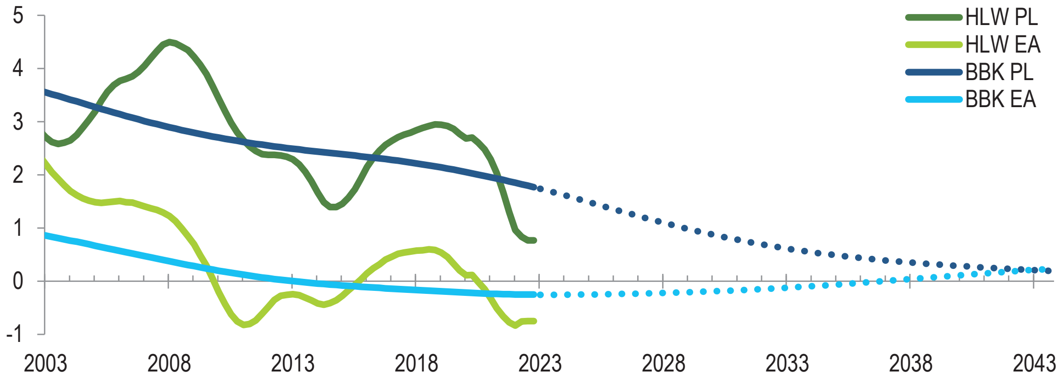
Structural model

While the semi-structural models can provide a proximate statistical decomposition of the movements in the NRI, their reliance on the random-walk assumption governing the underlying processes precludes a deeper economic interpretation of the obtained results. To interpret the observed decline in the NRI via the lens of measurable economic processes, we make use of the structural overlapping generations framework. Another advantage of this approach is to provide some intuition about the likely evolution of the NRI in the foreseeable future, as some trends that drive the data (especially demographic ones) are slow-moving or even largely pre-determined.

We first present the comparison of the "in-sample" estimates of the NRI obtained via the HLW method and from the BBK model. The HLW estimates fluctuate over the medium-run around a longer-run BBK trend, supporting our short-, medium- and long-run interpretation of the obtained results. The obtained estimates of the NRI from the BBK model for Poland suggest a decline from 3.6% in 2003Q1 to 1.8% in

2022Q4, an overall drop of around 1.8 pp. The decline in the euro area is more modest, from 0.9% in 2003Q1 to -0.3% in 2022Q4, an overall drop of more than 1 pp.

Figure 14 Estimates of the NRI in PL and the EA, BBK and HLW models

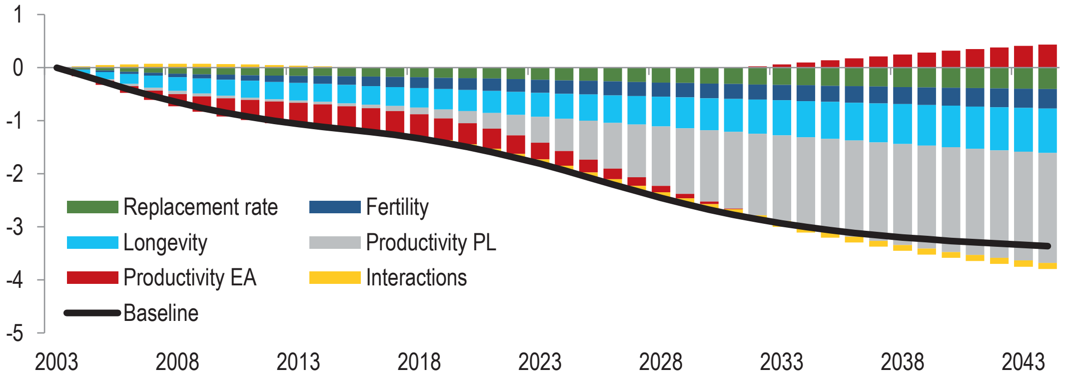


Source: Own calculations.

A unique advantage of the structural model is being able to provide a glimpse into future developments. First, according to our projections, the NRI in Poland will probably continue its decline, stabilizing slightly above 0% in the late 2030s. On the other hand, we do not expect a further decline in the EA NRI over the 2020-2040 period, in no small part due to the expected pick-up in the rate of technological progress compared to the 2010s. As a consequence, we expect a gradual convergence of the NRI in PL towards current EA levels, with most of the process completed by the mid-2030s.

While we cannot claim to be able to encompass all possible factors driving the evolution of the NRI, our model admits a rich framework to include several salient forces that have been established in the literature as suspects of the observed long-run decline in the NRI in the advanced, but also emerging, countries. The full life-cycle household sector allows us to take into account both changes in fertility and mortality rates, while the addition of the pension system makes households also react to the expected changes in the replacement rate (ratio of first pension benefit to last-received labour income). Moreover, the inclusion of the TFP catch-up process and the open economy environment enables us to incorporate the effects of expected decline in the pace of productivity growth as the Polish economy converges to the EA, as well as important spillovers from the forces governing the evolution of the NRI in advanced countries.

Figure 15 Structural decomposition of the decline in the NRI in Poland, BBK model



Source: Own calculations.

The bulk of the observed decline in the NRI over the 2003-2022 period can be attributed to two groups of forces. The first group is demographic in nature and reflects the gradual "greying" of society due to declines in fertility, increases in longevity and, in consequence, declines in the expected pension system replacement rates. All of these factors induce households to accumulate more assets towards their retirement period, shifting the asset supply schedule rightwards and reducing the equilibrium real interest rate. The second group consists of declines in the rate of growth of productivity, mainly due to a substantial EA productivity slowdown in the decade of the 2010s.[9]

For the next two decades, we expect that the strongest driver of the future decline in the Polish NRI will be the maturing of the convergence process, which will reduce the GDP trend growth rate in Poland to the growth rates of EA economies. This will

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[9] For a decomposition of the declines in the EA NRI see Bielecki et al. (2020).

reduce the relative attractiveness of investing physical capital in Poland, resulting in a leftward shift of the asset demand schedule, and a lower NRI.

