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The anatomy of standard DSGE models with financial frictions

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#### Abstract

In this paper we compare two standard extensions to the New Keynesian model featuring financial frictions. The first model, originating from Kiyotaki and Moore (1997), is based on collateral constraints. The second, developed by Carlstrom and Fuerst (1997) and Bernanke et al. (1999), accentuates the role of external finance premia. Our goal is to compare the workings of the two setups. Towards this end, we tweak the models and calibrate them in a way that allows for both qualitative and quantitative comparisons. Next, we make a thorough analysis of the two frameworks using moment matching, impulse response analysis and business cycle accounting. Overall, we find that the business cycle properties of the external finance premium framework are more in line with empirical evidence. In particular, the collateral constraint model fails to generate hump-shaped impulse responses and, for some important variables, shows moments that are inconsistent with the data by a large margin.

JEL: E30, E44

Keywords: financial frictions, DSGE models, business cycle accounting

# Non-technical summary

One of the important lessons from the 2007-2010 financial crisis was that financial markets matter for macroeconomic developments and should be taken into account when constructing macroeconomic models. This resulted in a surge of interest in theoretical frameworks incorporating financial frictions. Models with imperfect financial markets started to be widedely used by both academics and central bankers. As regards the latter, these models are expected to provide a workhorse setup for macroprudential analysis - a new concept linking financial and macroeconomic stability. This practical application is a very important reason to study the mechanics and business cycle properties of alternative modelling frameworks.

The current literature is mostly based on two alternative approaches developed long before the crisis. One important direction was introduced by the seminal paper of Kiyotaki and Moore (1997). This line of research introduces financial frictions via collateral constraints (CC). Agents are divided into lenders and borrowers. The financial sector intermediates between these groups and introduces frictions by requiring that borrowers provide collateral for their loans. Hence, this approach introduces frictions that affect directly the quantity of loans. The second stream of research originates from Carlstrom and Fuerst (1997) and Bernanke et al. (1999). In this setup frictions arise because monitoring a loan applicant is costly, which drives an endogenous wedge between the lending rate and the risk free rate (so-called external finance premium - EFP). This means that financial frictions affect the economy via prices of loans rather than via quantities as in models based on collateral constraints.

While both approaches allow for the introduction of financial frictions into the workhorse macro model, the propagation mechanisms in the two setups may substantially differ. For the development of a successful macro-financial framework it seems crucial to properly understand how price and quantity based frictions work, and to identify the common and distinct features of the two models. It is trivial to say that collateral constraints and finance premia affect the economy in not exactly the same ways. But what are exactly the differences? How and through which channels do the frictions amplify monetary policy effects? Do they result in intuitive or counterintuitive impulse response functions? What are the main propagation mechanisms?

In this paper we thoroughly compare the consequences of introducing collateral constraints versus external finance premia into a standard medium-sized New Keynesian DSGE model. Since we are interested in comparing the effect of two different types of financial frictions on the macroeconomy, we keep the CC and EFP versions as similar as possible in all aspects but the financial sector.

We find that both models add volatility to the New Keynesian (NK) framework, bringing it closer to the data. A notable exception are the price of and the return on capital, the volatilities of which are clearly overestimated in the CC model. Surprisingly, both models with a financial sector fail to capture the positive (even though rather weak) correlation between loans and GDP.

Looking at the model-implied inertia of the main macro-categories reveals that the EFP framework has significantly stronger internal propagation mechanisms than the CC setup.

This observation is confirmed by the impulse response analysis. Following any standard shock, the speed of return to the steady state is significantly slower in the EFP model and the responses display a hump-shaped pattern. This is in contrast to the CC version, where the strongest response usually occurs on impact. Both models generally dampen the impact of productivity shocks and amplify the impact of monetary policy shocks on investment and output. There are also several interesting qualitative differences and perhaps counterintuitive responses. For instance, in the CC setup there is a short-lived contraction in output following a positive productivity shock.

Finally, the business cycle accounting analysis confirms the superior performance of both models with financial frictions over the simple NK framework. In particular, in contrast to the NK model, both financial frictions setups give a significant role to the investment wedge, which reflects the difference between the intertemporal marginal rate of substitution and the return on capital.

All in all, we conclude that while both models improve in many areas the business cycle properties of the NK framework, the CC setup shows some features that might make its usage in policy analysis problematic. This comprises in particular its sharp but short-lived reactions to shocks and several moments that are by far inconsistent with empirical evidence.

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# 1 Introduction

The 2007-2010 financial crisis has shown how deep an impact financial markets can have on macroeconomic behaviour. Shocks originating from the American subprime mortgage market spread worldwide, affecting interbank markets and property markets in developed and developing economies. Financial institutions transmitted these shocks further, restricting lending and raising the cost of borrowing. As a result consumers reduced consumption, enterprises cut investment and the world economy witnessed the first recession since WWII.

One of the important lessons from the crisis was that financial markets matter for macroeconomic developments and should be taken into account when constructing macro models. This resulted in a surge of interest in theoretical frameworks incorporating financial frictions. Models with imperfect financial markets, previously at the margin of professional interest, promptly entered into the mainstream. They were used to answer questions important from the point of view of policymakers, like about (i) the impact of financial shocks on the economy (Gerali et al., 2010, Brzoza-Brzezina and Makarski, 2010), (ii) the optimal monetary policy in the presence of financial frictions (Cúrdia and Woodford, 2008; De Fiore and Tristani, 2009; Carlstrom et al., 2009; Kolasa and Lombardo, 2010), (iii) the effectiveness of alternative monetary policy tools (Lombardo and McAdam, 2010) or (iv) the impact of capital regulations and macroprudential policies on the economy (Angelini et al., 2010, Angeloni and Faia (2009), Benes et al. (2010)). Finally, it should be added that financial frictions have recently been added to models used for policy purposes at several central banks. This includes the Riksbank's model RAMSES (Christiano et al., 2010) and the European Central Bank's NAWM (Lombardo and McAdam, 2010). Looking ahead, it seems very likely that these models will provide a workhorse setup for macroprudential analysis - a new concept that is intended to make a connection between financial and macroeconomic stability. This practical application is a very important reason to study the mechanics and business cycle properties of alternative frameworks.

