

NATIONAL BANK OF POLAND WORKING PAPER №. 72

Central bank's macroeconomic projections and learning

Giuseppe Ferrero, Alessandro Secchi

Giuseppe Ferrero: giuseppe.ferrero@bancaditalia.it
Alessandro Secchi: alessandro.secchi@bancaditalia.it

The opinions expressed in this paper are those of the authors and do not necessarily reflect those of Bank of Italy. The authors thank Seppo Honkapohja, James Bullard, Jacek Suda, Petra Geraats, Giulio Nicoletti and participants at the National Bank of Poland conference „Publishing Central Bank forecast in theory and practice” and the Federal Reserve of St. Louis conference on learning for useful comments. The authors also thank two anonymous referees.

The paper was presented at the National Bank of Poland’s conference „Publishing Central Bank forecast in theory and practice” held on 5–6 November 2009 in Warsaw.

Design:
Oliwka s.c.

Layout and print:
NBP Printshop

Published by:
National Bank of Poland
Education and Publishing Department
00-919 Warszawa, 11/21 Świętokrzyska Street
phone: +48 22 653 23 35, fax +48 22 653 13 21

© Copyright by the National Bank of Poland, 2010

<http://www.nbp.pl>

Contents

Abstract	5
Non-technical summary	6
1. Introduction	8
2. The model	12
3. Central Bank interest rate path communication	16
3.1. E-Stability of the REE	16
3.2. Speed of convergence	20
4. Announcing expected inflation and output gap	27
4.1. Announcing only expected inflation and output gap	28
4.2. Announcing expected interest rate, inflation and output gap	28
5. Extensions	32
5.1. Publication of a longer path	32
5.2. Forward expectations in the policy rule	33
5.3. Announced path with a subjective judgemental component	33
6. Conclusions	35
References	36
Appendix: Proofs of propositions	38
Appendix 1) The REE under contemporaneous Taylor rules	38
Appendix 2) Proof of proposition 1 (Announcement of the policy path and E-stability of the REE)	39
Appendix 3) Proof of proposition 2 (Announcement of policy intentions and root-t convergence)	40
Appendix 4) Proof of proposition 3 (Root-t convergence under different weights to policy path announcement)	41
Appendix 5) Speed of convergence isoquants	42
Appendix 6) Proof of proposition 4 (Speed of convergence and communication of the path)	42
Appendix 7) Proof of proposition 5 (Announcing expected inflation and output gap)	43
Appendix 8) Proof of proposition 6 (Publishing interest rate, inflation and output gap projections)	45
Appendix 9) Proof of proposition 7 (Announcement of a T-period path)	46
Appendix 10) Proof of proposition 8 (Expectations-based policy rule)	48

List of Tables and Figures

Table 1.	Speed of convergence and simulations	24
Table 2.	Speed of convergence and simulations	30
Figure 1.	E-stability under no announcement, $(1 - \lambda_1) = 0$, and under a fully internalized announcement of the interest rate path, $(1 - \lambda_1) = 1$	20
Figure 2.	E-stability & root-t convergence under no announcement and under fully internalized announcement of expected interest rates	22
Figure 3.	The speed of learning isoquants for $\lambda_1 = 0$ (dotted line) and $\lambda_1 = 1$ (continue line)	26
Figure 4.	E-stability and root-t convergence under no announcement and under announcement only of expected inflation and output gap	29
Figure 5.	Weights to the projections and E-stability when the central bank announces interest rate, inflation and output gap paths	31
Figure 6.	E-stability and the announcement of a T-period interest rate path	32

Abstract

We study the impact of the publication of central bank's macroeconomic projections on the dynamic properties of an economy where: (i) private agents have incomplete information and form their expectations using recursive learning algorithms, (ii) the short-term nominal interest rate is set as a linear function of the deviations of inflation and real output from their target level and (iii) the central bank, ignoring the exact mechanism used by private agents to form expectations, assumes that it can be reasonably approximated by perfect rationality and releases macroeconomic projections consistent with this assumption.

Results in terms of stability of the equilibrium and speed of convergence of the learning process crucially depend on the set of macroeconomic projections released by the central bank. In particular, while the publication of inflation and output gap projections enlarges the set of interest rate rules associated with stable equilibria under learning and helps agents to learn faster, the announcement of the interest rate path exerts the opposite effect. In the latter case, in order to stabilize expectations and to speed up the learning process the response of the policy instrument to inflation should be stronger than under no announcement.

JEL Classification Numbers: E58, E52, E43, D83.

Keywords: Monetary policy, Communication, Interest rates, Learning, Speed of Convergence.

Non-technical summary

In the last two decades there has been a substantial change in the attitude of most central banks' communication strategies. Until the end of the 80s, the conventional wisdom was that "secrecy" about monetary policy decisions would make monetary policy more effective. In terms of communication, this translated into central bankers speaking in an opaque and convoluted language. Today the old view has been replaced by a new one which greatly emphasizes the need for clarity in the central banks communications and regards transparency as mandatory.

Despite the growing recognition of the importance of transparency in monetary policy-making, no consensus has emerged – either among academics or among central banks – on what the appropriate degree of transparency is, what constitutes an "optimal" communication strategy and what are the best instruments to enforce it.

Among the most debated aspects of central banks communication there is the degree of openness concerning the release of information about the future evolution of macroeconomic variables and, in particular, of the policy intentions. This can be done at different levels of precision, from the release of vague verbal hints to the publication of unambiguous numerical projections.

The main advantage associated with the use of more precise communication obviously lies in the fact that it allows a stricter control of private expectations and, in turn, greater macroeconomic stability. On the other hand, it has been argued that a communication strategy based on the release of precise information about macroeconomic expectations might involve a series of drawbacks. In particular recent studies have reported the possibility that an explicit announcement of central bank's expectations and of its policy intentions, might put credibility at risk, especially when the public ignores its conditional nature or misinterprets the precision of the received information.

The unresolved debate among central bankers and researchers about benefits and costs associated with the disclosure of information about central bank's macroeconomic expectations provides the key motivation for our analysis. Our contribution to this debate starts from the observation that an important issue that has attracted only marginal interest in the recent literature is the analysis of the effects of the announcement of macroeconomic projections in an environment where agents are learning.

We analyze an environment in which private agents have an incomplete understanding of the functioning of the economy and forecast its dynamics using a learning algorithm on past data. Moreover we assume that the central bank is endowed with complete information about the current state of the economy and that it publishes macroeconomic projections constructed under the hypothesis of rational expectations. We consider this last hypothesis to be a plausible

and realistic description of the way in which many of the institutions that announce macroeconomic projections obtain their forecasts. The public is then assumed to form its macroeconomic expectations as a weighted average of the projections released by the central bank and the prediction obtained through its learning algorithm.

We study the impact of the publication of central bank's macroeconomic projections on the macroeconomic stability and on the speed of convergence of the learning process of private agents.

It turns out that results crucially depend on the set of macroeconomic projections released by the central bank. In particular, we show that the release of interest rate projections slows the learning process and, if the policy rule of the central bank is not sufficiently aggressive against inflation, it amplifies initial expectations errors and generates instability. On the contrary, independently of the policy rule adopted by the central bank, the release of projections about inflation and output gap helps agents to learn faster and favors the stability of equilibria under learning.

Our analysis provides new results in favor of a prudential approach in disclosing information about expected interest rates and suggests that particular attention should be paid to those situations in which private agents, in forming their expectations, put a large weight on the announcement of the interest rate path. In those cases in fact the set of policy rules that guarantees stability and a fast process of convergence of expectations could be even smaller than the one associated with a communication strategy which is completely silent about central bank macroeconomic expectations and policy inclinations.

It is finally worthwhile to notice that our analysis is not necessarily against the publication of interest rate projections when agents are learning. If on one side we have concluded that in this case the publication of rational interest rate projection might threaten the stability of the economy, on the other we cannot exclude that there might exist alternative interest rate projections which can virtuously interact with private learning so to strengthen the stability of the economy and increase the speed of convergence of private expectations towards rationality.

1 Introduction

The current commonly-held view about monetary policy is that it influences economic decisions mainly through its impact on expectations (Blinder, 2000; Woodford, 2005; Svensson, 2006). One way in which a credible central bank can directly affect private expectations is through the release of information about its own view on the future evolution of macroeconomic variables and, in particular, of its policy intentions. This can be done at different levels of precision, from the release of vague verbal hints to the publication of unambiguous numerical projections.¹ It has been argued that the main advantage associated with the use of more precise communication is that it allows a stricter control of private expectations and, in turn, greater macroeconomic stability (Woodford, 2005; Rudebusch and Williams, 2008).² However, it has also been pointed out that the release of accurate information about macroeconomic expectations might involve a series of drawbacks. On top of the general claim that the provision of public information is not necessarily beneficial (Morris and Shin, 2002), recent studies have reported the possibility that an explicit announcement of central bank's expectations, and in particular of its policy intentions, might reduce credibility, especially when the public ignores its conditional nature or misinterprets the precision of the received information (Mishkin, 2004; Khan, 2007; Woodford, 2005; Rudebusch and Williams, 2008). The tension between benefits and costs associated with the disclosure of information about central bank's macroeconomic expectations remains unresolved.

Our contribution to this debate starts from the observation that an important issue that has received minor attention in the literature is the analysis of the effects of the announcement of macroeconomic projections in an environment where agents are learning. An exception in this respect is the work of Eusepi and Preston (2010). In their model, monetary policy stabilization is conducted in the presence of two informational frictions. First, the central bank has imperfect information about the state of the economy and sets the current interest rate as a function of its forecast of the current inflation and output gap. Second, private agents have an incomplete understanding of the functioning of the economy and forecast the variables which are relevant to their decision process using past data. In such an environment, where

¹For an empirical analysis of the effects of qualitative announcements of monetary policy intentions see Bernanke, Reinhart, and Sack, 2004; Gurkaynak, Sack, and Swanson, 2005; and Rudebusch, 2006. For the effects of the publication of numerical interest rate paths see, Archer, 2005; Moessner and Nelson, 2008; and Ferrero and Secchi, 2009.

²Other positive effects of the release of precise information regarding the future evolution of the economy are that (i) it also enhances the efficient pricing of financial assets (Archer, 2005; Kahn, 2007; Svensson, 2004), (ii) it increases the central bank's accountability (Mishkin, 2004) and (iii) it fosters the production of good forecasts by the central bank (Archer, 2005).

self-fulfilling expectations are possible, it is shown that the provision of detailed information about policy intentions favors the alignment of private and central bank's expectations – anchoring of expectations – thus restoring macroeconomic stability.

In this work we analyze an economy which shares with Eusepi and Preston (2010) the assumptions that the information available to private agents is incomplete and that they update their expectations using recursive learning algorithms. Moreover, also in our model the central bank implements monetary policy according to Taylor rules. However, we depart from their framework in assuming that the central bank is endowed with complete information about the current state of the economy and that it publishes macroeconomic projections based on the hypothesis that private agents are perfectly rational. We believe this hypothesis represents in a plausible and realistic way what is done by most of the central banks which disclose their expectations in the form of macroeconomic projections.³ The public is then assumed to form its macroeconomic expectations as a weighted average of the projections released by the central bank and the prediction obtained through its learning algorithm.⁴

We study the impact of the publication of central bank's macroeconomic projections on the stability of the equilibrium and on the speed of convergence of the learning process of private agents. It turns out that results crucially depend on the set of macroeconomic projections released by the central bank. In particular we show that the release of interest rate projections restricts the set of policy rules consistent with a stable equilibrium and reduces the speed of learning. This result overturns the main conclusion of Eusepi and Preston (2010) which states that more transparency about future policy rates favors macroeconomic stability. On the contrary the publication of projections about inflation and output gap helps agents to

³The Norges Bank produces forecasts using a core macroeconomic DSGE model with "*rational agents reacting to exogenous disturbances*" (Brubakk et al., 2006), the Swedish Riksbank uses a macroeconomic general equilibrium model derived under the assumptions of "*optimizing behaviors and rational expectations*" (Adolfson et al., 2007) and the Central Bank of Iceland uses a model where expectations "*are assumed to be rational, i.e. consistent with the model structure (model consistent expectations)*". The Reserve Bank of New Zealand represents, in part, an exception as at the core of its Forecasting and Policy System has a general equilibrium macro-model where expectations are modeled "*as some weighted combination of the model-consistent forecast and some other function of the recent data*" (Black et al., 1997). Learning, however, is not taken explicitly into account. For completeness it should also be noticed that central banks are aware of the limits of macroeconomic models and also of the rational expectation hypothesis. In describing the model used at the Reserve Bank of New Zealand, Black et al. (1997) observe that "*a valuable next step would be to specify how agents learn about the new policy rules, although as yet there is no generally-accepted theory of learning in macroeconomics*". For this reason macroeconomic projections are often "corrected" with judgmental factors before being disclosed to the public. The effect of this judgment component on the learning process of private agents is an interesting issue only partially addressed in this paper – see Section 5 – and it deserves further research.

⁴Similarly, we may assume that a fraction of private agents in the economy uses its own learning procedure to form expectations, while the remaining fraction fully internalizes the central bank's announcement.

learn faster and enlarges the set of monetary policies associated with stable equilibria under learning.