Finally, our projections envision a gradual pick-up in the pace of productivity improvements in the EA compared to the 2010s decade. These developments may become even more likely if the artificial intelligence revolution will indeed take place. However, these developments should not markedly influence the NRI gap between Poland and the EA, unless there will be significant differences in the pace of the adoption of new technological improvements.

5. Conclusions

The natural rate of interest is a central concept for central banking – it determines the boundary between contractionary and expansionary monetary policy. Its importance increases in periods of extraordinary developments, like the surge in inflation observed worldwide since 2021. In this paper, we use a wide range of models to provide a comprehensive view of the natural interest rates in Poland and in the euro area. The models applied range from statistical time series decompositions based on financial market data, via semi-structural models based on macroeconomic data to a structural life cycle model based on demographic and productivity data. Our main findings are as follows:

- NRIs declined significantly in both Poland and the euro area over the last 20 years.
- The main drivers of the declining NRIs were demographics and the productivity slowdown in the euro area. Low fertility rates and rising longevity in both the euro area and Poland contributed to higher per capita asset holdings and lower equilibrium interest rates. The productivity slowdown in the euro area affected the NRI in Poland due to openness to capital flows.
- We consider our estimates based on the structural model and one of the semi-structural models as the most likely indicators of where the NRI will be over the medium-run. These models point towards a mildly positive NRI in Poland and a slightly negative NRI in the euro area.
- The estimates of the time series model and our second semi-structural model that rely on additional information carried by short- and long-term nominal interest rates enable us to assess the likely level of the short-run NRI, with estimates pointing to mildly negative NRI in Poland, and appreciably more negative in the euro area. As both economies entered a period of strong aggregate demand after the Covid pandemic, we believe that the disturbances that were keeping the short-run NRI estimates comparatively

low have largely dissipated. We expect that the short-run measures are going to gradually converge upward towards their medium-run counterparts as more data points from the post-pandemic period will become available.

- The NRI in Poland remained consistently above the NRI in the euro area by an average of 2-3 pp throughout our entire sample period. This was mainly driven by the productivity catch-up in Poland. As the productivity gap has significantly narrowed by 2022, we expect a relative productivity slowdown in Poland in the future and, as a consequence, gradual convergence of the NRI levels. Conditional on this assumption, the NRI in Poland is expected to have converged to the euro area level to a large degree by the mid-2030s.
- We expect demographic forces to exert a downward pressure on NRIs both in Poland and in the euro area in the foreseeable future. Regarding productivity growth, the picture is less clear. Recent advances in the usage of artificial intelligence may have the potential to speed up productivity growth significantly. If this effect materialises, technology growth may start to raise the NRI which, as a result, may increase in the euro area. This effect seems, however, unlikely to outweigh the expected productivity slowdown in Poland, so that there the NRI is more likely to decline further in the future.

All in all, our paper delivers a robust set of findings regarding the NRI in Poland and in the euro area, which – we hope – will somewhat facilitate the conduct of monetary policy.

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Appendix

Data

Time series models

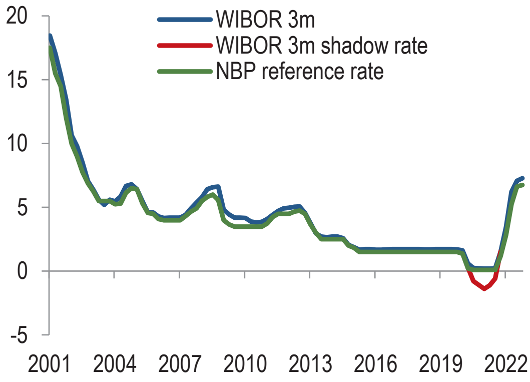
The dataset of financial and macroeconomic variables covers the period from 2001Q1 to 2022Q4 with the exception of Polish government bonds denominated in EUR (since 2013Q2). The data are taken from Bloomberg (by default), official statistics or research data sources (if explicitly mentioned) and recalculated from daily frequency to quarterly averages. The variables, except for interest rates, enter the models either as logarithms or first differences of their logs.

We use the 3-month WIBOR (Warsaw Interbank Offered Rate) as a popular short-term interest rate benchmark for the calculation of the natural interest rate in Poland. This money market rate, as it was subject to the effective zero lower bound (ZLB) in the period from 2020Q2 to 2021Q3, is replaced by the NBP estimates (Hertel et al. (2022)) of the shadow interest rate compiled with the method proposed by Krippner (2013). As we treat interbank deposits as a risk-bearing, non-liquid, and not a risk-free security, into the time series models we also incorporate a policy rate (NBP reference rate) for the period when NBP conducted the conventional open market operations. Lastly, we use 5- and 10-year Treasury bond interest rates denominated in PLN and EUR.

Disentangling nominal and real interest rate variability in the time series models we use the consumer price index (CPI) from Statistics Poland. As an alternative we applied the data on the (implicit and explicit) inflation target of Narodowy Bank Polski. In the beginning of the sample as long as long-term interest rates in Poland were influenced by strong expectations on the disinflation process (till 2003Q2), we adjusted the term structure of interest rates to allow for a positive time premium.