The current literature is mostly based on two alternative approaches developed long before the crisis. One important direction was introduced by the seminal paper of Kiyotaki and Moore (1997). This line of research introduces financial frictions via collateral constraints. Agents are heterogeneous in terms of their rate of time preference, which divides them into lenders and borrowers. The financial sector intermediates between these groups and introduces frictions by requiring that borrowers provide collateral for their loans. Hence, this approach introduces frictions that affect directly the quantity of loans. The original model of Kiyotaki and Moore (1997) has been recently developed by Iacoviello (2005), who introduced housing as collateral. Other recent applications relying on this framework include Calza et al. (2009), who analyse the impact of mortgage market characteristics on monetary transmission. Gerali et al. (2010) and Brzoza-Brzezina and Makarski (2010) use models with collateral constraints and monopolistic competition in the banking sector to examine the impact of financial frictions on monetary transmission and a credit crunch scenario. Iacoviello and Neri (2010) estimate a model with collateral constraints on US data in order to study the role of housing market shocks on the economy.

<sup>&</sup>lt;sup>1</sup>See Jonsson et al. (2010) for an extensive review.

The second stream of research originates from the seminal paper of Bernanke and Gertler (1989), where financial frictions have been incorporated into a general equilibrium model. This approach has been further developed by Carlstrom and Fuerst (1997) and merged with the New Keynesian framework by Bernanke et al. (1999), becoming the workhorse financial accelerator model in the 2000s. In this model frictions arise because monitoring a loan applicant is costly, which drives an endogenous wedge between the lending rate and the risk free rate (so-called external finance premium). This means that financial frictions affect the economy via prices of loans rather than via quantities as in models based on collateral constraints. The external finance premium setup has been extensively used i.a. by Christiano et al. (2003) to analyse the role of financial frictions during the Great Depression and by Christiano et al. (2010) to study business cycle implications of financial frictions. Goodfriend and McCallum (2007) provided an endogenous explanation for steady state differentials between lending and money market rates. In a similar framework, Cúrdia and Woodford (2008) derived optimal monetary policy in the presence of time-varying interest rate spreads in a simple model with heterogeneous households.

While both approaches allow for the introduction of financial frictions into the work-horse macro model, the propagation mechanisms in the two models may substantially differ. For the development of a successful macro-financial framework it seems crucial to properly understand how price and quantity based frictions work, and to identify the common and distinct business cycle properties of the two models. It is trivial to say that collateral constraints and external finance premia affect the economy in not exactly the same ways. But what are exactly the differences? How and through which channels do the frictions amplify monetary policy effects? Do they result in intuitive or counterintuitive impulse response functions? What are the main propagation mechanisms?

In this paper we thoroughly compare the consequences of introducing collateral constraints (referred to as CC) and external finance premia (EFP) into a standard medium-sized New Keynesian (NK) DSGE model. Since we are interested in comparing the effect of two different types of financial frictions on the macroeconomy, we keep the CC and EFP versions as similar as possible in all aspects but the financial sector. In particular, while calibrating the models to the data, we keep the non-financial structural parameters the same and set the financial parameters to match the key steady state proportions affected by the presence of financial frictions. Both models are subject to three non-financial shocks (productivity, monetary and government expenditure) and two financial sector shocks. We estimate the former outside of the models. Since the financial sectors differ across the models, so do the financial shocks. Therefore, to ensure comparability, we calibrate them so that both model versions match the autoregressions and the standard deviations of loans and the interest rate spreads observed in the data. We investigate the differences between the two modelling approaches using three tools: moment matching, impulse response analysis and business cycle accounting (as proposed by Chari et al. (2007)).

Both models add volatility to the NK framework, bringing it closer to the data. A notable exception are the price of and the return on capital, whose volatilities are clearly overestimated in the CC model. Surprisingly, both models with a financial sector fail to capture the positive correlation between loans and GDP.

Looking at the model-implied inertia of the main macro-categories reveals that the EFP framework has significantly stronger internal propagation mechanisms than the CC setup. This observation is confirmed by the impulse response analysis. Following any standard shock, the speed of return to the steady state is significantly slower in the EFP model and the responses display a hump-shaped pattern. This is in contrast to the CC version, where the strongest response usually occurs on impact. Both models generally dampen the impact of productivity shocks and amplify the impact of monetary policy shocks on investment and output. There are also several interesting qualitative differences and perhaps counterintuitive responses. For instance, in the CC setup there is a short-lived contraction in output following a positive productivity shock.

The business cycle accounting analysis confirms the superior performance of both models with financial frictions over the simple NK framework. In particular, in contrast to the NK model, both financial frictions setups give a significant role to the investment wedge, which reflects the difference between the intertemporal marginal rate of substitution and the return on capital.

All in all, we conclude that while both models improve in many areas the business cycle properties of the NK framework, the CC setup shows some features that might make its usage in policy analysis problematic. This comprises in particular its sharp but short-lived reactions to shocks and several moments that are by far inconsistent with empirical evidence.

The rest of the paper is structured as follows. Section two sketches the baseline NK model, section three presents the two versions of a financial sector. Section four discusses the calibration, section five presents the impulse response analysis and section six the results of business cycle accounting. Section seven concludes.

# 2 The benchmark NK model

Our baseline NK specification is a standard medium-sized closed economy DSGE model with sticky prices and a range of other frictions that have been found crucial for ensuring a reasonable empirical fit (see Christiano et al., 2005; Smets and Wouters, 2003). The model economy is populated by households, producers, fiscal and monetary authorities. Households consume, accumulate capital stock and work. Output is produced in several steps. Fiscal authorities use taxes to finance exogenously given government expenditure and monetary authorities conduct monetary policy according to a Taylor rule.

### 2.1 Households

The economy is populated by households of measure one. Each household h chooses consumption  $c_t^H$ , labour supply  $n_t$  and capital holdings for the next period  $k_t$ . The expected lifetime utility of a representative household is as follows

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[ \frac{\left( c_t^H(h) - \xi c_{t-1}^H \right)^{1-\sigma_c}}{1 - \sigma_c} - \frac{n_t(h)^{1+\sigma_n}}{1 + \sigma_n} \right]$$
 (1)

where  $\xi$  denotes the degree of external habit formation. The representative household uses labour income  $W_t n_t$ , capital income  $R_{k,t} k_{t-1}$ , dividends<sup>2</sup>  $\Pi_t$  and undepreciated capital holdings from the previous period  $(1 - \delta) k_{t-1}$  to finance its expenditure and lump sum taxes  $T_t$ . Each household faces the following budget constraint

$$P_{t}c_{t}^{H}(h) + Q_{t}k_{t}(h) + R_{t}^{-1}B_{t}(h) \leq W_{t}n_{t}(h) + (R_{k,t} + Q_{t}(1 - \delta))k_{t-1}(h) - P_{t}T_{t}(h) + \Pi_{t}(h) + B_{t-1}(h)$$
(2)

where  $P_t$  and  $Q_t$  denote, respectively, the price of consumption good and capital. As in Chari et al. (2002), we assume that agents have access to state contingent bonds  $B_t$ , paying the expected gross rate of return  $R_t$ , which allows to insure against idiosyncratic risk.