The result that the disclosure of the interest rate projections undermines the macroeconomic stability when the interest rule adopted by the central bank is not sufficiently aggressive against inflation can be explained as follows. In a New-Keynesian framework where private agents' are learning, an initial (positive) expectation bias leads to higher inflation both directly through the Phillips curve and indirectly through the real interest rate that affects the output gap in the IS curve. A policy rule that reacts to inflation (and output gap) introduces a feedback element in the IS curve that helps to offset the initial bias – if the response to inflation is sufficiently large. However, by publishing the interest rate projections obtained under the (incorrect) assumption that private agents are rational, the central bank is not taking into account the systematic mistakes that private agents are doing along the learning process and, therefore, reduces its ability to contrast the cumulative movement away from the rational expectation equilibrium (REE) through the interest rate rule – or in other terms it weakens the positive feedback element in the IS curve. As a result initial expectations biases tend to be amplified by the announcement, agents need a longer period of time to learn and the convergence toward the REE is slower. The overall system becomes more vulnerable to self-fulfilling expectations. This implies that in order to obtain stability under learning and to favor a fast convergence of the learning process, a central bank which decides to publish the interest rate path obtained under the assumption that private agents are fully rational should also choose a policy rule characterized by a response to inflation which is stronger than in the case of no announcement.

Publishing output gap and inflation projections has opposite implications. While the information about the policy rate (the instrument variable of the model) is indirectly exploited by private agents in order to form expectations about future inflation and output gap (the control variables of the model), information about these two variables is used directly to predict their future behaviors. Initial expectation biases are immediately reduced with no need for the stabilizing properties of interest rate rules that by responding to actual (or expected) inflation and output gap introduce the positive feedback in the IS curve. Therefore, by announcing its inflation and output gap expectations, the central bank helps agents to learn faster and enlarges the set of monetary policies associated with stable equilibria under learning.

The publication of interest rate projections is an aspect of monetary policy communication which has recently generated an extensive debate, both among policy makers and academics. Our analysis provides new results in favor of a prudential approach in disclosing information about expected interest rates. In fact they im-

ply that when the interest rate rule is not sufficiently aggressive against inflation the implementation of this communication strategy generates instability. It also emerges that the larger the weight given by private agents on central bank's interest rate projections, the more aggressive has to be the interest rate rule to preserve the system from instability. In particular it turns out that when such a weight is above a certain threshold the set of policy rules which generate instability becomes even larger than the one associated with the no disclosure benchmark.

From a more general point of view it is however useful to observe that our results are not necessarily against the publication of interest rate projections. In fact, even when private agents are learning, it cannot be excluded the possibility that the central bank, by taking into account the true mechanism used by private agents to form expectations, might devise interest rate projections which strengthen the stability of the economy and increase the speed of convergence of private expectations towards rationality.

The paper is organized as follows. In Section 2 we develop the baseline model; in Section 3 we analyze the effect of publishing the projections about the policy instrument in terms of stability under learning and speed of convergence; in Section 4 we analyze the alternative scenario where the central bank also publishes its expectations about the output gap and inflation; in Section 5 we consider some extensions. Section 6 concludes.

2 The model

We assume that under rational expectations the economy evolves according to the following standard New-Keynesian model:

$$x_t = E_t x_{t+1} - \varphi (i_t - E_t \pi_{t+1}) + g_t \quad (2.1)$$

$$\pi_t = \alpha x_t + \beta E_t \pi_{t+1} + u_t, \quad (2.2)$$

where x_t denotes the output gap, π_t is inflation and i_t the short-term nominal interest rate at time t . The operator E_t denotes rational expectations conditional to the information set available at t . Finally, g_t and u_t are, respectively, a demand and a cost-push shock. These two shocks evolve according to:

$$g_t = \rho_g g_{t-1} + \varepsilon_{gt} \quad \text{and} \quad u_t = \rho_u u_{t-1} + \varepsilon_{ut}, \quad (2.3)$$

where ε_{gt} and ε_{ut} are mutually orthogonal white noises with variances σ_g^2 , σ_u^2 .

We supplement equations (2.1)-(2.3) with a standard contemporaneous Taylor rule.⁵

$$i_t = \gamma + \gamma_x x_t + \gamma_\pi \pi_t. \quad (2.4)$$

The stochastic dynamic system (2.1)-(2.4) can be rewritten more compactly as:

$$y_t = Q + F \times E_t y_{t+1} + S w_t, \quad (2.5)$$

with

$$w_t = \Psi w_{t-1} + \varepsilon_t$$

where

$$y_t = \begin{bmatrix} \pi_t & x_t & i_t \end{bmatrix}' \quad w_t = \begin{bmatrix} u_t & g_t \end{bmatrix}' \quad \varepsilon_t = \begin{bmatrix} \varepsilon_{u,t} & \varepsilon_{g,t} \end{bmatrix}'$$

and the link between the parameters in equations (2.1)-(2.4) and matrices Q , F and S is derived in Appendix 1.

It is well known that under rational expectations, the linear system (2.5) has a unique non-explosive solution if and only if all eigenvalues of the F matrix are inside the unit circle.⁶ As shown in Bullard and Mitra (2002) this condition reduces to have

⁵In Section 5 we show that the results of our analysis do not change when the central bank is assumed to implement monetary policy through a Taylor rule based on forward looking variables.

⁶See for example McCallum, 2004.

$$\gamma_\pi > 1 - \frac{(1 - \beta)}{\alpha} \gamma_x. \quad (2.6)$$

When this condition is satisfied the unique non-explosive solution is of the minimum state variable (MSV) form

$$y_t = A + Bw_t \quad (2.7)$$

where A is a (3×1) vector and B a (3×2) matrix.

We now consider a departure from the hypothesis of rational expectations. In particular we assume that private agents know the sequence of shocks that hits the economy (up to the current time t) and the actual values of output gap, inflation and interest rates (up to time $t - 1$). We also assume that private agents are aware of the functional form of the MSV solution (2.7), but ignore the value of the A and B matrices.

Under these hypotheses, the economy evolves according to

$$y_t = Q + F \times E_t^* y_{t+1} + Sw_t, \quad (2.8)$$

$$w_t = \Psi w_{t-1} + \varepsilon_t$$

where the operator E_t^* denotes expectations conditional to the information set available at t and the "*" symbol is used to stress that expectations are not fully rational. In particular, we assume that, in each period t , private agents obtain estimates $\hat{A}_t$ and $\hat{B}_t$ of the corresponding matrices of equation (2.7) using a recursive learning algorithms as in Marcet and Sargent (1989) and Evans and Honkapohja (2001). These estimates are in turn used to form their own forecasts about the evolution of the endogenous variables at $t + 1$, $E_t^* y_{t+1}$. This procedure is an example of adaptive real-time learning, which basic idea is that agents follow a standard statistical or econometric procedure for estimating the perceived law of motion (PLM) of the endogenous variables.

Stacking estimates $\hat{A}_t$ and $\hat{B}_t$ in a matrix $\hat{\Gamma}_t$, and the constant term and the shocks u_t and g_t in vector z_t ,

$$\hat{\Gamma}_t = \begin{bmatrix} \hat{A}_t & \hat{B}_t' \end{bmatrix}' \text{ and } z_t = \begin{bmatrix} 1 & u_t & g_t \end{bmatrix}',$$

the matrix $\hat{\Gamma}_t$ is estimated recursively from past data according to

$$\hat{\Gamma}_t = \hat{\Gamma}_{t-1} + t^{-1} \hat{R}_t^{-1} z_{t-1} \left(y_{t-1}' - z_{t-1}' \hat{\Gamma}_{t-1} \right) \quad (2.9)$$

where

$$\widehat{R}_t = \widehat{R}_{t-1} + t^{-1} \left(z_{t-1} z'_{t-1} - \widehat{R}_{t-1} \right). \quad (2.10)$$

According to expressions (2.9)-(2.10), in each period private agents update their estimates of A and B by a term that depends on the last prediction errors.⁷ At the beginning of each period, when the public knows the realization of the shocks but endogenous variables are still to be determined, the law of motion perceived by private agents is

$$y_t^{PLM} = z_t' \widehat{\Gamma}_t, \quad (2.11)$$

which implies that private agents compute their forecasts of endogenous variables according to⁸

$$E_t^* y_{t+1} = \widehat{A}_t + \widehat{B}_t \Psi w_t. \quad (2.12)$$

In order to study whether the recursive least-squares estimates $\widehat{A}_t$ and $\widehat{B}_t$ converge to the corresponding matrices which define the MSV solution under RE we refer to the concept of expectation stability (E-stability) described in Evans and Honkapohja (2001).

The E-stability principle focuses on the mapping from the estimated parameters – the perceived law of motion (2.11) – to the true data generating process – the actual law of motion, ALM – obtained by inserting expectations (2.12) into the system (2.8),

$$y_t = Q + F \widehat{A}_t + \left(F \widehat{B}_t \Psi + S \right) w_t. \quad (2.13)$$

The resulting mapping from the PLM to the ALM is thus given by

$$T(\widehat{A}_t, \widehat{B}_t) = (Q + F \widehat{A}_t, F \widehat{B}_t \Psi + S). \quad (2.14)$$

Under some regularity conditions (here satisfied)⁹, the E-stability principle states that the MSV solution (2.7) is stable under least squares learning if it is locally asymptotically stable under the *ordinary differential equations* (ODE)

$$\frac{\partial}{\partial \tau}(A) = T_A(A) - A \quad (2.15)$$

$$\frac{\partial}{\partial \tau}(B) = T_B(B) - B, \quad (2.16)$$

where τ denotes “notional” or “artificial” time and $T_A(A) = Q + FA$ and $T_B(B) =$

⁷Note in particular that $\widehat{\Gamma}_t$ depends on information available up to $t - 1$.

⁸We assume that Ψ is known. This assumption is commonly adopted in the learning literature and does not affect the results (see for example Evans and Honkapohja, 2001).

⁹See Chapter 6 of Evans and Honkapohja (2001).

$FB\Psi + S$.

E-stability conditions are readily obtained by computing the derivative of the ODE's

$$\frac{d(T_A(A) - A)}{dA} \quad \text{and} \quad \frac{d(T_B(B) - B)}{dB}$$

and checking whether all their eigenvalues have negative real part. If this condition is satisfied, the economy described by equations (2.8)-(2.10), where agents form expectations using recursive learning algorithms, converges in the long run to the one described by equations (2.5) and (2.7), were agents are fully rational.

As shown in Bullard and Mitra (2002) expression (2.6) provides also necessary and sufficient condition for E-stability.

3 Central Bank interest rate path communication

The aim of this section is to analyze the effects of publishing the interest rate path in terms of E-stability and speed at which agents learn¹⁰. While agents' expectations evolve according to the learning procedure described in the previous section, we retain the assumption that the central bank produces its own forecasts assuming that private agents are perfectly rational. As we said in the introduction this assumption mostly reflects the fact that in practice the central banks that announce their policy path obtain their projections - to a large extent - from macroeconomic models solved under the rational expectation hypothesis.

In order to study the effects of the announcement we write the IS and the Phillips curve $T - 1$ periods ahead and substitute them in expressions (2.1) and (2.2)¹¹

$$x_t = E_t^* x_{t+T} - E_t^* \sum_{j=0}^{T-1} (\varphi i_{t+j} - \varphi \pi_{t+j+1} - g_{t+j}) \quad (3.1)$$

and

$$\pi_t = \beta^T E_t^* \pi_{t+T} + E_t^* \sum_{j=0}^{T-1} \beta^j (\alpha x_{t+j} + u_{t+j}) \quad (3.2)$$

It is worth to notice that in order to obtain equations (3.1) and (3.2) we are using the law of iterated expectations hypothesis, that holds under both RE and least square learning (Evans, Honkapohja and Mitra, 2003). This formulation points out the central role not only of actual real interest rate, but also of expected future short term real interest rates in determining today output and inflation.

3.1 E-Stability of the REE

For simplicity we assume that the central bank announces only the next period expected interest rate.¹² In this case we can write (3.1) and (3.2) for $T = 2$, as

$$\pi_t = \beta^2 E_t^* \pi_{t+2} + \alpha x_t + u_t + \beta \alpha E_t^* x_{t+1} + \beta E_t^* u_{t+1} \quad (3.3)$$

$$x_t = E_t^* x_{t+2} - \varphi (i_t - E_t^* \pi_{t+1} + E_t^* i_{t+1} - E_t^* \pi_{t+2}) + g_t + E_t^* g_{t+1}. \quad (3.4)$$

¹⁰In terms of determinacy, nothing changes since the model under rational expectations does not change when the central bank announces its interest rate projections.

¹¹Here, as in Rudebusch and Williams (2006), we substitute separately the T -period ahead IS and the Phillips curves into the time- t IS and the Phillips curve. Results do not change if we consider a more general forward representation of this system of equations.

¹²In Section 5 we consider also announcements over longer horizons.

Ferrero and Secchi (2009) study the case of the Reserve Bank of New Zealand, that publishes its own interest rate projections since 1999, and show that market expectations on short term interest rates respond in a significant and consistent way to the unexpected component of the published path, even though adjustment is not complete. In order to take into account the possibility that the public moves its expectations only partially in the direction of the announcement, we assume that private agents expectations about the expected interest rate depend on both central bank's announcement and their own estimates.¹³ Let $0 \leq (1 - \lambda_1) \leq 1$ be the weight that agents assign to the central bank's announcement,

$$E_t^* i_{t+1} = (1 - \lambda_1) E_t^{CB} i_{t+1} + \lambda_1 E_t^{RLS} i_{t+1} \quad (3.5)$$

where

$$E_t^{CB} i_{t+1} = a_i + \rho_u b_{u,i} u_t + \rho_g b_{g,i} g_t \quad (3.6)$$

and a_i , $b_{u,i}$ and $b_{g,i}$ are the coefficients that appear in the rational expectation equilibrium (2.7), while

$$E_t^{RLS} i_{t+1} = a_{i,t} + \rho_u b_{u,i,t} u_t + \rho_g b_{g,i,t} g_t \quad (3.7)$$

where $a_{i,t}$, $b_{u,i,t}$ and $b_{g,i,t}$ are estimated recursively.