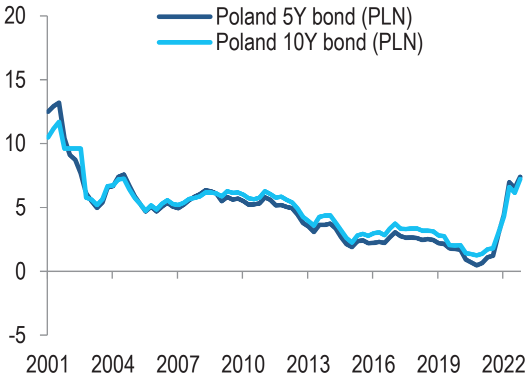
In the open economy setup, we extend the dataset with foreign financial market interest rates and bilateral PLNEUR exchange rate. In order to consistently capture the period of ECB unconventional monetary policy the estimates of natural interest rate are based on the policy rate from the Krippner webpage.[10] We use the shadow policy rate for the EA, which is very close to the main policy interest rate during the period of ECB conventional monetary policy, yet more sensitive to market expectations. The dataset for the euro area consists of inflation rates (harmonised index of consumer prices, i.e. HICP from Eurostat and inflation target from ECB) and long-term (5 and 10 years) government bond yields denominated in euro for euro zone countries and Poland. Finally, we consider the fundamental equilibrium exchange rate (FEER) as the indicator of the exchange rate trend. We use Global Value Chains extended FEER framework proposed by Kuziemska-Pawlak and Mućk (2022). Compared to Kuziemska-Pawlak and Mućk (2022), unitary demand elasticities of trade flows are assumed.

Figure 16 Short-term interest rates in Poland (%)



Source: Bloomberg, NBP.

Figure 17 Long-term interest rates in Poland (%)



Source: Bloomberg.

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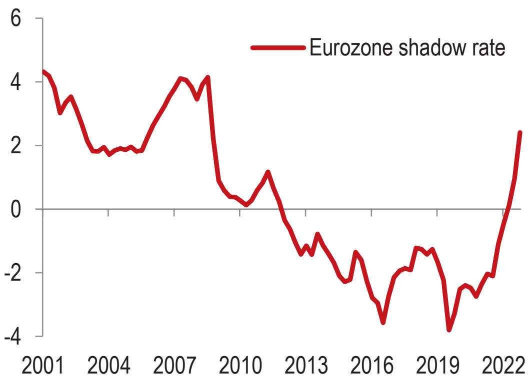
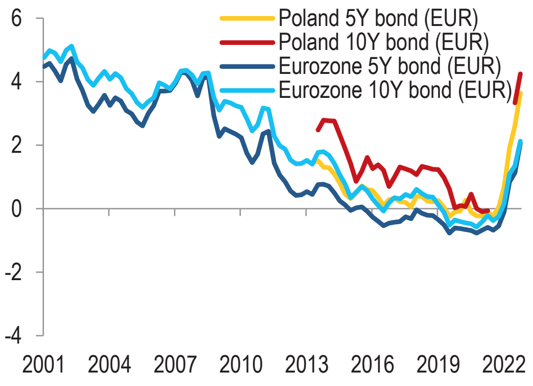
[10] https://www.ljkmfa.com/

Figure 18 Shadow policy rate for euro area (%)



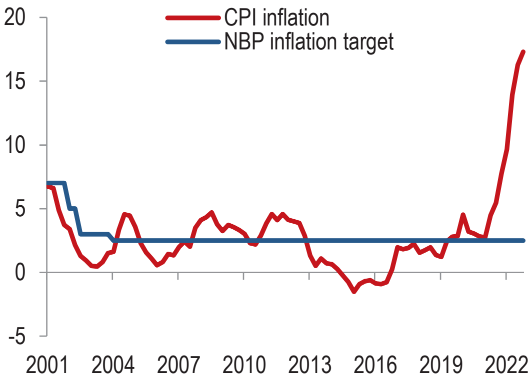
Source: Krippner (2013).

Figure 19 Long-term interest rates in Poland and euro area (%)



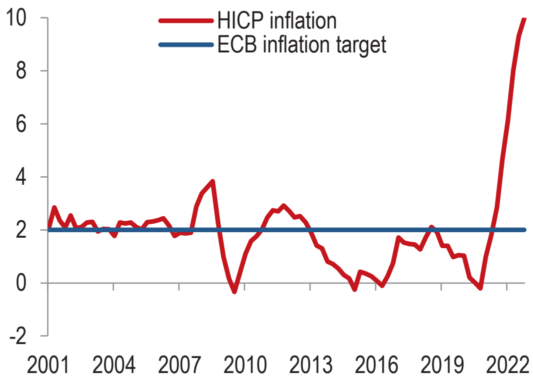
Source: Bloomberg.

Figure 20 Inflation in Poland (%, y-o-y)



Source: Statistics Poland, NBP.

Figure 21 Inflation in euro area (%, y-o-y)



Source: Eurostat, ECB.

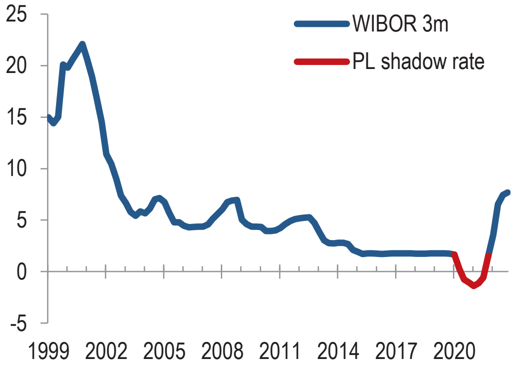
Semi-structural models

We use the following set of macroeconomic data from 1999Q1 to 2022Q4. Real GDP at constant prices for both PL and the EA, and the HICP core price index for the EA are taken from Eurostat. The CPI core price index for PL is taken from Statistics Poland. For the level of interest rates we use 3-month WIBOR and 3-month EURIBOR for PL and EA, respectively, downloaded from ECB SDW. For the ZLB periods we use the Krippner (2013) data for the EA and Hertel et al. (2022) for PL.