#### 2.1.1 Labour supply

We assume that each household has a unique labour type h. Labour services are sold to perfectly competitive aggregators, who pool all the labour types into one undifferentiated labour service with the following function

$$n_t = \left(\int_0^1 n_t \left(h\right)^{\frac{1}{\phi_w}} dh\right)^{\phi_w} \tag{3}$$

The problem of the aggregator gives the following demand for labour of type h

$$n_t(h) = \left\lceil \frac{W_t(h)}{W_t} \right\rceil^{\frac{-\phi_w}{\phi_w - 1}} n_t \tag{4}$$

<sup>&</sup>lt;sup>2</sup>Households own all firms in this economy.

where

$$W_t = \left(\int_0^1 W_t(h)^{\frac{-1}{\phi_w - 1}} dh\right)^{-(\phi_w - 1)} \tag{5}$$

is the aggregate wage in the economy.

Households set their wage rate according the the standard Calvo scheme, i.e. with probability  $(1 - \theta_w)$  they receive a signal to reoptimise and then set their wage to maximise their utility subject to the demand for their labour services. With probability  $\theta_w$  they do not receive the signal and index their wage according to the following rule

$$W_{t+1}(h) = ((1 - \zeta_w)\bar{\pi} + \zeta_w \pi_{t-1}) W_t(h)$$
(6)

where  $\pi_t = \frac{P_t}{P_{t-1}}$ ,  $\bar{\pi}$  is the steady state inflation rate and  $\zeta_w \in [0, 1]$ .

#### 2.2 Producers

There are several stages of production in the economy. Intermediate goods firms produce differentiated goods and sell them to aggregators. Aggregators combine differentiated goods into a homogeneous final good. The final good can be used for consumption or sold to capital goods producers.

#### 2.2.1 Capital good producers

Capital good producers act in a perfectly competitive environment. In each period a representative capital good producer buys  $i_t$  of final goods and old undepreciated capital  $(1 - \delta) k_{t-1}$ . Next she transforms old undepreciated capital one-to-one into new capital, while the transformation of the final goods is subject to an adjustment cost  $S(i_t/i_{t-1})$ . We adopt the specification of Christiano et al. (2005) and assume that in the deterministic steady state there are no capital adjustment costs (S(1) = S'(1) = 0), and the function is concave in its neighbourhood  $(S''(1) = \kappa > 0)$ . Thus, the technology to produce new capital is given by

$$k_t = (1 - \delta) k_{t-1} + \left(1 - S\left(\frac{i_t}{i_{t-1}}\right)\right) i_t$$
 (7)

The new capital is then sold in a perfectly competitive market to households and can be used in the next period production process. The real price of capital is denoted as  $q_t = Q_t/P_t$ .

#### 2.2.2 Final good producers

Final good producers play the role of aggregators. They buy differentiated products from intermediate goods producers y(j) and aggregate them into a single final good, which they sell in a perfectly competitive market. The final good is produced according to the following technology

$$y_t = \left(\int_0^1 y_t(j)^{\frac{1}{\phi}} dj\right)^{\phi} \tag{8}$$

The problem of the aggregator gives the following demands for differentiated goods

$$y_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{\frac{-\phi}{\phi-1}} y_t \tag{9}$$

where

$$P_t = \left[ \int P_t(j)^{\frac{-1}{\phi - 1}} dj \right]^{-(\phi - 1)} \tag{10}$$

is the price of the consumption good.

#### 2.2.3 Intermediate goods producers

There is a continuum of intermediate goods producers of measure one denoted by j. They rent capital and labour from the households and use the following production technology

$$y_t(j) = A_t k_t(j)^{\alpha} n_t(j)^{1-\alpha}$$
(11)

where  $A_t$  is the total factor productivity, the log of which follows an exogenous AR(1) process.<sup>3</sup>

Intermediate goods firms act in a monopolistically competitive environment and set their prices according to the standard Calvo scheme. In each period each producer receives with probability  $(1 - \theta)$  a signal to reoptimise and then sets her price to maximise the expected profits, subject to demand schedules given by (9). Those who are not allowed to reoptimise index their prices according to the following rule

$$P_{t+1}(j) = P_t(j) \left( (1 - \zeta) \,\bar{\pi} + \zeta \pi_{t-1} \right) \tag{12}$$

where  $\zeta \in [0, 1]$ .

#### 2.3 Government

The government uses lump sum taxes to finance government expenditure. For simplicity, we assume that the government budget is balanced each period so that

$$q_t = T_t. (13)$$

where  $g_t$  denotes government expenditure, driven by a simple AR(1) process

$$\ln g_t = (1 - \rho_g) \ln \mu_g + \rho_g \ln g_{t-1} + \varepsilon_{g,t}. \tag{14}$$

with i.i.d. normal innovations (the standard deviation is  $\sigma_g$ ),  $\rho_g \in (0,1)$  and  $\mu_g$  denoting the steady state level of government purchases.

<sup>&</sup>lt;sup>3</sup>The autoregressive coefficient is  $\rho_A$  and the standard deviation is  $\sigma_A$ .

#### 2.4 Central bank

As it is common in the New Keynesian literature, we assume that monetary policy is conducted according to a Taylor rule that targets deviations of inflation and GDP from the deterministic steady state, allowing additionally for interest rate smoothing

$$R_{t} = \left(\frac{R_{t-1}}{\bar{R}}\right)^{\gamma_{R}} \left(\left(\frac{\pi_{t}}{\bar{\pi}}\right)^{\gamma_{\pi}} \left(\frac{y_{t}}{\bar{y}}\right)^{\gamma_{y}}\right)^{1-\gamma_{R}} e^{\varphi_{t}}$$
(15)

where y denotes GDP and  $\varphi_t$  are i.i.d. normal innovations (the standard deviation is  $\sigma_R$ ).