We also assume that the central bank does not release information about its expected inflation and output gap¹⁴

$$E_t^* \pi_{t+i} = E_t^{RLS} \pi_{t+i} \text{ and } E_t^* x_{t+i} = E_t^{RLS} x_{t+i}.$$

Under these assumptions equations (3.3) and (3.4) can be written as

$$y_t = \tilde{Q} + \tilde{F} \times E_t^* y_{t+1} + \tilde{V} \times E_t^* y_{t+2} + \tilde{S} w_t, \quad (3.8)$$

where $\tilde{Q}$, $\tilde{F}$, $\tilde{V}$ and $\tilde{S}$ are derived in Appendix 2.

Private agents' forecasts under recursive learning are computed from the estimated PLM

$$y_t = A + B w_t$$

¹³Alternatively, we may assume that a fraction of private agents in the economy uses its own learning procedure to form expectations, while the remaining fraction, fully internalizes the central bank's announcement. The possibility of having a weight on the released information different than one is particularly relevant when we analyze the case in which the central bank announces both the policy path and the inflation and output gap projections (see Section 4).

¹⁴The case in which the central bank releases also inflation and output gap projections is analyzed in Section 4.

from which we compute the expectations

$$E_t y_{t+1} = A + B\Psi w_t \quad \text{and} \quad E_t y_{t+2} = A + B\Psi'\Psi w_t.$$

The actual law of motion of y_t is

$$y_t = \left(\tilde{Q} + \tilde{F}A + \tilde{V}A \right) + \left(\tilde{F}B\Psi + \tilde{V}B\Psi'\Psi + \tilde{S} \right) w_t, \quad (3.9)$$

the resulting mapping from the PLM to the ALM is

$$T(A, B) = \left(\tilde{Q} + \left(\tilde{F} + \tilde{V} \right) A, \tilde{F}B\Psi + \tilde{V}B\Psi'\Psi + \tilde{S} \right) \quad (3.10)$$

and the associated *ordinary differential equations* used to study E-stability are

$$\frac{\partial}{\partial \tau}(A) = \tilde{Q} + \left(\tilde{F} + \tilde{V} \right) A - A, \quad (3.11)$$

$$\frac{\partial}{\partial \tau}(B) = \tilde{F}B\Psi + \tilde{V}B\Psi'\Psi + \tilde{S} - B. \quad (3.12)$$

In the following proposition we state the conditions under which the REE is E-stable and compare them with those obtained under no announcement.

Proposition 1. *Let $\varphi\gamma_x + \alpha\varphi\gamma_\pi + 1 \neq 0$. In an economy that (i) evolves according to the system of equations (3.8), where (ii) at time t the central bank publishes the time $t+1$ interest rate projection consistent with the REE and (iii) private agents assign weight $0 \leq (1 - \lambda_1) \leq 1$ to these projections, revealing the interest rate path makes condition for stability under learning more stringent than under no announcement. In particular, the necessary and sufficient condition for E-stability of the equilibrium (2.7) is*

$$\gamma_\pi > \frac{2}{(1 + \lambda_1)} - \frac{1 - \beta}{\alpha} \gamma_x. \quad (3.13)$$

Proof. See Appendix 2.

The Phillips curves (2.2) and (3.2) being equilibrium conditions imply that each percentage point of permanently higher inflation determines a permanently higher output gap of $(1 - \beta) / \alpha$ percentage points. Therefore, when the policy maker does not announce future policy intentions, expression (2.6) states that necessary and sufficient condition for E-stability is that the long-run increase in the nominal interest rate prescribed by policy rules with contemporaneous endogenous variables should be larger than the permanent increase in the inflation rate. Applying a similar reasoning to the case where the central bank announces the next period expected interest rate, expression (3.13) states that necessary and sufficient condition

for E-stability is that the long-run increase in the nominal interest rate should be at least $2/(1 + \lambda_1)$ times as big as the permanent increase in the inflation rate. For $0 \leq (1 - \lambda_1) < 1$, this implies a larger response than under no announcement.

In a world where private agents are learning from past data – and along their learning process they produce biased predictions of the main macro variables – the result that E-stability conditions are more stringent under the announcement of the expected interest rate crucially depends on the assumption that the central bank's projections are obtained assuming that private agents are perfectly rational – that is a projection that in the long run, when the agents in the economy have enough data to estimate correctly the parameters of the model, will be (possibly) correct, but along the learning process will be inaccurate. As a result, initial expectations biases tend to be amplified by the announcement, the overall system becomes more vulnerable to self-fulfilling expectations and in order to stabilize expectations the long-run increase in the nominal interest rate should be at least $2/(1 + \lambda_1)$ times as big as the permanent increase in the inflation rate.¹⁵

Let's consider an example where private agents have an initial positive bias in expected inflation. This positive bias will lead to higher inflation both directly through the Phillips curve and indirectly through the real interest rate that affects the output gap in the IS curve and therefore inflation (in the Phillips curve). A policy rule that reacts directly to inflation (and output gap) introduces a feedback element in the IS curve that helps to offset the initial bias – if the response to inflation is sufficiently large, as stated in condition (2.6). By publishing the interest rate projections obtained under the (incorrect) assumption that private agents are rational, the central bank is not taking into account the systematic mistakes that private agents are doing along the learning process and, therefore, reduces its ability to contrast the cumulative movement away from REE through the interest rate rule – or in other terms it weakens the positive feedback element in the IS curve.

¹⁵Based on this argument we can correctly conclude that a central bank that takes into account the private agents learning process, by announcing the interest rate path consistent with the MSV solution would help to stabilize expectations. In fact, realizing that agents are learning means that previous beliefs, Γ_{t-1} , are an additional state variable of the system and the MSV solution would be a function also of it. An interest rate that responds directly to this variable would have the same stabilizing properties of a policy rule that respond to current (or expected) inflation and output gap, as it would be able to offset the initial deviations from the REE.

Figure 1: E-stability under no announcement, $(1 - \lambda_1) = 0$, and under a fully internalized announcement of the interest rate path, $(1 - \lambda_1) = 1$.

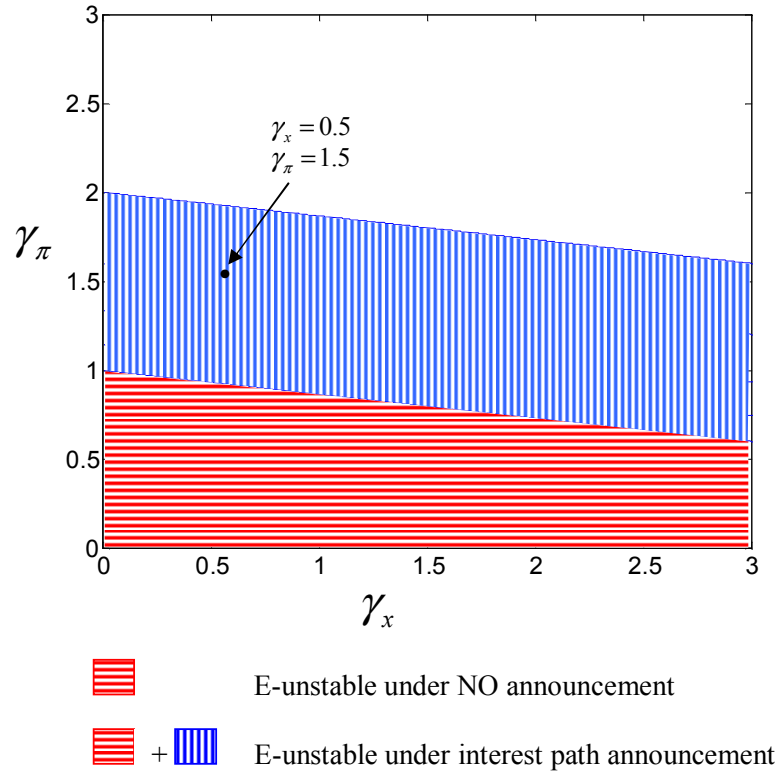


Figure 1 compares the regions of E-stability in the (γ_x, γ_π) space under no announcement and under announcement of the interest rate path. The lower region shows the set of policies that implies instability under learning when the central bank is silent about the interest rate path, $(1 - \lambda_1) = 0$. Publishing the path, the central bank enlarges the region of instability – the larger the weight the agents give to the announcement, the larger the region of instability under learning. In particular, when the weight that private agents give to the projection is larger than 0.65, the classical Taylor rule with $\gamma_x = 0.5$ and $\gamma_\pi = 1.5$ would fall in the region of instability under learning.

3.2 Speed of convergence

In the previous sections we have analyzed the effect of announcing the interest rate path on the long-run properties of the equilibrium under learning. Combinations of (γ_x, γ_π) that imply a determinate and E-stable REE are usually defined in the literature as "good" policies (Bullard and Mitra, 2002). The concept of speed of convergence can be used in order to refine further the set of these policies (see Ferrero, 2007). If convergence is rapid, we may think to focus on asymptotic behaviors,

because the economy would typically be close to the REE. In this case the publication of projections obtained under the assumption of fully rational private agents would have a minor effect on the stability of the economy. Conversely, if convergence is slow, the economy would be far from the REE for a long period of time and its behavior would be dominated by the transitional dynamics. In this case the consequences associated to the incorrect assumption that private agents are perfectly rational may result significantly more severe.

In the literature, the speed of convergence of recursive least square learning algorithms in stochastic models has been analyzed mainly through numerical procedures and simulations. The few analytical results on the transition to the rational expectations equilibrium environment are obtained by using a theorem of Benveniste, Metivier and Priouret (1990) that relates the speed of convergence of the learning process to the derivative of the associated ODE at the fixed point. In the present case, the ODE's to be analyzed are those described in expressions (3.11)–(3.12).

We define

$$S_1 = \left\{ \gamma_\pi, \gamma_x : \gamma_\pi > \max \left[\frac{\frac{(\beta^2 + 2\alpha\varphi)}{\alpha\varphi(1+\lambda_1+\beta)} - \frac{(1+\lambda_1-\beta^2)}{\alpha(1+\lambda_1+\beta)}\gamma_x}{\frac{4(2\beta+1)\alpha\varphi - (2\beta^2-1)}{(2\beta+1)(1+2\lambda_1)\alpha\varphi} - \frac{(1-2\beta^2)}{(1+2\beta)\alpha}\gamma_x} \right] \right\}$$

the set of policies – combinations of γ_π and γ_x – under which all the eigenvalues of the $\tilde{F} + \tilde{V}$ matrix have real part smaller than 0.5.

The following proposition, adapting arguments from Marcet and Sargent (1995), shows that by choosing the γ_π and γ_x , the policy-maker not only determines the level of inflation and output gap and their stability in the long run, but also the speed at which the economy converges to the REE, i.e. the speed at which agents learn.

Proposition 2. *In an economy that (i) evolves according to the system of equations (3.8), where (ii) private agents assign weight $0 \leq (1 - \lambda_1) \leq 1$ to the central bank's announcement, and (iii) the central bank chooses a policy $(\gamma_\pi, \gamma_x) \in S_1$, then*

$$\sqrt{t}(\Gamma_t - \Gamma) \xrightarrow{D} N(0, \Omega_\Gamma)$$

where the matrix Ω_Γ satisfies

$$\left[\frac{I}{2} (F + V - I) \right] \Omega_\Gamma + \Omega_\Gamma \left[\frac{I}{2} (F + V - I) \right]' + E[T(\Gamma') - \Gamma'] [T(\Gamma') - \Gamma']' = 0 \quad (3.14)$$

Proof. see Appendix 3. □

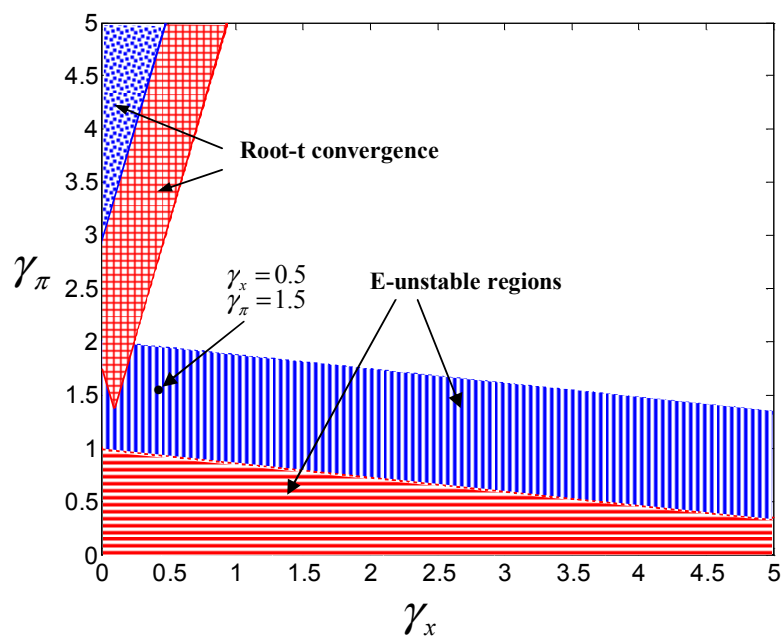
If the conditions of Proposition 2 are satisfied, the estimated Γ_t converges to the REE, Γ , at root- t speed. Root- t is the speed at which, in classical econometrics, the least square estimator converges to the true value of the parameters estimated. Note that the formula for the variance of the estimator Γ_t is modified with respect to the classical case. In particular, if $(\gamma_\pi, \gamma_x) \in S_1$, the higher the eigenvalues of $\tilde{F} + \tilde{V}$, the larger the asymptotic variance of the limiting distribution (Marcet and Sargent, 1995). In this case, convergence is slower in the sense that the probability that a shock will drive the estimates far away from the REE is higher and the period of time that agents need in order to learn it back is larger (see Ferrero, 2007).