The output gap is obtained outside of the models and calculated according to the Hamilton (2017) procedure.[11] Since both the shadow rate and output gaps are not directly observable, we admit a measurement error with a standard deviation of 1 pp in both cases. Additionally, due to the construction of the Hamilton filter, the output gaps in 2022 are increasingly contaminated by the unusual GDP levels in 2020 and they are not provided to the model, letting it obtain the output gaps endogenously based on the estimated properties of the processes behind the model.

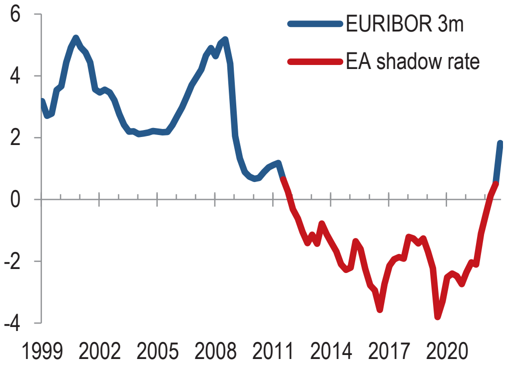
The BM method allows for specifying a time-varying inflation target. While we do not need to resort to this for the EA, the beginning of the sample in PL consists of a rapid disinflation process, during which the NBP inflation target was repeatedly adjusted. Since the results from the very beginning of our sample (1999-2003) are not crucial, we assume a linearly decreasing inflation target, stabilizing at 2.5% starting from 2001Q3. Additionally, since in PL the average core inflation rate was lower than the headline by around 0.5 pp throughout our sample period, we assumed that in terms of core inflation the inflation target was implicitly 0.5 pp lower.

Figure 22 Interest rates in PL (%)



Sources: ECB SDW, Hertel et al. (2022).

Figure 23 Interest rates in the EA (%)



Sources: ECB SDW, Krippner (2013).

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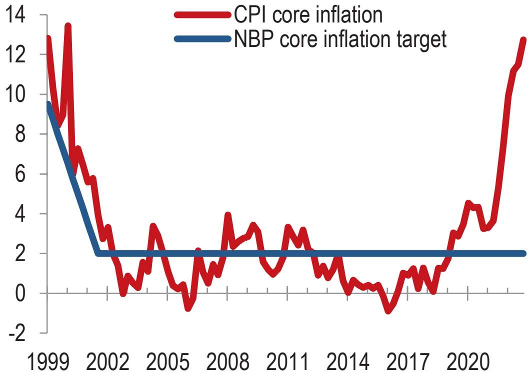
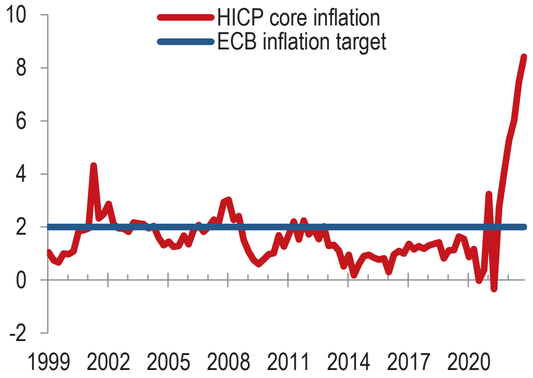
[11] Note: Hamilton output gap for the EA in 2020Q2 amounted to -15.4%.

Figure 24 Annualized, seasonally adjusted 3m core inflation in PL (%)



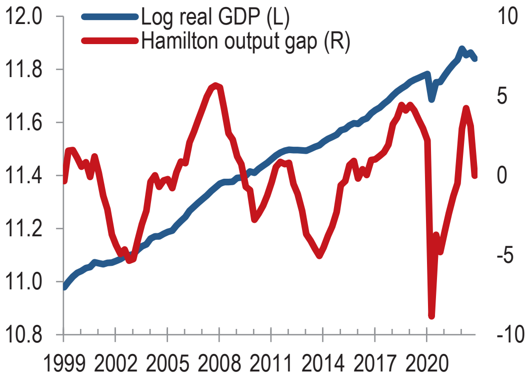
Sources: Statistics Poland, own calculations.

Figure 25 Annualized, seasonally adjusted 3m core inflation in the EA (%)



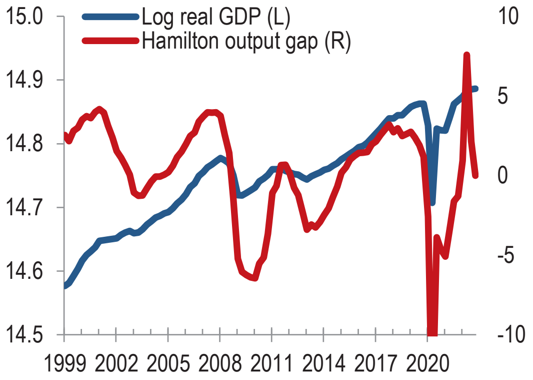
Source: Eurostat, own calculations.

Figure 26 Log real GDP (left axis) and Hamilton output gap (right axis, %) in PL



Sources: Eurostat, own calculations.

Figure 27 Log real GDP (left axis) and Hamilton output gap (right axis, %) in EA



Sources: Eurostat, own calculations.

Structural model

Demographic data and projections come from the 2022 edition of the United Nation's World Population Prospects (WPP). The assumptions behind the evolution of TFP for the EA are based on the data provided by the Area Wide Model database (AWM) and the European Commission's Ageing Working Group 2018 projections (AWG). The corresponding data for PL are constructed based on the Penn World Tables database version 10 (PWT) and we assume that the TFP growth rate in PL will converge to the EA one by 2045, at which point the TFP gap in levels is nearly closed.

Additionally, we approximate the process of the gradual integration of PL in world financial and trade markets by a smoothly time-varying country risk premium multiplier, so that the model replicates the observed Net Foreign Assets evolution. The NFA data for PL for years 1995-2021 were taken from NBP.