### 2.5 Market clearing

To close the model, we need the market clearing condition for the final goods market

$$c_t^H + i_t + g_t = y_t \tag{16}$$

Finally, the factor markets need to clear as well

$$k_{t-1} = \int_0^1 k_t(j) \, dj \tag{17}$$

$$n_t = \int_0^1 n_t(j) \, dj \tag{18}$$

### 3 Financial frictions

In the NK model presented above, financial markets work perfectly. In particular, agents can make deposits and take loans in any quantity at the risk free rate  $R_t$ , fully controlled by the central bank. This will no longer be the case in the extensions discussed in this section.

Implementing any credit imperfections requires distinguishing between borrowers and lenders. As in both EFP and CC versions financial frictions emerge at the level of capital management, its ownership needs to be separated from the households. Therefore, one introduces a new type of agents, named entrepreneurs, who specialise in capital management.<sup>4</sup> Entrepreneurs finance their operations, i.e. renting capital services to firms, by taking loans from the banking sector, which refinances them by accepting deposits from households. The financial intermediation between households and entrepreneurs is subject to frictions, which result in interest rate spreads or quantity constraints.

# 3.1 External finance premium version (EFP)

In the EFP version, financial frictions arise because management of capital is risky. Individual entrepreneurs are subject to idiosyncratic shocks, which are observed by them for free, while the lenders can learn about the shocks' realisations only after paying monitoring costs. This costly state verification problem (Townsend, 1979) results in a financial contract featuring an endogenous premium between the lending rate and the risk-free rate, which depends on borrowers' leverage. Since the banking sector is perfectly competitive and entrepreneurs are risk neutral, banks pay interest on household deposits equal to the risk-free rate and break even every period.

#### 3.1.1 Entrepreneurs

There is a continuum of risk-neutral entrepreneurs, indexed by  $\iota$ . At the end of period t, each entrepreneur purchases installed capital  $k_t(\iota)$  from capital producers, partly using her own financial wealth  $V_t(\iota)$  and financing the remainder with a bank loan  $L_t(\iota)$ 

$$L_t(\iota) = Q_t k_t(\iota) - V_t(\iota) \ge 0 \tag{19}$$

After the purchase, each entrepreneur experiences an idiosyncratic productivity shock, which converts her capital to  $a_E(\iota)k_t(\iota)$ , where  $a_E$  is a random variable, distributed independently over time and across entrepreneurs, with a cumulative density function  $F(\iota)$  and a unit mean. Following Christiano et al. (2003), we assume that this distribution is log normal, with a time-varying standard deviation of  $\log a_E$  equal to  $\varepsilon_{E,t}\sigma_{a_E}^{\ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ }$ , known to entrepreneurs before their capital decisions.

Next, each entrepreneur rents out capital services, treating the rental rate  $R_{k,t+1}$  as given. Since the mean of an idiosyncratic shock is equal to one, the average rate of return on capital earned by entrepreneurs can be written as

<sup>&</sup>lt;sup>4</sup>This means in particular that terms related to capital management drop out from households' budget constraint (2).

<sup>&</sup>lt;sup>5</sup>The autoregressive coefficient is  $\rho_E$  and the standard deviation is  $\sigma_E$ .

$$R_{E,t+1} = \frac{R_{k,t+1} + (1-\delta)Q_{t+1}}{Q_t} \tag{20}$$

and the rate of return earned by an individual entrepreneur is  $a_E(\iota)R_{E,t+1}$ .

Since lenders can observe the return earned by borrowers only at a cost, the optimal contract between these two parties specifies the size of the loan  $L_t(\iota)$  and the gross non-default interest rate  $R_{L,t+1}(\iota)$ . The solvency criterion can also be defined in terms of a cutoff value of idiosyncratic productivity, denoted as  $\tilde{a}_{E,t+1}(\iota)$ , such that the entrepreneur has just enough resources to repay the loan<sup>6</sup>

$$\tilde{a}_{E,t+1}R_{E,t+1}Q_tk_t(\iota) = R_{L,t+1}L_t(\iota) \tag{21}$$

Entrepreneurs with  $a_E$  below the threshold level go bankrupt. All their resources are taken over by the banks, after they pay a proportional monitoring costs  $\mu$ .

#### 3.1.2 Banks

Banks finance their loans by issuing time deposits to households at the risk-free interest rate  $R_t$ . The banking sector is assumed to be perfectly competitive and owned by risk-averse households. This, together with risk-neutrality of entrepreneurs implies a financial contract insulating the lender from any aggregate risk.<sup>7</sup> Hence, interest paid on a bank loan by entrepreneurs is state contingent and guarantees that banks break even in every period. The aggregate zero profit condition for the banking sector can be written as

$$(1 - F_{1,t+1}) R_{L,t+1} L_t + (1 - \mu) F_{2,t+1} R_{E,t+1} Q_t k_t = R_t L_t$$
(22)

or equivalently (using (21))

$$R_{E,t+1}Q_tk_t\left[\tilde{a}_{E,t+1}(1-F_{1,t+1}) + (1-\mu)F_{2,t+1}\right] = R_tL_t \tag{23}$$

where

$$F_{1,t} = \int_{0}^{\tilde{a}_{E,t}} dF(a_{E}) \tag{24}$$

$$F_{2,t} = \int_0^{\tilde{a}_{E,t}} a_E dF(a_E)$$
 (25)

#### 3.1.3 Optimal contract

The equilibrium debt contract maximises welfare of each individual entrepreneur. It is defined in terms of expected end-of-contract net worth relative to the risk-free alternative, which is holding a domestic bond

<sup>&</sup>lt;sup>6</sup>In order to save on notation, in what follows we use the result established later on, according to which the cutoff productivity  $\tilde{a}_E(z_E)$  and the non-default interest paid on a bank loan  $R_{B,t+1}(z_E)$  is identical across entrepreneurs.