Proposition 3. *In an economy that (i) evolves according to the system of equations (3.8), where (ii) private agents assign weight $0 \leq (1 - \lambda_1) \leq 1$ to the central bank's announcement, and (iii) the central bank chooses a policy $(\gamma_\pi, \gamma_x) \in S_1$, revealing the path makes condition for root- t convergence more stringent than under no announcement. In particular, the smaller the weight to the announcement, the larger the set of policies under which private agents learn at root- t speed.*

Proof. see Appendix 4.

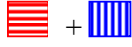
In Figure 2 we focus on the two extreme cases where there is no announcement, $(1 - \lambda_1) = 0$, and where private agents fully internalize the announcement, $(1 - \lambda_1) = 1$. Figure 2 shows that (i) the set of combinations (γ_x, γ_π) resulting in

Figure 2: E-stability & root- t convergence under no announcement and under fully internalized announcement of expected interest rates





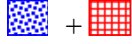
E-unstable - NO announcement



E-unstable - interest path announcement



Root-T convergence - interest path announcement



Root-T convergence - NO announcement

root- t convergence is much smaller than the one under which E-stability holds and (ii) the region of "fast" convergence (i.e. root- t convergence) is smaller when the central banks announces its policy (the smallest region in the upper-left corner) than under no announcement (the sum of the two upper-left corner regions).

Let's now define

$$S_2 = \left\{ (\gamma_\pi, \gamma_x) \in R_+^2 : \frac{2}{(1+\lambda_1)} - \frac{(1-\beta)}{\alpha} \gamma_x < \gamma_\pi < \max \left[\frac{\frac{(\beta^2+2\alpha\varphi)}{\alpha\varphi(1+\lambda_1+\beta)} - \frac{(1+\lambda_1-\beta^2)}{\alpha(1+\lambda_1+\beta)}}{\frac{4(2\beta+1)\alpha\varphi-(2\beta^2-1)}{(2\beta+1)(1+2\lambda_1)\alpha\varphi}} - \frac{(1-2\beta)}{(1+2\beta)} \right] \right\}$$

the set of policies under which all the eigenvalues of $\tilde{F} + \tilde{V}$ have real part less than one but not all have real part less than 0.5.

Although Propositions 2 and 3 do not apply when $(\gamma_\pi, \gamma_x) \in S_2$, it can be shown by Monte Carlo calculations that under those policies the effects of initial conditions fail to die out at an exponential rate (as it is needed for root- t convergence) and agents' beliefs converge to rational expectations at a rate slower than root- t . In particular, also when $(\gamma_\pi, \gamma_x) \in S_2$, the link between the derivative of the ODE and the speed of convergence holds.

Marcet and Sargent (1995) suggest a numerical procedure to obtain an estimate of the rate of convergence when $(\gamma_\pi, \gamma_x) \in S_2$. In this case it is possible to define the rate of convergence, δ , for which

$$t^\delta (\Gamma_t - \Gamma) \xrightarrow{D} F \quad (3.15)$$

for some non-degenerate well-defined distribution F with mean zero and variance Ω_F .

Expression (3.15) can be used to obtain an approximation of the rate of convergence¹⁶ for large t . Since $E[t^\delta (\Gamma_t - \Gamma) (\Gamma_t - \Gamma)'] = \Omega_F$ as $t \rightarrow \infty$, we have that

¹⁶The calculation of the rate of convergence is based on the assumption that such a δ exists.

$$\frac{E \left[t^{2\delta} (\Gamma_t - \Gamma) (\Gamma_t - \Gamma)' \right]}{E \left[(tz)^{2\delta} (\Gamma_{tz} - \Gamma) (\Gamma_{tz} - \Gamma)' \right]} \rightarrow 1 \quad \text{or} \quad \delta = \frac{1}{2 \log z} \log \frac{E \left[(\Gamma_t - \Gamma) (\Gamma_t - \Gamma)' \right]}{E \left[(\Gamma_{tz} - \Gamma) (\Gamma_{tz} - \Gamma)' \right]}.$$

The expectations can be approximated by simulating a large number of independent realizations of length t and tz , and calculating the mean square distance from Γ across realizations for each coefficient. Table 1 reports the rate of convergence, δ , the real part of the largest eigenvalue of the $(\tilde{F} + \tilde{V})$ matrix, k , the number of quarters needed in order to halve the initial expectation error, $T_{1/2}$, and the number of quarters needed in order to reduce to one third the initial error, $T_{1/3}$, for different values of $(\gamma_\pi, \gamma_x) \in S_2$ ¹⁷.

Calculations show that (i) for a given response to inflation, γ_π , the larger the response to output gap, γ_x , the higher the real part of the larger eigenvalue, the smaller δ and the lower the speed of convergence; (ii) the opposite relation holds for

Table 1: Speed of convergence and simulations

		$\gamma_\pi = 1.5$		$\gamma_\pi = 2.5$		$\gamma_\pi = 3.5$	
		$\lambda = 1$	$\lambda = 0$	$\lambda = 1$	$\lambda = 0$	$\lambda = 1$	$\lambda = 0$
$\gamma_x = 0.25$	k	0.76	>1	0.19	0.86	0.07	0.62
	δ	0.24	N.A.	0.5	0.14	0.5	0.37
	$T_{1/2}$	72	N.A.	11	>400	7	20
	$T_{1/3}$	>400	N.A.	23	>400	12	66
$\gamma_x = 0.5$	k	0.85	>1	0.63	0.90	0.42	0.73
	δ	0.14	N.A.	0.36	0.10	0.5	0.26
	$T_{1/2}$	>400	N.A.	21	>400	10	40
	$T_{1/3}$	>400	N.A.	67	>400	24	195
$\gamma_x = 1$	k	0.91	>1	0.79	0.92	0.67	0.82
	δ	0.08	N.A.	0.21	0.07	0.32	0.17
	$T_{1/2}$	>400	N.A.	78	>400	24	147
	$T_{1/3}$	>400	N.A.	>400	>400	89	>400

NOTE: Initial expectation error is 10 per cent of the REE. In all simulations we compute the rate of convergence, δ , with 1000 independent realizations for $t=9000$ and $tz=10000$ periods; k is the real part of the largest eigenvalue of the $F + V$ matrix; $T_{1/2}$ indicates the quarters needed in order to reduce the inflation forecast error to one half of the initial bias; $T_{1/3}$ indicates the quarters needed in order to reduce the inflation forecast error to one third of the initial bias.

the response to inflation: for a given response to output gap, γ_x , the larger γ_π , the higher the speed of convergence; (iii) for a given (γ_π, γ_x) policy, the announcement

¹⁷Simulations are obtained under Clarida, Galí and Gertler (CGG, 2000) calibration: US data, $\varphi = 4$, $\alpha = 0.075$, $\beta = 0.99$; We use quarterly interest rates and we measure inflation as quarterly changes in the log of prices. Therefore our CGG calibration divides by 4 the α and multiplies by 4 the φ reported by CGG (see also Honkapohja and Mitra, 2004).

of the interest rate path has a large impact on the speed of convergence. For the Taylor rule's parameter ($\gamma_\pi = 1.5, \gamma_x = 0.5$), under no announcement we need more than 100 years in order to halve the initial expectation error, while when the announcement is fully internalized agents never learn. A stronger response to inflation speeds up the learning process, but differences between announcing or not the interest rate path remain substantial: under no announcement, for $\gamma_\pi = 3.5$ and $\gamma_x = 0.5$, the initial error is halved in about 2.5 years, but still we need about 10 years under announcement.

In order to formally map elements of the set of policy rules into a measure of the speed of convergence we define the *speed of convergence isoquants*.¹⁸

Definition 1. A speed of convergence isoquant- k is a curve in R^2 along which all points – combinations (γ_π, γ_x) – imply that the largest eigenvalue of $\tilde{F} + \tilde{V}$ has real part equal to k . In an economy that evolves according to the system of equations (3.8), the k -isoquant satisfies

$$\gamma_\pi = \max \left[\frac{\frac{(1-2k+\beta^2+2\alpha\varphi)}{(2k+\beta+\lambda_1)\alpha\varphi} - \frac{(-\beta^2+2k+\lambda_1)}{(2k+\beta+\lambda_1)\alpha}\gamma_x}{\frac{2}{(k+\lambda_1)} - \frac{(\beta^2-k)(1-k)}{\alpha\varphi(k+\beta)(k+\lambda_1)} - \frac{(k-\beta^2)}{\alpha(k+\beta)}\gamma_x} \right]. \quad (3.16)$$

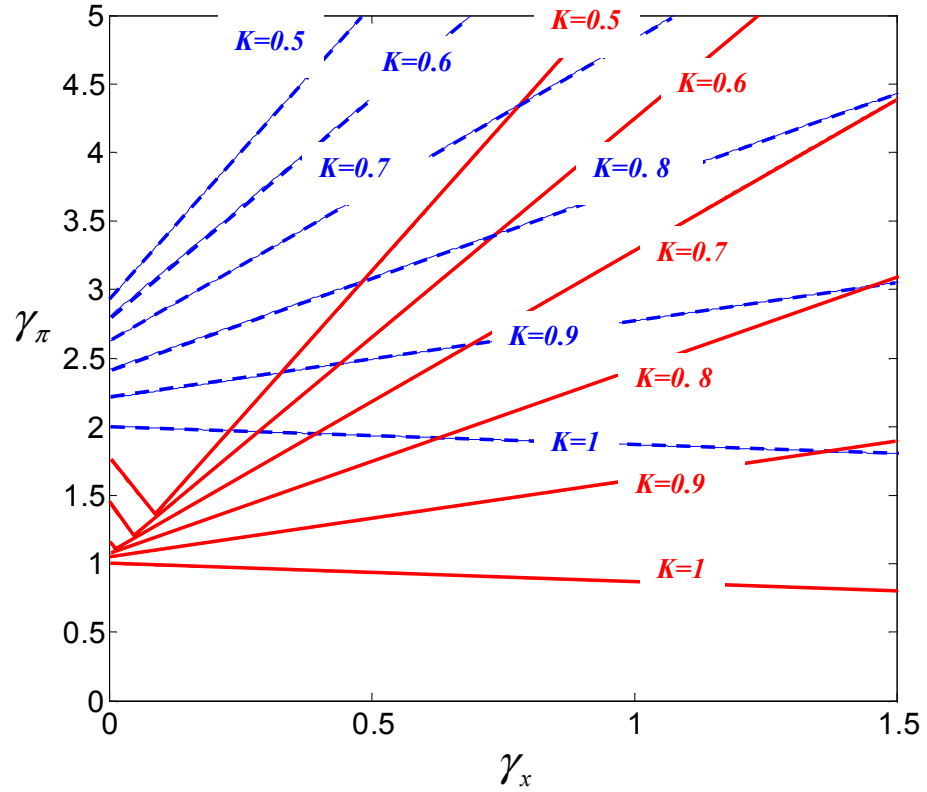
Figure 3 shows the map of the speed of convergence isoquants in the two extreme cases where there is no announcement, $(1 - \lambda_1) = 0$, and where the agents fully internalize the announcement, $(1 - \lambda_1) = 1$. We observe that, for a given λ_1 , the lower the isoquant, the slower the convergence. In fact, from Marcet and Sargent (1995), the larger the real part of the largest eigenvalue of $\tilde{F} + \tilde{V}$, the slower the convergence and the lower the isoquant. Moreover, for a given policy, the speed at which agents learn is lower if the central banks announces its policy path.

For example, consider the point $(\gamma_\pi, \gamma_x) = (1.5, 0.5)$ in the isoquant map. Being this point below the $k = 1$ isoquant in the mapping obtained under announcement (dotted lines), private agents never learn. Under no announcement they learn, but very slowly, as the $(\gamma_\pi, \gamma_x) = (1.5, 0.5)$ point is close to the 0.8 isoquant in the continuous-line mapping. Increasing γ_π to 2.5 we reach the E-stable region under announcement, but learning is very slow (the point $(\gamma_\pi, \gamma_x) = (2.5, 0.5)$ is between the 0.8 and the 0.9 isoquant in the dotted mapping); under no announcement convergence is much faster, close to root-t (the 0.5 isoquant in the continuous-line mapping).

The next proposition formalizes these results.

¹⁸In the definition we tie up speed of convergence with the eigenvalues of the matrix $\tilde{F} + \tilde{V}$. In general, the speed of convergence depends on the eigenvalues of the derivatives of the mapping from PLM to ALM, $T(A)$. In this case, the derivative is equal to $\tilde{F} + \tilde{V}$ (see Ferrero, 2003).

Figure 3: The speed of learning isoquants for $\lambda_1 = 0$ (dotted line) and $\lambda_1 = 1$ (continue line)



The next proposition formalizes these results.

Proposition 4. *In an economy that (i) evolves according to the system of equations (3.8), where (ii) private agents assign weight $0 \leq (1 - \lambda_1) \leq 1$ to the central bank's announcement, and (iii) the central bank chooses a policy $(\gamma_\pi, \gamma_x) \in S_2$, for a given γ_x , the smaller the weight to the policy path projections, the smaller has to be γ_π in order to reach the same speed of convergence. Or in other terms, for a given combination of (γ_x, γ_π) , the smaller the weight that private agents assign to the policy path projections, the faster the learning process.*

Proof. see Appendix 5. □

4 Announcing expected inflation and output gap

In the previous sections we have shown that in a world where private agents are learning from past data – along their learning process they produce biased predictions of the main macro variables – a central bank that publishes its projection obtained under the incorrect assumption that private agents are perfectly rational reduces the speed at which agents learn¹⁹.