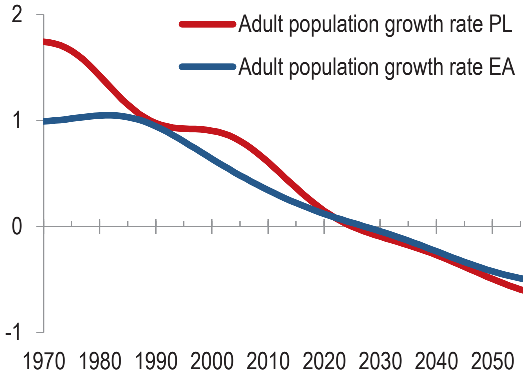
The model features an age-dependent productivity profile, which we base on the Household Finance and Consumption Survey data (HFCS) on household labour income for the EA countries. Since we do not have corresponding data for PL, we assume an identical profile. Since in PL the AWG projects starkly declining pension replacement rates, we include them as an exogenous driver.

The model parameters were calibrated symmetrically for PL and the EA. Households' discount rate was chosen so that the model real interest rate in the EA for the 1999-2008 period was equal to the data average.

Table 1 Calibrated BBK model parameters

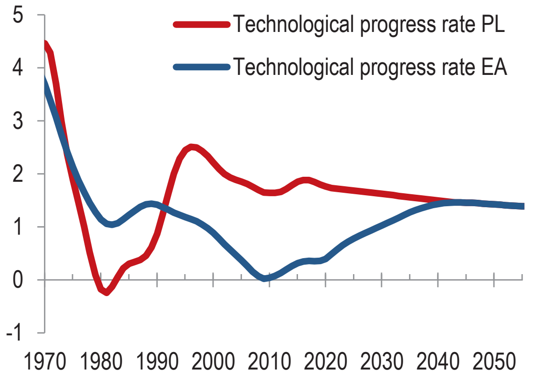
Parameter	Description	Value
ρ	Households' discount rate	-0.0065
J	Maximum household age	80
JR	Exogenous retirement age	43
α	Elasticity of output w.r.t. capital	0.25
δ	Capital depreciation rate	0.10
μ	Product markup	1.25
ξ	Country risk premium	0.025
G/Y	Government spending to GDP ratio	0.2
B/Y	Government debt to GDP ratio	0.6

Figure 28 Historical and projected adult population growth rates, EA and PL (%)



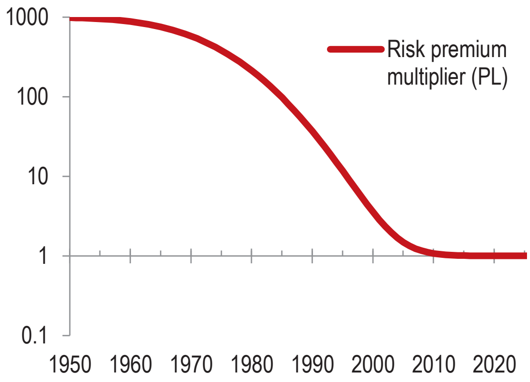
Sources: WPP, own calculations.

Figure 29 Historical and projected technological progress rates, EA and PL (%)



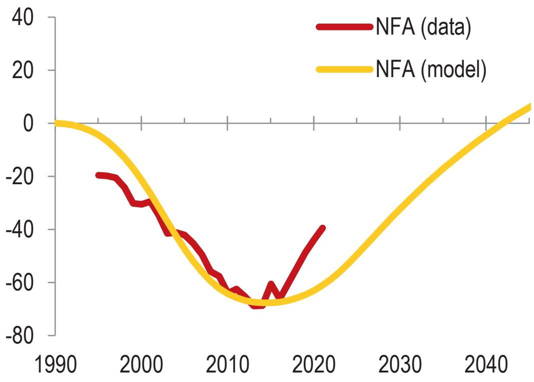
Sources: AWM, AWG, PWT, own calculations.

Figure 30 Time-varying risk premium multiplier, only PL (log scale)



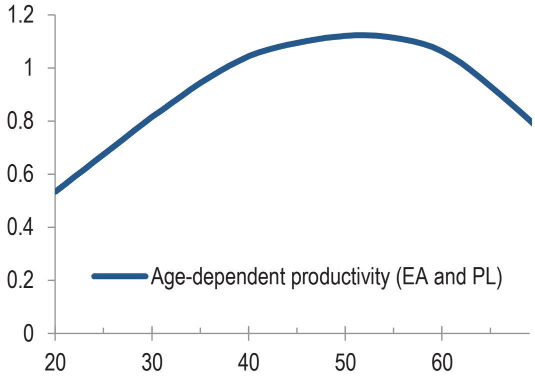
Source: own calculations.

Figure 31 Net foreign assets position for PL, data and model (% of GDP)



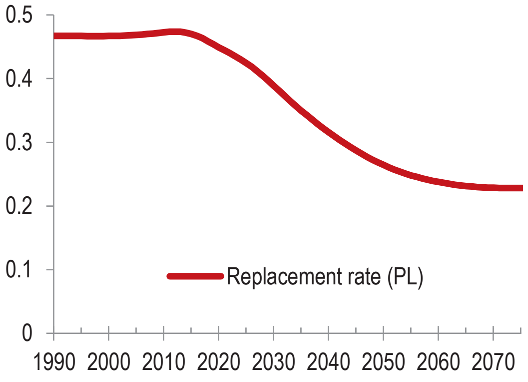
Sources: NBP, own calculations.

Figure 32 Age productivity profile



Sources: HFCS, own calculations.

Figure 33 Pension replacement rate, only PL



Sources: AWG, own calculations.