<sup>&</sup>lt;sup>7</sup>Given the infinite number of entrepreneurs, the risk arising from idiosyncratic shocks is fully diversifiable.

$$E_t \left\{ \frac{\int_{\tilde{a}_{E,t}}^{\infty} \left( R_{E,t+1} Q_t k_t(\iota) a_E(\iota) - R_{L,t+1} L_t(\iota) \right) dF(a_E(\iota))}{R_t V_t(\iota)} \right\}$$
(26)

The first-order condition to this optimisation problem can be written as

$$E_{t} \left\{ \begin{array}{c} \frac{R_{E,t+1}}{R_{t}} \left[ 1 - \tilde{a}_{E,t+1} (1 - F_{1,t+1}) - F_{2,t+1} \right] + \\ + \frac{1 - F_{1,t+1}}{1 - F_{1,t+1} - \mu \tilde{a}_{E,t+1} F'_{1,t+1}} \left( \frac{R_{E,t+1}}{R_{t}} \left[ \tilde{a}_{E,t+1} (1 - F_{1,t+1}) + (1 - \mu) F_{2,t+1} \right] - 1 \right) \end{array} \right\} = 0 \quad (27)$$

As can be seen from (27), if  $\mu$  is set to zero, the expected rate of return on capital is equal to the risk-free interest rate and so the financial markets work without frictions.

Equation (27), together with the bank zero profit constraint (23), defines the optimal debt contract in terms of the cutoff value of the idiosyncratic shock  $\tilde{a}_{E,t+1}$  and the leverage ratio  $\varrho_t$ , defined as:

$$\varrho_t = \frac{Q_t k_t}{V_t} \tag{28}$$

These two contract parameters are identical across entrepreneurs. Similarly, the rate of interest paid to the bank is the same for each non-defaulting entrepreneur:

$$R_{L,t+1} = \frac{\tilde{a}_{E,t+1} R_{E,t+1} \varrho_t}{\varrho_t - 1} \tag{29}$$

We will refer to the difference between this rate and the risk-free rate  $R_t$  as the credit spread. It is easy to show that the credit spread depends positively on the leverage.

#### 3.1.4 Net worth evolution and resource constraint

Proceeds from selling capital, net of interest paid to the banks, constitute end of period net worth. To capture the phenomenon of ongoing entries and exits of firms and to ensure that entrepreneurs do not accumulate enough wealth to become fully self-financing, it is assumed that each period a randomly selected and time-varying fraction  $1 - \varepsilon_{\nu,t} v^8$  of them go out of business, in which case all their financial wealth is rebated to the households. At the same time, an equal number of new entrepreneurs enter, so that the total number of entrepreneurs is constant. Those who survive and enter receive a fixed transfer  $T_E$  from households. This ensures that entrants have at least a small but positive amount of wealth, without which they would not be able to buy any capital.

Aggregating across all entrepreneurs and using (23) yields the following law of motion for net worth in the economy

$$V_{t} = \varepsilon_{\nu,t} v \left[ R_{E,t} Q_{t-1} k_{t-1} - \left( R_{t-1} + \frac{\mu F_{2,t} R_{E,t} Q_{t-1} k_{t-1}}{L_{t-1}} \right) L_{t-1} \right] + T_{E}$$
 (30)

The term in the square brackets represents the total revenue from renting and selling capital net of interest paid on bank loans, averaged over both bankrupt and non-bankrupt entrepreneurs.

<sup>&</sup>lt;sup>8</sup>The autoregressive coefficient is  $\rho_v$  and the standard deviation is  $\sigma_v$ .

Finally, as monitoring costs are real, the aggregate resource constraint from the NK model (16) needs to be modified so that it becomes

$$c_t^H + i_t + g_t + \mu F_{2,t} R_{E,t} Q_{t-1} k_{t-1} = y_t \tag{31}$$

# 3.2 Collateral constraint version (CC)

The key financial friction in the CC version is introduced by assuming that borrowers need collateral to take a loan. The restrictiveness of this constraint is perturbed stochastically in the form of a shock to the required loan-to-value (LTV) ratio. Additionally, to ensure comparability with the EFP version, we assume that the interest rates on loans differ from the risk free rate. The difference is due to monopolistic competition in the banking sector and is subject to exogenous shocks.

Similarly as in the case of the goods producers, banking activity is divided into several steps. First, banks collect deposits from households and use them to offer differentiated loans to financial intermediaries. Financial intermediaries aggregate all differentiated loans into a homogeneous loan that is offered to entrepreneurs.

#### 3.2.1 Entrepreneurs

There is a continuum of entrepreneurs in the model, indexed by  $\iota$ . They draw utility only from their consumption  $c_t^E$ 

$$E_0 \sum_{t=0}^{\infty} (\beta_E)^t \left( \frac{\left( c_t^E(\iota) - \xi c_{t-1}^E \right)^{1-\sigma_c}}{1 - \sigma_c} \right)$$
 (32)

Entrepreneurs cover their consumption and capital expenditures with revenues from renting capital services to wholesale goods producers, financing the remainder by bank loans  $L_t$ , on which the interest to pay is  $R_{L,t}$ .

$$P_{t}c_{t}^{E}(\iota) + Q_{t}k_{t}(\iota) + R_{L,t-1}L_{t-1}(\iota) + \tilde{T}_{E} = (R_{k,t} + Q_{t}(1-\delta))k_{t-1}(\iota) + L_{t}(\iota)$$
(33)

where  $T_E$  denotes transfers between households and entrepreneurs. Loans taken by the entrepreneurs are subject to the following collateral constraint

$$R_{L,t}L_t(\iota) \le m_t E_t \left[ Q_{t+1} \left( 1 - \delta_k \right) k_t(\iota) \right] \tag{34}$$

where  $m_t$  is firm's loan-to-value ratio, the log of which follows an AR(1) process with i.i.d. normal innovations.<sup>9</sup>

Since entrepreneurs consume, the modified aggregate resource constraint is

$$c_t^H + i_t + g_t + c_t^E = y_t (35)$$

<sup>&</sup>lt;sup>9</sup>The autoregressive coefficient is  $\rho_m$  and the standard deviation is  $\sigma_m$ .

#### 3.2.2 Financial intermediaries

Financial intermediaries take differentiated loans from lending banks  $L_t(i)$  at the interest rate  $R_{L,t}$  and aggregate them into one undifferentiated loan  $L_t$  that is offered to entrepreneurs at the rate  $R_{L,t}$ . The technology for aggregation is

$$L_{t} = \left[ \int_{0}^{1} L_{t} (i)^{\frac{1}{\phi_{L,t}}} di \right]^{\phi_{L,t}}$$
(36)

where  $\phi_{L,t}$  denotes mark-up shock, the log of which follows an exogenous AR(1) process.<sup>10</sup> Note that this shock introduces time varying spreads thus, throughout the paper, we call it spread shock. Financial intermediaries operate in a competitive market, thus they take the interest rates as given and maximise profits

$$R_{L,t}L_t - \int_0^1 R_{L,t}(i) L_t(i) di$$
 (37)

subject to (36).