In this section we analyze the implications in terms of E-stability and speed of convergence when the central bank announces its projections about inflation and output gap, possibly in addition to the interest rate path. We assume that also these projections are obtained under the incorrect assumption that private agents are perfectly rational,

$$E_t^{CB} y_{t+1} = A + B\Psi w_t.$$

The main difference between the announcement of the policy path and the announcement of inflation and output gap paths is that, while the interest rate (the instrument variable of the model) is implicitly used by private agents in order to form expectations about future inflation and output gap (the control variables of the model), information about output gap and inflation are used directly to predict those variables. Initial expectation biases are immediately reduced with no need for the stabilizing properties of interest rate rules that respond to actual (or expected) inflation and output gap. Therefore, by announcing its inflation and output gap expectations, the central bank helps agents to learn faster and enlarges the set of monetary policies associated with stable equilibria under learning.

Let $0 \leq (1 - \lambda_2) \leq 1$ be the weight that private agents give to the central bank's announcement of inflation and output gap

$$\begin{aligned} E_t^P \pi_{t+1} &= (1 - \lambda_2) E_t^{CB} \pi_{t+1} + \lambda_2 E_t^* \pi_{t+1} \\ E_t^P x_{t+1} &= (1 - \lambda_2) E_t^{CB} x_{t+1} + \lambda_2 E_t^* x_{t+1}. \end{aligned}$$

Under learning the economy evolves according to the system of equation

$$y_t = \tilde{Q} + \tilde{F} \times E_t^* y_{t+1} + \tilde{V} \times E_t^* y_{t+2} + \tilde{S} w_t. \quad (4.1)$$

where matrices $\tilde{Q}$, $\tilde{F}$, $\tilde{V}$ and $\tilde{S}$ are derived in Appendix 7.

¹⁹Here we are not analyzing the important implications in terms of welfare. In particular we are not saying that a slower convergence will necessarily imply a lower social welfare. For an analysis of speed of convergence and welfare in a New-Keynesian model see Ferrero 2007.

4.1 Announcing only expected inflation and output gap

First we focus on the case in which the central bank only announces the inflation and output gap projections and private agents partly internalize this announcement to form their expectations, i.e. $\lambda_1 = 1$ and $0 < (1 - \lambda_2) \leq 1$.²⁰

Proposition 5. *In an economy that (i) evolves according to the system of equation (4.1), where (ii) the central bank publishes only the projection about inflation and output gap consistent with the REE and (iii) private agents assign weight $0 < (1 - \lambda_2) \leq 1$ to those projections, revealing the path makes condition for stability under learning less stringent and the region of root-t convergence larger than under no announcement. In particular, the necessary and sufficient condition for E-stability of the REE is*

$$\gamma_\pi > \frac{2\lambda_2(\beta\lambda_2 + 1)\alpha\varphi - (\beta^2\lambda_2 - 1)(\lambda_2 - 1)}{(\beta\lambda_2 + 1)(1 + \lambda_2)\alpha\varphi} - \frac{(1 - \beta^2\lambda_2)}{(\beta\lambda_2 + 1)\alpha}\gamma_x \quad (4.2)$$

Proof. See Appendix 7. □

Announcing inflation and output gap projections has opposite implications with respect to publish the interest rate path. The former enlarges the region of policies that imply an E-stable equilibrium and root-t convergence, while the latter enlarges the region of instability and slow convergence.

Figure 4 shows the region of E-stability and root-t convergence when the central bank announces only the inflation and output gap and the private sector gives weight 0.25 to this announcement. It is sufficient that private agents give a little weight to inflation and output gap projections in order to shrink substantially the region of instability under learning and to increase the speed of convergence.

4.2 Announcing expected interest rate, inflation and output gap

In practice, all central banks that publish their own interest rate projections were previously publishing their inflation and output gap expectations. Therefore, we also focus on the case where the central bank announces both the interest rate path and the inflation and output gap projections and the private agents at least partially internalize these announcements to form their expectations.²¹

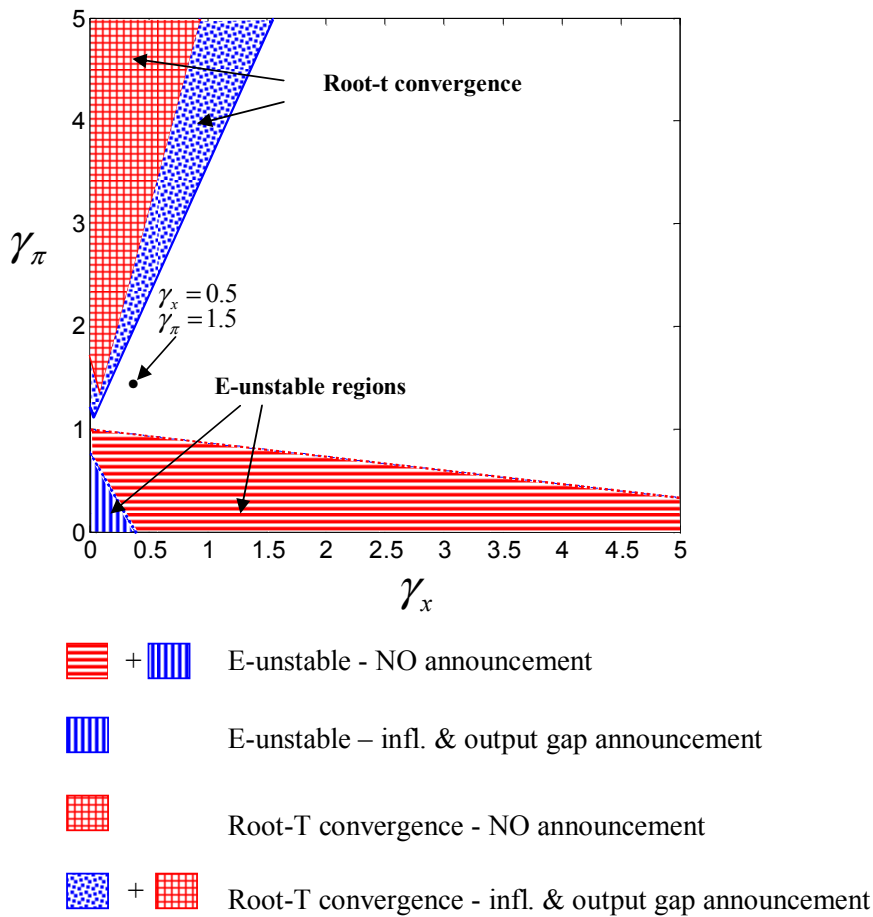
²⁰Again, we may assume that a fraction of private agents in the economy uses its own learning procedure to form expectations, while the remaining fraction fully internalizes the central bank's announcement.

²¹By allowing for a $\lambda_1 \neq \lambda_2$ we are able to analyze the case in which different typology of agents

Clearly the two types of announcement have opposite effects in the dynamics of the economy.

In Table 2 we report for different combinations of λ_1 and λ_2 , the speed of convergence and the number of periods needed in order to reduce to 1/3 the initial forecast error, when the central bank sets the interest rate according to a Taylor rule with coefficients $(\gamma_\pi, \gamma_x) = (1.5, 0.5)$. We observe that (i) for a given λ_2 , the larger the weight to the interest rate announcement (lower λ_1), the slower the convergence;

Figure 4: E-stability and root-t convergence under no announcement and under announcement only of expected inflation and output gap



Note: Here we are assuming that central bank's announcement about inflation and output gap expectations, is partially internalized by private agents ($1 - \lambda_2 = 0.25$)

in the economy pay different attention to the announcements. For example market operators may pay more attention to interest rate announcements, while workers to inflation expectations. We are also able to study the case in which the communication strategy of the central bank focuses more on a particular macroeconomic variable, depending on the specific macroeconomic environment in which it operates.

moreover, (ii) it is sufficient a low weight to the announcement of inflation and output gap in order to increase significantly the speed of convergence; finally (iii) for sufficiently low λ_1 and high λ_2 the negative effects of the interest rate path dominate the positive effects of the inflation and output announcement. Proposition 6 states formally this last result.²²

Table 2: Speed of convergence and simulations

		$\lambda_2 = 0.5$	$\lambda_2 = 0.6$	$\lambda_2 = 0.7$	$\lambda_2 = 0.8$	$\lambda_2 = 0.9$	$\lambda_2 = 1.0$
$\lambda_1 = 0.0$	δ	0.46	0.35	0.24	0.13	0.02	N.A.
	$T_{1/3}$	13	28	110	>400	>400	N.A.
$\lambda_1 = 0.1$	δ	0.49	0.38	0.27	0.16	0.06	N.A.
	$T_{1/3}$	12	24	75	>400	>400	N.A.
$\lambda_1 = 0.2$	δ	0.50	0.41	0.30	0.19	0.09	N.A.
	$T_{1/3}$	12	21	57	>400	>400	N.A.
$\lambda_1 = 0.3$	δ	0.50	0.43	0.32	0.22	0.11	0.00
	$T_{1/3}$	11	19	46	255	>400	>400
$\lambda_1 = 0.4$	δ	0.50	0.45	0.34	0.24	0.13	0.03
	$T_{1/3}$	10	17	38	167	>400	>400
$\lambda_1 = 0.5$	δ	0.50	0.47	0.36	0.26	0.16	0.05
	$T_{1/3}$	10	16	33	121	>400	>400
$\lambda_1 = 0.6$	δ	0.50	0.48	0.38	0.28	0.18	0.07
	$T_{1/3}$	9	15	30	94	>400	>400
$\lambda_1 = 0.7$	δ	0.50	0.49	0.40	0.29	0.20	0.09
	$T_{1/3}$	9	14	27	76	>400	>400
$\lambda_1 = 0.8$	δ	0.50	0.50	0.41	0.31	0.21	0.10
	$T_{1/3}$	9	13	24	64	>400	>400
$\lambda_1 = 0.9$	δ	0.50	0.50	0.42	0.32	0.22	0.12
	$T_{1/3}$	9	13	23	55	287	>400
$\lambda_1 = 1.0$	δ	0.50	0.50	0.43	0.33	0.24	0.14
	$T_{1/3}$	8	12	21	49	218	>400

NOTE: Initial expectation error is 10 per cent of the REE. In all simulations we compute the rate of convergence, δ , with 1000 independent realizations for $t=9000$ and $tz=10000$ periods; $T_{1/3}$ indicates number of quarters needed in order to reduce the inflation forecast error to one third of the initial bias. Interest rate rule coefficients are $(\gamma_\pi, \gamma_x) = (1.5, 0.5)$.

Proposition 6. *In an economy that (i) evolves according to the system of equation (4.1), where (ii) the central bank publishes both the interest rate path and the inflation and output gap projections consistent with the REE and (iii) private agents give weight $0 < (1 - \lambda_1) \leq 1$ to the former and $0 < (1 - \lambda_2) \leq 1$ to the latter, in order*

²²In any case also when the positive effect of the inflation and output gap announcement dominates, the publication of the interest rate path worsen the properties of the equilibrium in terms of stability and speed of convergence with respect to the case in which the central bank is silent about its own interest rate projections obtained under the erroneous assumption that private agents are perfectly rational.

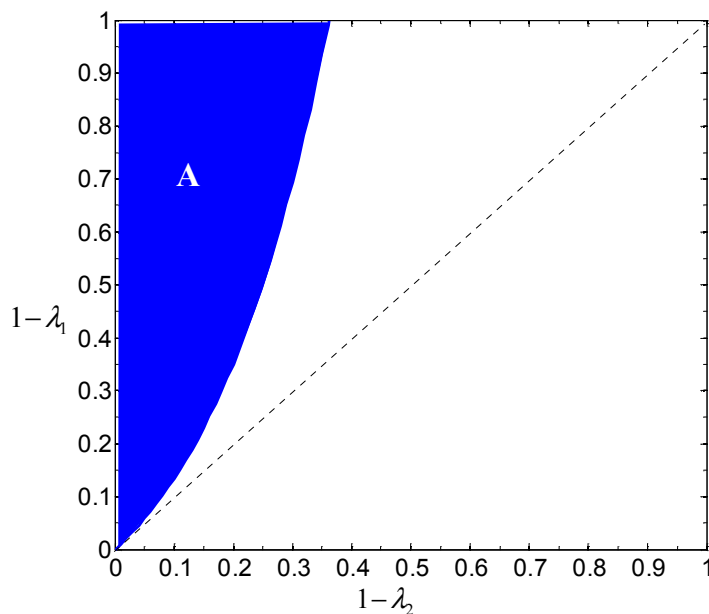
to have condition for *E*-stability less stringent under no announcement than under announcement, the necessary condition is that

$$0 \leq \lambda_1 < 2 - \frac{(1 - \lambda_2)(1 - \beta^2 \lambda_2)}{\lambda_2 \alpha \varphi (1 + \beta \lambda_2)} - \frac{1}{\lambda_2} \quad (4.3)$$

Proof. See appendix 8. $\square$

In Figure 5, region A shows the combination of weights to the two projections under which the announcement of the paths reduces the region of stability under learning. In particular when private agents give relatively high weights to the interest rate path and low weights to the inflation and output gap projections, transparency increases the region of instability under learning.