Method and additional results

Time series models

The DGGT model (1-4) can be cast into a Bayesian framework and estimated using Gibbs sampler after a prior elicitation. The detailed two-step sampling procedure (with a simulation smoother necessary to obtain draws for the latent states) is described in the Appendix of Del Negro et al. (2017). A relatively tight prior, $p(\Sigma_e)$, which is inverse-Wishart distributed, guarantees the smoothness of the common trends in $\bar{y}_t$ (Table 2). In turn, there is a relatively loose prior on $p(\Sigma_\varepsilon)$ with a standard Minnesota prior in the stationary VAR part using indicator function, $\varphi = vec(\Phi)$, $p(\varphi|\Sigma_\varepsilon) = N\left(\underline{\varphi}, \Sigma_\varepsilon \otimes \underline{\Omega}\right) I(\varphi)$. An indicator, $I(\varphi)$, is equal to 0 if characteristic roots of VAR polynomial lie outside the unit circle, and 1 otherwise, which excludes the possibility of explosive paths in VAR.

Table 2 Hyperparameters on the random walks for common trends in the DGGT model

scale factor	$\overline{\pi}$	$\overline{r}$	$\overline{cy}$	$\overline{tp}$	$\overline{r}^{EA}$	$\overline{cy}^{EUR}$	$\overline{x}$	$\overline{\pi}^{EA}$
$\frac{1}{400}\times$	4.0	2.5	0.25	0.5	2.0	0.25	4.0	2.0

Note: The elements in the table are standard deviations of prior distributions on the diagonal of Σ_e, their squares are multiplied by the common scale factor.

The initial conditions for normally distributed latent states: $\bar{y}_0$, $\tilde{y}_{0:-p+1}$ should be of less importance given the simulation smoother step if one omits a few initial observations from the posterior analysis as we do. Finally, we adjusted the Matlab procedures from Del Negro et al. (2017) available at https://github.com/FRBNY-DSGE/rstarBrookings2017 to draw 5000 replications from marginal posterior distribution for each of the models.

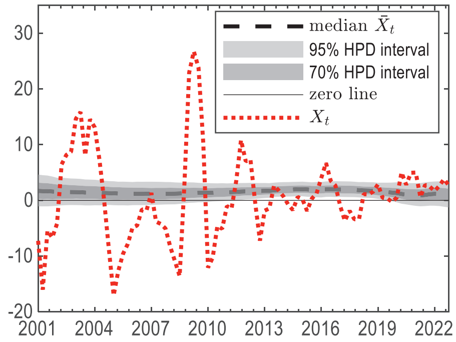
In the main text we discuss the results of the baseline version of the DGGT model. Figure 34 presents the six remaining common trends (additionally to NRI) and their uncertainty. For the EURPLN exchange rate we accompany the trend with the observables to stress the difference in the scales of fluctuations in the persistent and stationary components.

In the first alternative version of the open economy DGGT model (10Y) we changed the maturities of all the bonds from 5 to 10 years. In this approach compared with the baseline the poor availability of the quotations for the Polish Treasury bonds denominated in EUR from 2021Q2 to 2022Q2 further decreased the overlap with EA bonds by 5 quarters. Hence, it is not a preferred version of the model, which introduces a lot of uncertainty in the end of the sample. As a result, the uncertainty of the NRI in 2022Q4 increases both sides and the 70% HPD interval is from -3.0 to 1.9 (left panel in Figure 35). The median estimate of NRI from the baseline version, however, is still within the 70% HPD interval of the 10Y version. At the end of the sample, the cyclical component of the interest rate, $\tilde{r}$, is positive with a tendency to abate and the median inflation trend is elevated to 3.3 pp (the left panel in Figure 35).

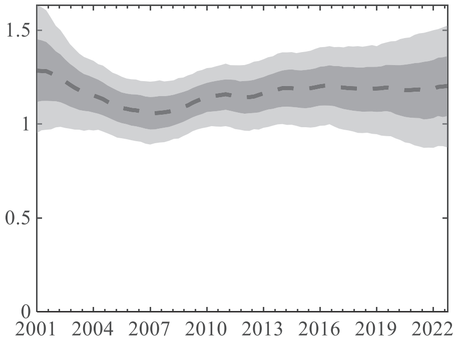
In the second alternative version of the open economy DGGT model (FEER trend) we additionally use FEER to better account for long-run trends in the exchange rates. The approach introduces some economic rationale behind the evolution of the exchange rate which, probably due to the assumption of UIP, translates into less uncertainty about NRI in Poland (right panel in Figure 35). The median estimate of NRI from the baseline version is within the 70% HPD interval of the FEER version in the full sample, and at the end of the sample the NRI from the FEER version is not significantly negative (the upper bound of the 70% credible interval is from 0.0 to 0.2 from 2022Q1 to 2022Q4 – c.f. the right panel in Figure 35).

Figure 34 The estimates of trend components (median, and 95% and 70% HPD intervals)

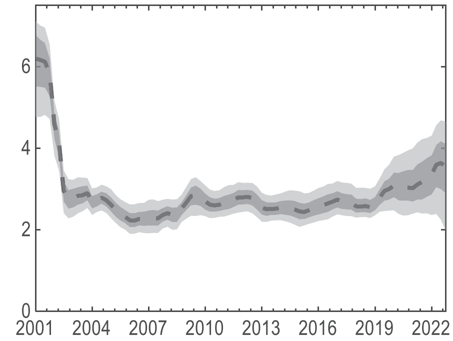
EURPLN (X_t)