Solving the problems above we get the demand for the lending banks' loans

$$L_t(i) = \left(\frac{R_{L,t}(i)}{R_{L,t}}\right)^{\frac{-\phi_{L,t}}{\phi_{L,t}-1}} L_t,$$
 (38)

and from the zero profit condition we get the interest rate on loans to entrepreneurs

$$R_{L,t} = \left( \int_0^1 R_{L,t} (i)^{\frac{-1}{\phi_{L,t}-1}} di \right)^{-(\phi_{L,t}-1)}$$
(39)

#### 3.2.3 Lending banks

The lending bank i collects deposits  $D_t(i)$  from households at the risk-free rate  $R_t$ , and uses them for lending to financial intermediaries  $L_t(i)$  at the interest rate  $R_{L,t}(i)$ . Furthermore, the bank can borrow (or lend) on the interbank market  $L_{IB,t}(i)$  at the policy rate  $R_t$ .

$$L_t(i) = D_t(i) + L_{IB,t}(i)$$
 (40)

The bank operates in a monopolistically competitive market, so it sets its interest rate to maximise its profits

$$R_{L,t}(i) L_t(i) - R_t(D_t(i) + L_{IB,t}(i))$$
 (41)

subject to the demand for loans (38) and (40).

The autoregressive coefficient is  $\rho_{\phi_L}$  and the standard deviation is  $\sigma_{\phi_L}$ . Its steady state level we denote as  $\phi_L$ 

### 4 Calibration

The main goal of calibration is to achieve the highest possible comparability between the EFP and CC specifications. This task is not trivial since both versions have different forms of financial frictions and thus we cannot assume equal parameters and stochastic processes across the models. Nevertheless, we are able to succeed by applying a precise calibration procedure, the results of which are presented in Tables 1 to 3.

We start with the structural parameters unrelated to the financial sector and so common across the NK, EFP and CC versions. We take their values directly from the literature, relying mainly on Smets and Wouters (2007), or set them to match the key steady state proportions of the US data.

In each of our extensions to the NK setup, the financial sector is governed by four parameters<sup>11</sup>, which pin down four steady state proportions: investment share in GDP, interest rate spread, capital to debt ratio and the output share of monitoring costs (EFP) / entrepreneurs' consumption (CC).<sup>12</sup> The first three have their natural empirical counterparts, which we match exactly. The target value for the last one is consistent with Christiano et al. (2010).

We apply a similar procedure to calibration of stochastic shocks. We first calibrate the shocks that are common for the NK, EFP and CC models. The parameters of the technology shock are taken from Cooley and Prescott (1995) and those describing the monetary shock come from Smets and Wouters (2007). For the government expenditure shock, we set the autoregression coefficient at 0.95, which is standard in the literature, and we calibrate the standard deviation to match it with the data on real government spending. Next, we calibrate the financial shocks in the EFP and the CC models. In the former, we have net worth and entrepreneur riskiness shocks, while in the latter we have LTV and spread shocks. These shocks are different but they govern the behaviour of two financial variables appearing in both models: spreads and loans to firms. While calibrating these shocks, we simulate both models with the three standard shocks (already calibrated) and the two financial sector shocks. We set the parameters of the latter to match autocorrelations and standard deviations of spreads and loans observed in the data. This procedure anchors the magnitude and inertia of financial sector shocks, thus enabling us to calibrate models with different financial sector structures in a coherent way.

To see what our calibration strategy implies for the role played by different shocks in each model version, in Tables 4 to 6 we present the results of the variance decomposition. In the NK variant, virtually all volatility of the main macroaggregates can be attributed to productivity shocks. This is no longer the case once we add financial frictions. It is worth noting that in both versions of our models with imperfect financial markets, financial shocks are very important, which is consistent with recent findings of Jermann and Quadrini (2009). The overall picture is very similar for the EFP and CC models. The highest share of variance in standard macroeconomic variables is driven by productivity and net worth/LTV shocks.

<sup>&</sup>lt;sup>11</sup>These parameters are  $\mu$ ,  $\nu$ ,  $\sigma_{a_E}$ ,  $T_E$  (EFP model), and  $\beta_E$ ,  $\phi_L$ ,  $m_f$ ,  $\tilde{T}_E$  (CC model).

<sup>&</sup>lt;sup>12</sup>Monitoring costs and entrepreneurs' consumption are treated as a part of government spending in the simulation and business cycle accounting analyses, so our calibration implies that the steady state value of this wedge is identical in the EFP and CC versions.

The notable difference is relatively low importance of non-financial shocks for investment in the EFP version. As regards financial variables (i.e. loans, spreads, the price of and the return on capital), financial shocks are even more important, accounting for at least two-thirds of fluctuations. By construct, spread in the CC model is entirely driven by spread shocks, but this variable is hardly affected by non-financial shocks also in the EFP version. An important difference across our two financial sector specifications concerns the role of productivity shocks for fluctuations of loans (negligible in the CC model) and the price of capital (relatively small in the EFP variant).

Given our method of calibration, we can compare the performance of the models along the dimensions that were not used in the calibration process. Table 7 documents several important features of the models. First, both financial frictions models improve upon the NK baseline when matching volatilities of output, consumption, investment and labour, though the EFP framework does a somewhat better job here. Second, all three models clearly overestimate the volatility of inflation and interest rate. Finally, the CC model generates fluctuations in the price of and return on capital that exceeds those in the data by an order of magnitude.

There are further differences between the models when one looks at autocorrelations. The most notable one is very high autocorrelation of GDP, labour, consumption and investment in the EFP and the NK models (in most cases higher than in the data) and substantially lower autocorrelation of the same variables in the CC model (usually lower than in the data). Moreover, while the real price of and return on capital are moderately autocorrelated in the NK and EFP models (as in the data), they are close to white noise in the CC model.

There are also important differences in correlations of the main macro variables with GDP between the models and the data. It is well known that a standard NK model with a dominant role of productivity shocks implies countercyclicality of labour, which is clearly in contrast with the data. Both versions of financial frictions improve upon it in this respect, but none gets the sign right. A similar picture emerges with respect to inflation and the return on capital, which are procyclical in the data, but countercyclical in all three models. Furthermore, both financial frictions models clearly underestimate the procyclicality of consumption and, interestingly, neither of the two models is able to reproduce the procyclicality of loans. All in all, in terms of correlations of the main macrovariables with GDP, the financial frictions models do not improve upon the NK model.