Figure 5: Weights to the projections and E-stability when the central bank announces interest rate, inflation and output gap paths



5 Extensions

In this section we focus on some extensions of the baseline model. In particular we analyze how our main results are affected by: (i) the announcement of a longer interest rate path, (ii) a policy rule that responds to expected inflation and output gap and (iii) the announcement also includes a subjective judgmental component.

5.1 Publication of a longer path

One interesting extension considers the case where the central bank announces a longer path. The following proposition states that the longer the horizon T of the announced path, the larger the region of instability.

Proposition 7. *In an economy that (i) evolves according to equations (3.8), where (ii) private agents assign weight $0 \leq (1 - \lambda_1) \leq 1$ to the central bank's announcement, the longer is the path revealed the more stringent are conditions for E-stability. In particular, the necessary and sufficient condition for the MSV solution to be E-stable when the central bank announces the T -period path is*

$$\gamma_\pi > \frac{T}{(1 + \lambda_1(T - 1))} - \frac{(1 - \beta)}{\alpha} \gamma_x \quad (5.2)$$

Proof. See Appendix 9.

Figure 6: E-stability and the announcement of a T -period interest rate path

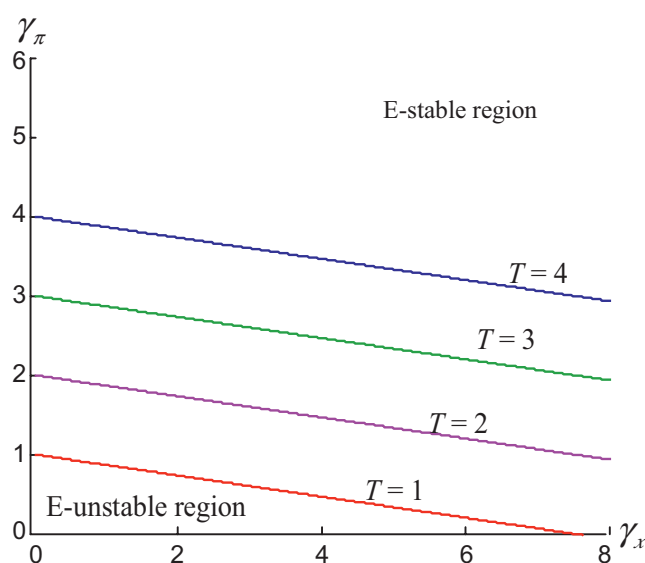


Figure 7 reports for different T s the regions where the REE is E-stable and shows that for T sufficiently large, no realistic policy will be able to drive the economy back to the REE.

5.2 Forward expectations in the policy rule

We consider now an expectation-based policy rule:

$$i_t = \gamma + \gamma_x E_t^* x_{t+1} + \gamma_\pi E_t^* \pi_{t+1} \quad (5.3)$$

so that i_t is set by the monetary authority in response to forecast of next period inflation and output gap.

As shown in Bullard and Mitra necessary and sufficient conditions for the REE to be E-stable is

$$\gamma_\pi > 1 - \frac{1 - \beta}{\alpha} \gamma_x \quad (5.6)$$

Proposition 8 shows that under expectation-based policy rules conditions for E-stability when the central bank announces the interest rate path are the same of those obtained under contemporaneous policy rules. In particular, also under the hypothesis that the central bank follows an expectations-based policy rule, revealing the path makes conditions for E-stability more stringent than under no announcement.

Proposition 8. *Let $\varphi\gamma_x + \alpha\varphi\gamma_\pi + 1 \neq 0$. In an economy where (i) the central bank follows an expectations-based policy rule (5.3), (ii) at time t the central bank publishes the time $t+1$ interest rate projection consistent with the REE and (iii) private agents give weight $0 \leq (1 - \lambda_1) \leq 1$ to these projections, revealing the path makes condition for E-stability more stringent than under no announcement. In particular, the necessary and sufficient condition for stability under learning (E-stability) of the equilibrium (2.6) is*

$$\gamma_\pi > \frac{2}{(1 + \lambda_1)} - \frac{1 - \beta}{\alpha} \gamma_x. \quad (5.7)$$

Proof. See appendix 10. □

5.3 Announced path with a subjective judgmental component

In the previous sections we have assumed that central banks projections are model consistent and derived under the assumption of rational expectations. Here we

investigate the effect of introducing a subjective and judgemental component in the projection. We model this component by introducing a shock in the policy path,²³

$$E_t^{CB}i_{t+1} = a_i + \rho_u b_{u,i} u_t + \rho_g b_{g,i} g_t + z_t \quad (5.8)$$

with

$$z_t = \rho_z z_{t-1} + \varepsilon_{zt} \quad (5.9)$$

where ε_{zt} is white noises with variances σ_z^2 . Under this assumption the only elements that change in the system of equations describing the evolution of the economy

$$y_t = \tilde{Q} + \tilde{F} \times E_t^* y_{t+1} + \tilde{V} \times E_t^* y_{t+2} + \tilde{S} w_t, \quad (5.10)$$

are the matrix $\tilde{S}$ and the vector of exogenous state variables w_t , that now includes also the z_t variable.

$$w_t' = \begin{bmatrix} u_t & g_t & z_t \end{bmatrix}.$$

As the matrices $\tilde{F}$ and $\tilde{V}$ are not affected by the introduction of the additional term in the announced path, all the results in terms of E-stability and speed of convergence obtained in the previous sections hold also when we introduce a judgmental component modeled as an exogenous shock in the projections.

²³This is only one way to introduce the judgmental component. A more general analysis is left for future research.

6 Conclusions

In this paper we have studied a standard New-Keynesian economy where private agents are learning, the central bank implements monetary policy through an interest rate rule and it publishes its macroeconomic projections. To stay close to the current practice of most of the central banks which follow this communication strategy we have assumed that projections are obtained under the assumption that the economy is populated by perfectly rational agents.

Our analysis has focused on the effects of such a publication on the stability of the equilibrium and on the speed of convergence of private expectations towards rationality. The main result is that the effect depends on the economic variable which is published. In particular when the central bank reveals information about its own expected interest path, conditions for stability under learning become more stringent and the speed of convergence slows down. In such a situation macroeconomic stability and a faster process of convergence of expectations toward rationality can be restored only if the central bank selects an interest rule that responds more aggressively against inflation. On the contrary, the announcement of expected inflation and output gap enlarges the set of policy rules which are consistent with stability and a fast process of convergence.

Our analysis suggests that a central bank, before deciding to publish interest rate projections, should investigate the expectation formation process of private agents and assess how far such a mechanism is from perfect rationality. The greater the distance, the higher the risk that the release of interest rate projections might exert adverse effects on the economy. Our analysis suggests that particular attention should be paid to those situations in which private agents, in forming their expectations, are likely to put a larger weight on the announcement of the interest rate path than on the inflation and output gap projections. In those cases, in fact, the set of policy rules that guarantees stability and a fast convergence of expectations could be even smaller than in the no announcement case.

It is finally worthwhile to notice that our analysis is not necessarily against the publication of interest rate projections when agents are learning. If on one side we have concluded that in this case the publication of rational interest rate projection exerts a negative effect on the economy, on the other the central bank by providing to the public alternative interest rate projections that take into account the true expectation formation process of private agents, could strengthen the macroeconomic stability and increase the speed of convergence of private expectations towards perfect rationality.

References

- [1] Adolfson, M., Laseen, S., Lindé, J., and Mattias Villani "RAMSES – a new general equilibrium model for monetary policy analysis", 2007, Economic Review 2, Sveriges Riksbank.
- [2] Archer D., "Central Bank communication and the publication of interest rate projections", 2005, BIS Working paper.
- [3] Benveniste, A., Metivier, M. and P. Priouret, "Adaptive Algorithms and Stochastic Approximations", 1990, Springer-Verlag, Berlin Heidelberg.
- [4] Black, R., Cassino, V., Drew, A., Hansen, E., Hunt, B., Rose, D., and Alasdair Scott, "The Forecasting and Policy System: the core model", 1997, Research paper No. 43, Reserve Bank of New Zealand.
- [5] Blinder, A.S., "Central-Bank Credibility: Why Do We Care? How Do We Build It?," 2000, American Economic Review, vol. 90(5), pp 1421-1431.
- [6] Brubakk, L., Husebø, T.A., Maih, J., Olsen, K., and Magne Østnor. "Finding NEMO: Documentation of the Norwegian economy model", 2006, Staff Memo, Norges Bank, 2006/6.
- [7] Bullard, J. and K. Mitra, "Learning about Monetary Policy Rules.", 2002, Journal of Monetary Economics 49, 1105-1129.
- [8] Evans, G.W. and S. Honkapohja, "Learning and Expectations in Macroeconomics", 2001, Princeton University Press, Princeton, New Jersey.
- [9] Evans, G.W. Honkapohja, S., and K. Mitra, "Notes on Agents' Behavioral Rules Under Adaptive Learning and Recent Studies of Monetary Policy", 2003, mimeo.
- [10] Eusepi S. and B. Preston, "Central Bank Communication and Expectations Stabilization", 2010, American Economic Journal: Macroeconomics, Forthcoming.
- [11] Ferrero, G., and A. Secchi "The Announcement of Future Policy Intentions", 2009, Bank of Italy Working Paper No. 720.
- [12] Goodhart, C. "The Interest rate conditioning assumption", 2005, Working paper.
- [13] Kahn, G.A., "Communicating a policy path: the next frontier in central bank transparency?", 2007, Federal Reserve Bank of Kansas City, Economic Review, Q1.

- [14] Marcet, A. and T.J. Sargent, "Convergence of Least Squares Learning Mechanism in Self-Referential Linear Stochastic Models.", 1989, *Journal of Economic Theory* 48, 337-368.
- [15] Marcet, A. and T.J. Sargent, "Speed of Convergence of Recursive Least Squares: Learning with Autoregressive Moving-Average Perceptions." In: Kirman, A., Salmon, M. (Eds.), *Learning and Rationality in Economics*, 1997, Basil Blackwell, Oxford, 179-215.
- [16] McCallum B.T., "On the relationship between determinate and MSV solutions in linear RE models", 2004, *Economics Letter* 84, 55-60.
- [17] Mishkin F., "Can Central Bank transparency go too far?", 2004 in *The future of inflation targeting*, Federal Reserve Bank of Australia, pp.51-80.
- [18] Morris S. and H. S. Shin, "The Social Value of Public Information", 2002, *American Economic Review* 92, pp. 1521-1534.
- [19] Rudebusch G.D. and J.C. Williams, "Revealing the Secrets of the Temple: The Value of Publishing Central Bank Interest Rate Projections", 2006, NBER Working Papers 12638, National Bureau of Economic Research, Inc.
- [20] Svensson, L., "The Instrument-rate projection under inflation targeting: the Norwegian example", 2006, CEPS Working paper No. 127.
- [21] Woodford M., "Central Bank Communication and Policy Effectiveness", 2005, NBER working Paper 11898.

Appendix: Proofs of propositions

Appendix 1) The REE under contemporaneous Taylor rules

Under the set of policies

$$\dot{i}_t = \gamma + \gamma_x x_t + \gamma_\pi \pi_t$$

the economy evolves according to the following stochastic dynamic system:

$$Hy_t = K + L \times E_t^* y_{t+1} + Mw_t,$$

with

$$y_t = \begin{bmatrix} \pi_t \\ x_t \\ \dot{i}_t \end{bmatrix}, \quad w_t = \begin{bmatrix} u_t \\ g_t \end{bmatrix}, \quad \Psi = \begin{bmatrix} \rho_u & 0 \\ 0 & \rho_g \end{bmatrix}, \quad \varepsilon_t \sim N(0, \Omega_\varepsilon)$$

$$H = \begin{bmatrix} 1 & -\alpha & 0 \\ 0 & 1 & \varphi \\ -\gamma_\pi & -\gamma_x & 1 \end{bmatrix}, \quad K = \begin{bmatrix} 0 \\ 0 \\ \gamma \end{bmatrix}, \quad L = \begin{bmatrix} \beta & 0 & 0 \\ \varphi & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad M = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

To calculate the minimum state variable (MSV) solution, we rewrite the system as

$$y_t = Q + F \times E_t^* y_{t+1} + Sw_t,$$

where

$$F = H^{-1}L, \quad Q = H^{-1}K, \quad S = H^{-1}M$$

The MSV solution satisfies

$$\begin{aligned} A &= Q + FA \\ B &= FB\Psi + S \end{aligned}$$

and, therefore,

$$\begin{aligned} A &= (I - F)^{-1}Q \\ \text{vec}(B) &= (I - \Psi' \otimes F)^{-1} \text{vec}(S). \end{aligned}$$

For determinacy we need all the eigenvalues of the F matrix to be inside the unit circle (Bullard and Mitra, 2001). One of the eigenvalues is equal to 0 and the other two can be obtained as a solution of the characteristic polynomial

$$X^2 + a_1X + a_2.$$

The necessary and sufficient condition for determinacy is $|a_2| < 1$ and $|a_1| < 1 + a_2$. This reduces to have

$$\gamma_\pi > 1 - \frac{(1 - \beta)}{\alpha} \gamma_x.$$

For E-stability we need the eigenvalues of the $F - I$ matrix and those of the $(\frac{dFB\Psi}{dB} - I)$ matrix to have the real part smaller than zero.

Since one of the eigenvalues of the $F - I$ matrix is equal to -1 and the others can be obtained from the characteristic polynomial

$$X^2 + a_1X + a_2,$$

where

$$a_1 = a_2 + c, \text{ with } c > 0$$

the necessary and sufficient condition for E-stability, $a_2 > 0$ and $a_1 > 0$, reduces to have $a_2 > 0$, that is

$$\gamma_\pi > 1 - \frac{(1 - \beta)}{\alpha} \gamma_x$$

It is easy to show that as ρ_u and ρ_g are smaller than 1 the previous condition is also sufficient for the eigenvalues of the $(\frac{dFB\Psi}{dB} - I)$ matrix to have the real part smaller than zero.