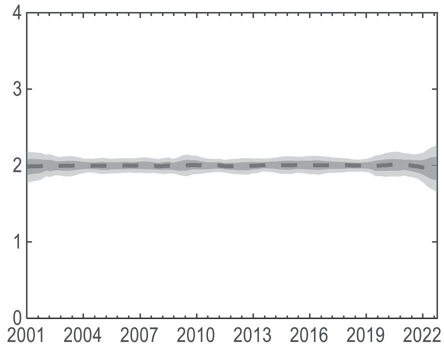
5-year time premium



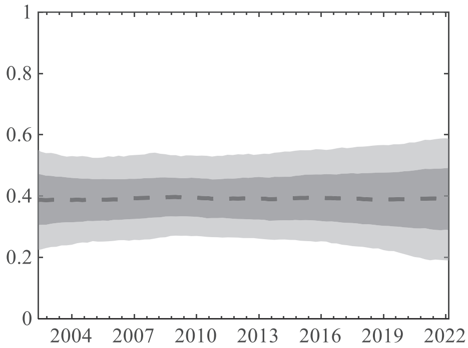
Inflation (PL)



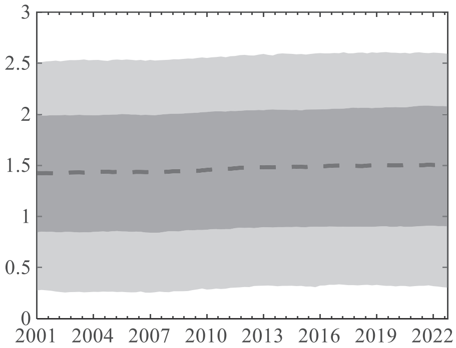
Inflation (EA)



Convenience yield (PL)

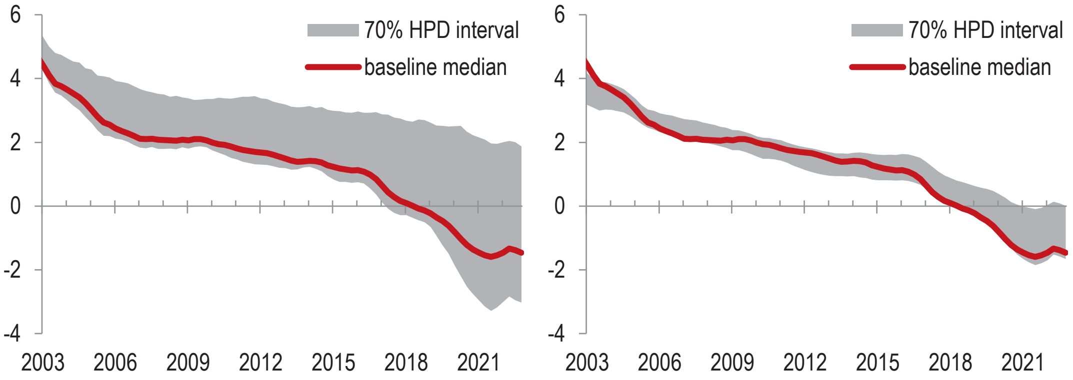


Convenience yield (EA)



Source: Own calculations.

Figure 35 Estimates of the NRI in PL in the alternative versions of open economy DGGT model



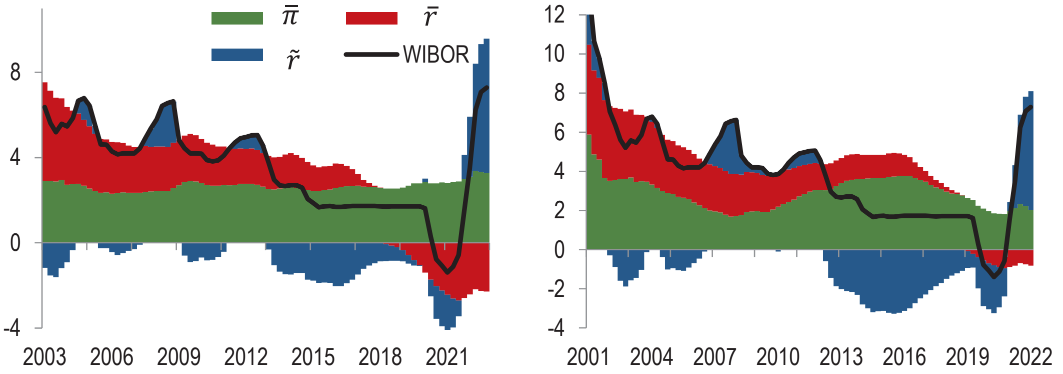
10Y government bonds

Source: Own calculations.

5Y government bonds and FEER trend

Source: Own calculations.

Figure 36 Time series decomposition of WIBOR (in pp) in the alternative versions of time series models



10Y government bonds

Source: Own calculations.

5Y government bonds and FEER trend

Source: Own calculations.

Semi-structural models

The majority of parameter priors were designed to be only weakly informative, imposing a standardized normal distribution. The single exception was the a_r parameter in the IS curve, where due to the pile-up problem a beta prior with the mean of 0.1 and standard deviation of 0.05 was imposed. The priors for standard deviation of shocks follow the inverse gamma distribution with standard deviations on g and z based on the HLW model estimated on the pre-pandemic EA sample. Both the BM and HLW models were estimated using 2 million MCMC draws with an acceptance rate around 0.3.

Table 3 Priors and posterior estimates of the parameters in the BM and HLW models