To sum up, adding financial market imperfections improves the NK framework in terms of matching volatilities with the data. However, the CC setup raises the volatilities of some variables substantially too much. In terms of autocorrelations, the EFP model generally increases the persistence of the NK framework, while the CC version brings it much below what is observed empirically. Finally, as regards corelations with GDP, all three models look alike and show several features inconsistent with the data.

# 5 Impulse response analysis

A natural way to explain the results reported in the previous section is to compare the impulse responses of the analysed models to shocks. Such an analysis highlights and helps to understand the key differences in the propagation mechanisms embedded in various setups. We begin with the standard macroeconomic shocks (productivity and monetary policy), common to all model versions. We next move to shocks specific to the financial frictions literature (net worth, LTV, riskiness and spreads). As we have already mentioned, financial shocks differ in our two model variants by construction and so are not fully comparable. However, there is some conceptual analogy between the net worth shock in the EFP model and the LTV shock from the CC version. Similarly, a natural counterpart of the riskiness shock (EFP) is the spread shock (CC). Therefore, we present the impulse response analysis for the financial shocks by grouping them into these two pairs. In all Figures 1 to 4, the impulse response functions for the EFP model are denoted with the solid line, for the CC model with the dashed line and for the standard NK model with the dotted line. Moreover, since there are no financial frictions in the NK model, we only present its impulse response functions to the productivity and monetary shocks.

# 5.1 Productivity shock

Figure 1 shows the reactions to a positive productivity shock. In all models, the shock lowers the marginal cost and drives inflation down. This process has a non-standard impact on the CC model. Lower inflation raises the real value of debt and forces the constrained agents to decumulate capital. Demand for capital falls, bringing down investment and the real price of capital. This in turn results in a further tightening of the collateral constraint, decreasing investment and output even more. As a result, the initial reactions of the real variables are non-standard, only in later periods the usual positive effects of higher productivity prevail.

In the EFP model, lower inflation also raises the real value of loans, thus increasing leverage. This results in higher spreads between the lending rate and the risk free rate. However, entrepreneurs do not face direct collateral constraints, so higher spreads make them cut capital spending only marginally on impact and the effect of higher productivity prevails already in the next period. Still, as tighter lending standards do not die out quickly, the expansion in investment is significantly weaker than in the standard NK model and in the medium run it also falls short of that in the CC version. The flip side of an increase in the real value of loans is an increase in the real value of households' deposits. This wealth effect boosts their consumption and ensures a positive response of output already from the beginning.

Overall, since debt in both models with financial frictions is nominal, a decrease in inflation resulting from a positive productivity shocks leads to a debt deflation (Fisher) effect, which dampens the response of investment and output compared to the NK model. The dampening impact of the CC setup is mostly pronounced in the short run, while that of the EFP is spread over time.

# 5.2 Monetary policy shock

Figure 2 presents the impulse responses to a monetary policy shock. Following the shock, nominal and real interest rates rise and, as in the standard NK model, aggregate demand declines. Lower demand for capital makes its price go down, which has an amplifying effect in models with financial frictions. In the CC version, lower value of collateral forces the constrained agents to save on investment. This drives the price of capital further down, tightening the lending constraint even more. Since inflation falls, the demand side is additionally weakened by the debt deflation channel. As a result, investment and output sharply decline on impact, but then recover relatively quickly (comparing to the EFP model) following the rise in the price of capital.

In the EFP model, the fall in the price of capital subtracts from entrepreneurs' net worth, which, together with a rising real value of their debt, translates into a higher spread between the lending and the risk free rate. This mechanism amplifies the initial impact of monetary tightening on investment, though by substantially less than the direct collateral effect in the CC model. The positive effect of unexpected deflation on households' wealth further increases the difference between the output response across the two alternative approaches. On the other hand, as entrepreneurs' balance sheets (and so lending conditions) improve only gradually, the speed of reversion to the steady state is much lower in the EFP variant.

#### 5.3 Net worth and loan-to-value ratio shock

In Figure 3 we compare the impact of a shock to net worth (implemented as an increase in the survival rate of entrepreneurs) on the EFP model to the impact of an LTV shock on the CC model. The shock definitions are not fully equivalent. Nevertheless, there are some similarities between them. First of all, both shocks have an expansionary impact on the economy. A higher LTV ratio allows entrepreneurs to demand more loans and increase investment. Higher net worth increases entrepreneurs' stake in financing capital expenditures and so allows them to borrow at a lower premium over the risk free rate. It is also worth noting that the two shocks are important sources of fluctuations in standard macro-variables in both models.

The transmission of an LTV shock in the CC model is fairly intuitive. Entrepreneurs increase borrowing and use the proceeds to invest more, which raises output. Higher demand for capital sharply increases its price, relaxing the collateral constraint further. The boom translates into an increase in inflation. The reactions are very fast but short-lived, with output and private demand components peaking in the first quarter, but then turning negative already in the second year.

The story behind the reaction of the EFP model to a net worth shock differs in several vital respects. As in the case of an LTV shock, the responses of investment and output are positive, but exhibit a hump-shaped pattern, gaining momentum and dying out very slowly. The second striking difference across the models concerns the reactions of consumption and debt. Since a positive net worth shock is defined in the EFP version as a decrease in the number of exiting entrepreneurs, it implies lower wealth transfers to households. As a result, households cut consumption and savings, the latter bringing about a fall in debt. Finally,

in contrast to an increase in the LTV ratio, a boost to net worth leads to a fall in inflation, indicating that the two shocks imply a different balance of supply and demand effects.

#### 5.4 Riskiness and spread shock

While the net worth and LTV shocks affected primarily the available quantity of loans, the entrepreneur riskiness (EFP) and spread (CC) shocks directly affect their cost. As can be seen from the response of spreads in both models (see Figure 4), the degree of comparability between these two shocks is very high. Therefore, even though they contribute relatively less to the variance of non-financial variables, their inspection is useful in highlighting the differences between the two financial sector variants considered in this paper.

In the CC model, higher spreads tighten the collateral constraint, affecting negatively loans, consumption and investment. As a result, output and labour input fall as well. As the demand for capital decreases, its price goes down. The story in the EFP model is similar. Higher riskiness of projects run by entrepreneurs makes banks demand a higher premium on loans to entrepreneurs, which discourages the latter from investing. As a result, credit, output and the price of capital decrease. In both models, shocks affecting spreads act like cost push shocks, driving output and inflation in opposite directions.