Appendix 2) Proof of proposition 1 (Announcement of the policy path and E-stability of the REE)

To study how the economy evolves under learning we rewrite the system as

$$\tilde{H}y_t = \tilde{K} + \tilde{L} \times E_t^* y_{t+1} + \tilde{P} \times E_t^* y_{t+2} + \tilde{M}w_t,$$

where

$$y_t = \begin{bmatrix} \pi_t \\ x_t \\ i_t \end{bmatrix}, \quad \tilde{K} = \begin{bmatrix} 0 \\ -\varphi(1 - \lambda_1)a_i \\ \gamma \end{bmatrix}, \quad \tilde{P} = \begin{bmatrix} \beta^2 & 0 & 0 \\ \varphi & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{aligned}\tilde{H} &= \begin{bmatrix} 1 & -\alpha & 0 \\ 0 & 1 & \varphi \\ -\gamma_\pi & -\gamma_x & 1 \end{bmatrix}, \quad \tilde{L} = \begin{bmatrix} 0 & \beta\alpha & 0 \\ \varphi & 0 & -\varphi\lambda_1 \\ 0 & 0 & 0 \end{bmatrix}, \\ \tilde{M} &= \begin{bmatrix} 1 + \beta\rho_u & 0 \\ -\varphi(1 - \lambda_1)\rho_u b_{u,i} & 1 + (1 - \varphi(1 - \lambda_1)b_{g,i})\rho_g \\ 0 & 0 \end{bmatrix}\end{aligned}$$

or

$$y_t = \tilde{Q} + \tilde{F} \times E_t^* Y_{t+1} + \tilde{V} \times E_t^* Y_{t+2} + \tilde{S} w_t,$$

where

$$\tilde{F} = \tilde{H}^{-1}\tilde{L} \text{ and } \tilde{V} = \tilde{H}^{-1}\tilde{P}.$$

For E-stability we need the eigenvalues of the $\tilde{F} + \tilde{V} - I$ matrix and of the $\left(\frac{d(FB\Psi + VB\Psi'\Psi)}{dB} - I\right)$ matrix to have the real part smaller than zero.

The characteristic polynomial of the $\tilde{F} + \tilde{V} - I$ matrix is

$$(X + 1)(X^2 + a_1X + a_2)$$

and the necessary and sufficient conditions for E-stability are $a_2 > 0$ and $a_1 > 0$. As

$$a_1 = a_2 + c, \quad \text{with } c > 0$$

the necessary and sufficient condition for E-stability is just $a_2 > 0$, that is

$$\gamma_\pi > \frac{2}{(1 + \lambda_1)} - \frac{(1 - \beta)}{\alpha}\gamma_x$$

We also need the eigenvalues of the $\left(\frac{d(\tilde{F}\tilde{B}\Psi + \tilde{V}\tilde{B}\Psi'\Psi)}{d\tilde{B}} - I\right)$ matrix to be smaller than zero.

Proceeding as we have done for the matrix $\tilde{F} + \tilde{V} - I$, it is easy to show that whenever the previous condition is satisfied also the eigenvalues of $\left(\frac{d(\tilde{F}\tilde{B}\Psi + \tilde{V}\tilde{B}\Psi'\Psi)}{d\tilde{B}} - I\right)$ have real part smaller than zero.

Appendix 3) proof of proposition 2 (Announcement of policy intentions and root-t convergence)

From Marcet and Sargent (1992) it follows that a necessary condition for root- t convergence is that the eigenvalues of $\tilde{F} + \tilde{V}$ and those of $\frac{d(\tilde{F}\tilde{B}\Psi + \tilde{V}\tilde{B}\Psi'\Psi)}{d\tilde{B}}$ have the real part smaller than $\frac{1}{2}$. On the other hand, agents beliefs will not converge to the

MSV solution at root- t speed if any eigenvalue of $\tilde{F} + \tilde{V}$ or $\frac{d(\tilde{F}\tilde{B}\Psi + \tilde{V}\tilde{B}\Psi'\Psi)}{dB}$ has real part more than $1/2$. Similarly to the proof in Appendix 2, it turns out that it is sufficient to have the real part of the eigenvalues of $\tilde{F} + \tilde{V}$ smaller than $\frac{1}{2}$.

The characteristic polynomial of $\tilde{F} + \tilde{V} - \frac{1}{2}I$ is

$$(X + 1)(X^2 + a_1X + a_2)$$

By applying the Routh theorem, the necessary and sufficient condition for root- t convergence that both eigenvalues of $\tilde{F} + \tilde{V}$ have real parts less than $1/2$ (i.e. both eigenvalues of $\tilde{F} + \tilde{V} - \frac{1}{2}I$ have negative real parts) reduces to have $a_1 > 0$ and $a_2 > 0$. Under the assumption that γ_x and γ_π are non negative, the necessary and sufficient condition is:

$$\gamma_\pi > \max \left[\frac{\frac{(\beta^2 + 2\alpha\varphi)}{\alpha\varphi(1+\lambda_1+\beta)} - \frac{(1+\lambda_1-\beta^2)}{\alpha(1+\lambda_1+\beta)}\gamma_x}{\frac{4(2\beta+1)\alpha\varphi - (2\beta^2-1)}{(2\beta+1)(1+2\lambda_1)\alpha\varphi}} + \frac{(2\beta^2-1)}{(1+2\beta)\alpha}\gamma_x \right]$$

Appendix 4) proof of proposition 3 (Root- t convergence under different weights to policy path announcement)

Given the set

$$S_1 = \left\{ \gamma_\pi, \gamma_x : \gamma_\pi > \max \left[\frac{\frac{(\beta^2 + 2\alpha\varphi)}{\alpha\varphi(1+\lambda_1+\beta)} - \frac{(1+\lambda_1-\beta^2)}{\alpha(1+\lambda_1+\beta)}\gamma_x}{\frac{4(2\beta+1)\alpha\varphi - (2\beta^2-1)}{(2\beta+1)(1+2\lambda_1)\alpha\varphi}} - \frac{(1-2\beta^2)}{(1+2\beta)\alpha}\gamma_x \right] \right\}$$

under which we have root- t convergence, we take derivatives of the two terms inside the max operator with respect to λ_1 and we show that the larger the λ_1 , that is the smaller the weight to the announcement (the extreme cases of $\lambda_1 = 1$ coincide with the case of no announcement), the smaller is the S_1 set, since the larger has to be γ_π in order to stay in the S_1 set.

As

$$\frac{\partial \frac{(1+\lambda_1-\beta^2)}{\alpha(1+\lambda_1+\beta)}}{\partial \lambda_1} > 0$$

and

$$\frac{\partial \frac{(\beta^2 + 2\alpha\varphi)}{(1+\beta+\lambda_1)\alpha\varphi}}{\partial \lambda_1} < 0,$$

the first term of the max function is larger for smaller λ_1 .

And since

$$\frac{\partial \frac{4(2\beta+1)\alpha\varphi - (2\beta^2-1)}{(2\beta+1)(1+2\lambda_1)\alpha\varphi}}{\partial \lambda_1} < 0$$

for

$$\alpha\varphi > 0.08,$$

condition that is always satisfied for all reasonable values of $\alpha\varphi$ (for example under CGG (1999) parametrization $\alpha\varphi = 0.3$; under Woodford (2003) parametrization $\alpha\varphi = 0.15$), the second term of the max function is larger for smaller λ_1 .

Therefore, the larger λ_1 , the smaller the set of (γ_π, γ_x) combinations under which we have root-T convergence.

Appendix 5) Speed of convergence isoquants

We look at combinations of (γ_π, γ_x) that results in the same value for the real part of the largest eigenvalue of the $\tilde{F} + \tilde{V}$ matrix. Let $0 < k < 1$, then

$$z^2 + a_1^k z + a_2^k$$

is the characteristic polynomial of $\tilde{F} + \tilde{V} - kI$.

Both eigenvalues of $\tilde{F} + \tilde{V}$ have real parts less than k (i.e. both eigenvalues of $\tilde{F} + \tilde{V} - kI$ have negative real parts) if and only if $a_1^k > 0$ and $a_2^k > 0$, that is,

$$\gamma_\pi = \max \left[\frac{\frac{(1-2k+\beta^2+2\alpha\varphi)}{(2k+\beta+\lambda_1)\alpha\varphi} - \frac{(-\beta^2+2k+\lambda_1)}{(2k+\beta+\lambda_1)\alpha}}{\frac{2\alpha\varphi(k+\beta) - (\beta^2-k)(1-k)}{\alpha\varphi(k+\beta)(k+\lambda_1)} - \frac{(k-\beta^2)}{\alpha(k+\beta)}} \gamma_x \right].$$

Appendix 6) proof of proposition 4 (Speed of convergence and communication of the path)

Let's consider an isoquant in the S_2 set:

$$\gamma_\pi^k = \max \left[\frac{\frac{(1-2k+\beta^2+2\alpha\varphi)}{(2k+\beta+\lambda_1)\alpha\varphi} - \frac{(-\beta^2+2k+\lambda_1)}{(2k+\beta+\lambda_1)\alpha}}{\frac{2}{(k+\lambda_1)} - \frac{(\beta^2-k)(1-k)}{\alpha\varphi(k+\beta)(k+\lambda_1)} - \frac{(k-\beta^2)}{\alpha(k+\beta)}} \gamma_x \right] \quad \text{for } \frac{1}{2} \leq k \leq 1$$

We take derivatives of the two terms inside the max operator with respect to λ_1 and we show that for a given k , the larger the λ_1 , that is the smaller the weight to the announcement (the extreme cases of $\lambda_1 = 1$ coincide with the case of no announcement), the smaller is γ_π^k .

As

$$\frac{\partial \frac{(-\beta^2 + 2k + \lambda_1)}{(2k + \beta + \lambda_1)\alpha}}{\partial \lambda_1} > 0$$

and

$$\frac{\partial \frac{(1 - 2k + \beta^2 + 2\alpha\varphi)}{(2k + \beta + \lambda_1)\alpha\varphi}}{\partial \lambda_1} < 0$$

for

$$k < \frac{1 + \beta^2}{2} + \alpha\varphi$$

and since

$$\frac{1 + \beta^2}{2} + \alpha\varphi > 1$$

for all reasonable values of α, φ, β , it is $\frac{\partial}{\partial \lambda_1} \frac{(1 - 2k + \beta^2 + 2\alpha\varphi)}{(2k + \beta + \lambda_1)\alpha\varphi} < 0$. Therefore, first term of the max function is larger for smaller λ_1 .

Moreover, for $\frac{1}{2} \leq k \leq 1$,

$$\frac{\partial \frac{2\alpha\varphi(k + \beta) - (\beta^2 - k)(1 - k)}{\alpha\varphi(k + \beta)(k + \lambda_1)}}{\partial \lambda_1} < 0$$

for

$$\alpha\varphi > 0.08,$$

condition that we have already seen is always satisfied for all reasonable values of $\alpha\varphi$. Therefore, also the second term of the max function is larger for smaller λ_1 .

Therefore for a given γ_x , the larger is λ_1 , the smaller has to be γ_π in order to reach the same speed of convergence denoted by the k-isoquant. Or in other terms, for a given combination of γ_x, γ_π , the larger is λ_1 the smaller is k , and the fastest the learning process.

Appendix 7) proof of proposition 5 (Announcing expected inflation and output gap)

Under learning the economy evolves according to the system of equation

$$\tilde{\tilde{H}}y_t = \tilde{\tilde{K}} + \tilde{\tilde{L}} \times E_t^* y_{t+1} + \tilde{\tilde{P}} \times E_t^* y_{t+2} + \tilde{\tilde{M}}w_t,$$

and the $\tilde{\tilde{H}}$, $\tilde{\tilde{L}}$ and $\tilde{\tilde{P}}$ matrices that are relevant in order to study stability under learning are

$$\begin{aligned}\tilde{\tilde{H}} &= \begin{bmatrix} 1 & -\alpha & 0 \\ 0 & 1 & \varphi \\ -\gamma_\pi & -\gamma_x & 1 \end{bmatrix}, \quad \tilde{\tilde{P}} = \begin{bmatrix} \lambda_2 \beta^2 & 0 & 0 \\ \varphi \lambda_2 & \lambda_2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \\ \tilde{\tilde{L}} &= \begin{bmatrix} 0 & \beta \alpha \lambda_2 & 0 \\ \varphi \lambda_2 & 0 & -\varphi \lambda_1 \\ 0 & 0 & 0 \end{bmatrix}\end{aligned}$$

or

$$y_t = \tilde{\tilde{Q}} + \tilde{\tilde{F}} \times E_t^* y_{t+1} + \tilde{\tilde{V}} \times E_t^* y_{t+2} + \tilde{\tilde{S}} w_t,$$

where

$$\tilde{\tilde{F}} = \tilde{\tilde{H}}^{-1} \tilde{\tilde{L}}, \quad \tilde{\tilde{V}} = \tilde{\tilde{H}}^{-1} \tilde{\tilde{P}}.$$

Necessary and sufficient condition for E-stability is that the eigenvalues of the $\tilde{\tilde{F}} + \tilde{\tilde{V}} - I$ matrix have the real part smaller than zero.