Param.	Prior			HLW PL		BM PL		HLW EA		BM EA	
	shape	mean	std. dev.	mean	95% HPD	mean	95% HPD	mean	95% HPD	mean	95% HPD
a_1	normal	0.00	1.00	0.71	0.49 - 0.93	0.70	0.48 - 0.93	0.74	0.53 - 0.95	0.74	0.53 - 0.95
a_2	normal	0.00	1.00	0.13	-0.09 - 0.34	0.12	-0.10 - 0.35	0.08	-0.14 - 0.29	0.08	-0.13 - 0.29
a_r	**beta**	**0.10**	**0.05**	0.06	0.01 - 0.13	0.09	0.02 - 0.17	0.06	0.01 - 0.13	0.10	0.02 - 0.20
b_1	normal	0.00	1.00	0.82	0.62 - 1.02	0.90	0.80 - 0.99	0.79	0.58 - 0.99	0.93	0.84 - 0.99
b_y	normal	0.00	1.00	0.10	-0.00 - 0.21	0.12	0.01 - 0.22	0.04	-0.01 - 0.10	0.04	-0.01 - 0.10
ρ_1	normal	0.00	1.00			0.88	0.79 - 0.95			0.64	0.43 - 0.85
ρ_π	normal	0.00	1.00			1.26	0.51 - 2.06			0.53	0.05 - 1.05
ρ_y	normal	0.00	1.00			1.17	0.39 - 2.09			0.28	0.07 - 0.49
$\sigma_{\tilde{y}}$	inv. gamma	1.00	Inf	1.45	1.22 - 1.68	1.42	1.20 - 1.65	1.82	1.55 - 2.11	1.79	1.53 - 2.07
$\sigma_{\bar{y}}$	inv. gamma	1.00	Inf	0.43	0.24 - 0.64	0.50	0.28 - 0.70	0.49	0.28 - 0.70	0.55	0.37 - 0.74
σ_π	inv. gamma	1.00	Inf	1.31	1.13 - 1.50	1.34	1.15 - 1.53	0.79	0.68 - 0.91	0.76	0.66 - 0.87
σ_r / σ_i	inv. gamma	1.00	Inf	0.80	0.69 - 0.92	0.78	0.62 - 0.94	0.44	0.36 - 0.52	0.30	0.20 - 0.40
σ_g	inv. gamma	**0.10**	**Inf**	0.11	0.03 - 0.19	0.08	0.02 - 0.15	0.09	0.03 - 0.17	0.06	0.03 - 0.10
σ_z	inv. gamma	**0.30**	**0.10**	0.45	0.10 - 1.11	0.45	0.15 - 1.07	0.45	0.11 - 1.10	0.38	0.21 - 0.57

Structural model

The BBK model is solved using a fully nonlinear, deterministic solver available in Dynare. Below we reproduce the full set of model equilibrium conditions that jointly determine the evolution of real per capita allocations and real prices, for given initial conditions and for given paths of exogenous deterministic variables: growth rate of 20-year-olds $n_{1,t}$, mortality rate $\omega_{j,t}$ ($j = 1, \ldots, J-1$), labour productivity profile z_j, TFP gap $A_t \leq 1$, rate of technological progress x_t, pension replacement rate ϱ_t, country risk premium multiplier γ_t, and world real interest rate r_t^*.

Households

$$\begin{aligned}
c_{j,t} + (1+x_{t+1})a_{j+1,t+1} &= \left(\mathbf{1}_{j<JR}\right)\left(1-\tau_t+\frac{\pi_t}{w_t h_t}\right)w_t z_j \\
&\quad +(1-\tau_t)\left(\mathbf{1}_{j\geq JR}\right)pen_t + beq_t + (1+r_t)a_{j,t} \\
a_{0,t} &= 0 \\
a_{J,t} &= 0 \\
(1+x_{t+1})c_{j+1,t+1} &= \beta(1-\omega_{j,t})(1+r_{t+1})c_{j,t}
\end{aligned}$$

Demographics

$$\begin{aligned}
n_{1,t+1} &= \frac{N_{1,t+1}}{N_{1,t}} - 1 \\
N_{j+1,t+1} &= (1-\omega_{j,t})N_{j,t} \\
N_t &= \sum_{j=1}^{J} N_{j,t} \\
n_{t+1} &= \frac{N_{t+1}}{N_t} - 1
\end{aligned}$$

Aggregation over households

$$\begin{aligned}
c_t &= \sum_{j=1}^{J} \frac{N_{j,t} c_{j,t}}{N_t} \\
h_t &= \sum_{j=1}^{JR-1} \frac{N_{j,t} z_j}{N_t} \\
a_{t+1} &= \sum_{j=1}^{J} \frac{N_{j,t} a_{j+1,t+1}}{N_{t+1}} \\
beq_t &= \sum_{j=1}^{J} \frac{(N_{j,t-1} - N_{j,t})(1 + r_t) a_{j,t}}{N_t}
\end{aligned}$$

Firms

$$\begin{aligned}
(1 + n_{t+1})(1 + x_{t+1}) k_{t+1} &= (1 - \delta) k_t + i_t \\
r_t &= \frac{\alpha}{\mu} A_t k_t^{\alpha-1} h_t^{1-\alpha} - \delta \\
w_t &= \frac{1 - \alpha}{\mu} A_t k_t^{\alpha} h_t^{-\alpha} \\
y_t &= A_t k_t^{\alpha} h_t^{1-\alpha} \\
\pi_t &= y_t - w_t h_t - i_t
\end{aligned}$$

Government

$$\begin{aligned}
pen_t &= \varrho_t w_t \frac{\sum_{j=1}^{JR-1} N_{j,t}\, z_j}{\sum_{j=1}^{JR-1} N_{j,t}} \\
\tau_t w_t \sum_{j=1}^{JR-1} \frac{N_{j,t}}{N_t} z_j + (1 + n_{t+1})(1 + x_{t+1}) \frac{B}{Y} &= \frac{G}{Y} + (1 - \tau_t) pen_t \sum_{j=JR}^{J} \frac{N_{j,t}}{N_t} + (1 + r_t) \frac{B}{Y}
\end{aligned}$$

External sector

$$1 + r_{t+1} = \left[1 + \gamma_t \xi \left(\exp(-b_t^*/y_t) - 1\right)\right](1 + r_{t+1}^*)$$

Market clearing

$$\begin{aligned}
a_t &= k_t + b_t + b_t^* \\
(1 + n_{t+1})(1 + x_{t+1}) b_{t+1}^* &= (1 + r_t) b_t^* + y_t - c_t - i_t - \frac{G}{Y}
\end{aligned}$$

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