The characteristic polynomial of this matrix is

$$(X + 1)(X^2 + a_1 X + a_2).$$

All eigenvalues of $\tilde{\tilde{F}} + \tilde{\tilde{V}} - I$ have real parts smaller than zero if and only if $a_1 > 0$ and $a_2 > 0$, but since

$$a_1 = a_2 + c$$

with $c > 0$, the necessary and sufficient condition for E-stability is just $a_2 > 0$, that is

$$\gamma_\pi > \frac{2\lambda_2}{(1 + \lambda_1)} - \frac{(1 - \beta^2 \lambda_2)(1 - \lambda_2)}{\alpha \varphi (1 + \beta \lambda_2)(1 + \lambda_1)} - \frac{(1 - \beta^2 \lambda_2)}{\alpha (1 + \beta \lambda_2)} \gamma_x$$

In order to determine the effect of announcing only the expected inflation and expected output gap, we impose $\lambda_1 = 1$ and obtain the following condition:

$$\gamma_\pi > h - z \gamma_x$$

with

$$\begin{aligned}h &= \lambda_2 - \frac{(1 - \beta^2 \lambda_2)(1 - \lambda_2)}{2\alpha \varphi (1 + \beta \lambda_2)} \\ z &= \frac{(1 - \beta^2 \lambda_2)}{\alpha (1 + \beta \lambda_2)}.\end{aligned}$$

Since

$$\frac{\partial h}{\partial \lambda_2} > 0 \text{ and } \frac{\partial z}{\partial \lambda_2} < 0,$$

we have that the larger the weight to the projection (the lower λ_2), the lower the intercept h and the steeper the slope (in absolute value) z .

Therefore, for all values of the parameters, we have that by announcing the expected inflation and output gap, the combinations of (γ_x, γ_π) that imply E-instability is a subset of those obtained when the central bank does not announce the inflation and output gap.

In order to study the speed of convergence, we consider the k-isoquant obtained from the $\tilde{\tilde{F}} + \tilde{\tilde{V}} - kI$ matrix. We look at combinations of (γ_π, γ_x) that result in the same value for the real part of the largest eigenvalue of the $\tilde{\tilde{F}} + \tilde{\tilde{V}}$ matrix. Let $0 < k < 1$, then

$$(z + k)(z^2 + a_1^k z + a_2^k)$$

is the characteristic polynomial of $\left(\tilde{\tilde{F}} + \tilde{\tilde{V}} - kI\right)$. Both eigenvalues of $\tilde{\tilde{F}} + \tilde{\tilde{V}}$ have real parts less than k (i.e. both eigenvalues of $\tilde{\tilde{F}} + \tilde{\tilde{V}} - kI$ have negative real parts) if and only if $a_1^k > 0$ and $a_2^k > 0$, that is

$$\gamma_\pi > \frac{(1 + \beta^2 + 2\alpha\varphi)\lambda_2 - 2k}{\alpha\varphi(2k + \lambda_1 + \beta\lambda_2)} - \frac{2k + \lambda_1 - \beta^2\lambda_2}{\alpha(2k + \lambda_1 + \beta\lambda_2)}\gamma_x$$

and

$$\gamma_\pi > \frac{2\lambda_2}{(\lambda_1 + k)} - \frac{(\lambda_2 - k)(\beta^2\lambda_2 - k)}{\alpha\varphi(\beta\lambda_2 + k)(\lambda_1 + k)} - \frac{(k - \beta^2\lambda_2)}{\alpha(\beta\lambda_2 + k)}\gamma_x.$$

A k-isoquant in the S_2 set satisfies:

$$\gamma_\pi^k = \max \left[\begin{array}{l} \frac{(1 + \beta^2 + 2\alpha\varphi)\lambda_2 - 2k}{\alpha\varphi(2k + \lambda_1 + \beta\lambda_2)} - \frac{2k + \lambda_1 - \beta^2\lambda_2}{\alpha(2k + \lambda_1 + \beta\lambda_2)}\gamma_x, \\ \frac{2\lambda_2}{(\lambda_1 + k)} - \frac{(\lambda_2 - k)(\beta^2\lambda_2 - k)}{\alpha\varphi(\beta\lambda_2 + k)(\lambda_1 + k)} - \frac{(k - \beta^2\lambda_2)}{\alpha(\beta\lambda_2 + k)}\gamma_x \end{array} \right] \text{ for } \frac{1}{2} \leq k \leq 1$$

Following the steps suggested in the proof of proposition 4, we obtain that the larger the λ_2 , the larger is γ_π^k .

Appendix 8) proof of proposition 6 (Publishing interest rate, inflation and output gap projections)

Condition for E-stability becomes

$$\gamma_\pi > h - z\gamma_x$$

with

$$h = \frac{2\lambda_2}{(1+\lambda_1)} - \frac{(1-\beta^2\lambda_2)(1-\lambda_2)}{\alpha\varphi(1+\beta\lambda_2)(1+\lambda_1)}$$

$$z = \frac{(1-\beta^2\lambda_2)}{\alpha(1+\beta\lambda_2)}\gamma_x$$

Here we are interested on the combinations of λ_1 and λ_2 that make announcement worse than no announcement.

Under no announcement we have

$$\gamma_\pi > 1 - \frac{(1-\beta)}{\alpha}\gamma_x.$$

comparing the slope under announcement and under no announcement we have

$$\frac{(1-\beta^2\lambda_2)}{(\beta\lambda_2+1)\alpha} - \frac{(1-\beta)}{\alpha} = \frac{(1-\lambda_2)\beta}{(\beta\lambda_2+1)\alpha} \geq 0$$

for all values of $0 \leq \lambda_2 \leq 1$.

The intercept

$$1 - \frac{2\lambda_2}{(1+\lambda_1)} + \frac{(1-\beta^2\lambda_2)(1-\lambda_2)}{\alpha\varphi(1+\beta\lambda_2)(1+\lambda_1)} < 0$$

for

$$0 \leq \lambda_1 < 2 - \frac{(1-\lambda_2)(1-\beta^2\lambda_2)}{\lambda_2\alpha\varphi(1+\beta\lambda_2)} - \frac{1}{\lambda_2}$$

where

$$2 - \frac{(1-\lambda_2)(1-\beta^2\lambda_2)}{\lambda_2\alpha\varphi(1+\beta\lambda_2)} - \frac{1}{\lambda_2} \leq 1$$

for all values of $\lambda_2 \leq 1$, since

$$(1-\beta^2\lambda_2)(1-\lambda_2) + \alpha\varphi(1-\lambda_2) + (1-\lambda_2)\alpha\beta\varphi\lambda_2 \geq 0$$

Appendix 9) Proof of proposition 7 (Announcement of a T-period path)

We rewrite

$$\pi_t = \beta^T E_t^* \pi_{t+T} + E_t^P \sum_{j=0}^{T-1} \beta^j (\alpha x_{t+j} + u_{t+j})$$

$$x_t = E_t^* x_{t+T} - E_t^P \sum_{j=0}^{T-1} (\varphi i_{t+j} - \varphi \pi_{t+j+1} - g_{t+j})$$

$$E_t^P y_{t+j} = (1 - \lambda_i) E_t^{CB} y_{t+j} + \lambda_i E_t^* y_{t+j}, \text{ where } i = 1 \text{ for the } i_t \text{ and } i = 2 \text{ for } \pi_t \text{ and } x_t$$

as

$$A_0 Y_t = K + A_1 E_t^* Y_{t+1} + \dots + A_{T-1} E_t^* Y_{t+T-1} + A_T E_t^* Y_{t+T} + S w_t$$

where

$$\begin{aligned} A_0 &= \begin{bmatrix} 1 & -\alpha & 0 \\ 0 & 1 & \varphi \\ -\gamma_\pi & -\gamma_x & 1 \end{bmatrix}, \\ A_{T-1} &= \begin{bmatrix} 0 & \lambda_2 \alpha \beta^{T-1} & 0 \\ \lambda_2 \varphi & 0 & -\lambda_1 \varphi \\ 0 & 0 & 0 \end{bmatrix} \\ A_T &= \begin{bmatrix} \lambda_2 \beta^T & 0 & 0 \\ \lambda_2 \varphi & \lambda_2 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \end{aligned}$$

Since the perceived law of motion (PLM) is

$$E_t^* Y_{t+T} = A + B \Psi^T w_t$$

we can rewrite the previous expression as

$$A_0 Y_t = K + A_1 (A + B \Psi w_t) + \dots + A_{T-1} (A + B \Psi^{T-1} w_t) + A_T (A + B \Psi^T w_t) + S w_t.$$

Therefore, the actual law of motion (ALM) is

$$Y_t = A_0^{-1} K + A_0^{-1} (A_1 + \dots + A_T) A + (A_0^{-1} A_1 B \Psi + \dots + A_0^{-1} A_T B \Psi^T + A_0^{-1} S) w_t,$$

And the mappings from the PLM to the ALM are

$$T(A) - A = A_0^{-1} (A_1 + \dots + A_T) A - A$$

$$T(B) - B = A_0^{-1} (A_1 B \Psi + \dots + A_T B \Psi^T) - B$$

To study the E-stability conditions we just need to focus on the $A_0^{-1} (A_1 + \dots + A_T)$ matrix, where

$$(A_1 + \dots + A_T) = \begin{bmatrix} \beta^T \lambda_2 & \alpha \lambda_2 \Sigma_\beta & 0 \\ T \varphi \lambda_2 & \lambda_2 & -(T-1) \varphi \lambda_1 \\ 0 & 0 & 0 \end{bmatrix}$$

and $\Sigma_\beta = (\beta + \beta^2 + \dots + \beta^{T-1})$.

In particular necessary and sufficient condition for E-stability is that all eigenvalues of the $[A_0^{-1}(A_1 + A_2 + \dots + A_{T-1} + A_T) - I]$ matrix have the real part smaller than zero.

The characteristic polynomial of the matrix $[A_0^{-1}(A_1 + A_2 + \dots + A_{T-1} + A_T) - I]$ is

$$\frac{(X+1)(X^2 + a_1X + a_2)}{(\varphi\gamma_x + \alpha\varphi\gamma_\pi + 1)^2}$$

Necessary and sufficient conditions for E-stability are

$$a_1, a_2 > 0$$

and since

$$a_1 = a_2 + c$$

with $c > 0$, it is sufficient that $a_2 > 0$, that is

$$\gamma_\pi > \frac{T\lambda_2}{(1 + (T-1)\lambda_1)} - \frac{(1 - \lambda_2)(1 - \beta^T\lambda_2)}{\alpha\varphi(1 + (T-1)\lambda_1)(1 + \lambda_2\Sigma_\beta)} - \frac{(1 - \beta^T\lambda_2)}{\alpha(1 + \lambda_2\Sigma_\beta)}\gamma_x.$$

In particular, for $\lambda_2 = 1$, i.e. no announcement of inflation and output gap, we have

$$\gamma_\pi > \frac{T}{(1 + (T-1)\lambda_1)} - \frac{(1 - \beta)}{\alpha}\gamma_x.$$

and since the derivative of the intercept with respect to T

$$\frac{\partial \frac{T}{(1 + (T-1)\lambda_1)}}{\partial T} = \frac{1 - \lambda_1}{(1 + (T-1)\lambda_1)^2} \geq 0$$

for $0 \leq \lambda_1 \leq 1$, the longer the path announced, the larger the region of E-instability.

Appendix 10) Proof of proposition 8 (Expectations-based policy rule)

Under the set of policies

$$i_t = \gamma + \gamma_x E_t x_{t+1} + \gamma_\pi E_t \pi_{t+1}$$

the economy evolves according to the following stochastic dynamic system:

$$HY_t = K + L \times E_t^* Y_{t+1} + Mw_t,$$

with

$$\begin{aligned} y_t &= \begin{bmatrix} \pi_t \\ x_t \end{bmatrix}, \quad w_t = \begin{bmatrix} u_t \\ g_t \end{bmatrix}, \quad \Psi = \begin{bmatrix} \rho_u & 0 \\ 0 & \rho_g \end{bmatrix}, \quad M = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ H &= \begin{bmatrix} 1 & -\alpha \\ 0 & 1 \end{bmatrix}, \quad K = \begin{bmatrix} 0 \\ -\varphi\gamma \end{bmatrix}, \quad L = \begin{bmatrix} \beta & 0 \\ \varphi(1-\gamma_\pi) & (1-\varphi\gamma_x) \end{bmatrix} \end{aligned}$$

or

$$Y_t = Q + F \times E_t^* Y_{t+1} + Sw_t,$$

with

$$F = H^{-1}L, \quad Q = H^{-1}K, \quad S = H^{-1}M$$

As shown in Bullard and Mitra (2002), necessary and sufficient condition for E-stability is

$$\gamma_\pi > 1 - \frac{(1-\beta)}{\alpha} \gamma_x.$$

Under announcement of the policy path we rewrite the system as

$$Y_t = \tilde{Q} + \tilde{F} \times E_t^* Y_{t+1} + \tilde{V} \times E_t^* Y_{t+2} + \tilde{S}w_t.$$

Necessary and sufficient condition for E-stability is that the eigenvalues of the $\tilde{F} + \tilde{V} - I$ matrix have real part smaller than zero.

Since the characteristic polynomial of $\tilde{F} + \tilde{V} - I$ is

$$X^2 + a_1X + a_2$$

the necessary and sufficient condition for the eigenvalues of the $\tilde{F} + \tilde{V}$ matrix to have real part smaller than one is $a_1, a_2 > 0$. As $a_1 = a_2 + c$, where $c > 0$, the necessary and sufficient condition for the eigenvalues of the $\tilde{F} + \tilde{V}$ matrix to have real part smaller than one is just $a_2 > 0$, that is

$$\gamma_\pi > \frac{2}{(1+\lambda_1)} - \frac{(1-\beta)}{\alpha} \gamma_x.$$