Again, the main difference between the two alternative specifications concerns how the responses are spread over time. In the CC variant, all real variables are most strongly affected on impact. In the EFP version, consumption, investment and debt display an inverted hump-shaped pattern and there is even a short-lived increase in output and labour input. The latter, somewhat counterintuitive effect results from an increase in the bankruptcy rate, which means that more resources are needed to cover monitoring costs.

#### 5.5 Summary

Several more general observations can be drawn from the analysis of impulse response functions presented above. First, in all cases the reaction of the CC model is much faster than that of the EFP setup. In particular, the CC model usually generates reaction functions with the deepest impact occurring in the first quarter of the shock. This seems inconsistent with VAR evidence on monetary transmission, where the reactions are usually hump-shaped, more like in the EFP model. Second, in the CC model all shocks exercise a very strong influence on the price of capital, driving it down or up by as much as 10-20% after a standard shock. This is inconsistent with empirical estimates of the price of capital behaviour discussed before and also translates into excessive fluctuations in the rate of return on capital. Finally, some of the impulse response functions are counterintuitive in sign. One notable example is the initial decline of output after a productivity shock in the CC model.

# 6 Business cycle accounting

To shed more light on the differences between the EFP and CC setups, we filter the artificial data generated from these two model versions through the business cycle accounting procedure developed by Chari et al. (2007). In a nutshell, this approach considers a standard real business cycle model with time-varying wedges that resemble productivity, labour and investment taxes, and government spending. The wedges are assumed to follow a first-order vector autoregressive process. The original idea of the Chari et al. (2007) paper was to take this prototype economy to the observed data and examine the role of each wedge in accounting for fluctuations in the main macro-variables. The outcomes could then be used to judge which frictions are quantitatively important for business cycle fluctuations. We do the same exercise on simulated data, with the purpose to highlight the main differences between the propagation mechanisms embedded in our alternative models and evaluate consistency of these mechanisms with actual data.

# 6.1 Setup

We design our prototype economy in a similar way as in Chari et al. (2007). The only difference is incorporation of habits and capital adjustment costs, which we define consistently with our baseline model structure presented in section 2.

The households' problem is to maximise their discounted lifetime utility

$$E_0 \sum_{t=0}^{\infty} \beta^t \left( \frac{\left( c_t^H - \xi c_{t-1}^H \right)^{1-\sigma_c}}{1 - \sigma_c} - \frac{n_t^{1+\sigma_n}}{1 + \sigma_n} \right) \tag{42}$$

subject to the budget constraint

$$c_t^H + (1 + \tau_{i,t})i_t = r_{k,t}k_{t-1} + (1 - \tau_{l,t})w_t n_t - T_t$$
(43)

and capital accumulation

$$k_t = (1 - \delta)k_{t-1} + \left(1 - S\left(\frac{i_t}{i_{t-1}}\right)\right)i_t$$
 (44)

Firms are perfectly competitive and maximise their profits

$$y_t - r_{k,t}k_{t-1} - w_t n_t (45)$$

subject to production function

$$y_t = A_t k_{t-1}^{\alpha} n_t^{1-\alpha} \tag{46}$$

The aggregate resource constraint is

$$y_t = c_t^H + i_t + g_t \tag{47}$$

All variables are defined as in our baseline setup, except that factor prices are now expressed in real terms and so denoted by lower case letters. The four wedges are  $A_t$ ,  $\tau_{l,t}$ ,

 $\tau_{i,t}$  and  $g_t$ . We will refer to them as efficiency, labour, investment and government wedges, respectively.

We calibrate the structural parameters of the prototype economy as in section 4 and estimate the stochastic process for wedges using 25000 observations simulated from each model, as well as actual data for the US economy, covering the period 1970q1-2008q4. As in the original business cycle accounting procedure, the observable variables are output, labour, investment and government spending. In the data simulated from models with financial frictions, we treat monitoring costs (EFP) / entrepreneurs' consumption (CC) as a part of government spending. This together with incorporation of habits and investment adjustment costs into our prototype economy ensures that: the efficiency wedge in all three models coincides with total factor productivity, fluctuations in the labour wedge are, to first order, only due to wage and price stickiness, while first-order movements in the investment wedge are only due to financial frictions and price stickiness.<sup>13</sup> The estimation method is maximum likelihood.

#### 6.2 Results

To see the role of each wedge in each of our datasets, we run the estimated business cycle accounting models with all wedges and with each wedge separately. The results of these exercises are presented in Table 8.

In the first three panels we present, respectively, standard deviations, autocorrelations, and correlations with output for the components of output implied by each single wedge. Starting from the standard deviations, one can see that all three models generate similar and excessive volatility of the efficiency component of output and underestimate the role of the labour component.<sup>14</sup> As in the data, the role of the government spending wedge is marginal in all three models.<sup>15</sup> Important differences concern the investment wedge. In the NK model, its output component results only from price stickiness and is very small. In both financial frictions models, the contribution of this wedge to output fluctuations is substantial, consistently with the data.

As regards autocorrelations, the differences between the NK and EFP models are very small. Both feature too much inertia of the efficiency component and too little inertia of that resulting from movements in the labour wedge. The latter shortcoming is even more pronounced in the CC variant, which additionally underestimates the autocorrelation related to the investment wedge.

The analysed models have substantially different implications for the correlations of individual wedge components with output. Productivity shocks account for virtually all output fluctuations in the NK model, so it should come as no surprise that the efficiency component

<sup>&</sup>lt;sup>13</sup>As demonstrated by Sustek (2011), sticky prices act as equal labour and investment wedges.

<sup>&</sup>lt;sup>14</sup>Total factor productivity is described by the same stochastic process in the NK, EFP and CC models, hence the properties of the efficiency components of output also do not differ. On the other hand, even though the degree of wage stickiness (i.e. the Calvo probability and indexation) is identical in all three model variants, their shock propagation mechanisms are not the same, which makes the labour wedge evolve differently.

<sup>&</sup>lt;sup>15</sup>Given our treatment of the government spending data explained above, we do not give any interpretation to this wedge.

is much more correlated with output in this simple version than in the case for our two model variants with financial frictions. On the other hand, both financial friction models underestimate the procyclical behaviour of this wedge. Moving to the labour wedge, its output component is countercyclical in all three models, which is clearly at odds with the data. This result is consistent with the negative correlation of labour and GDP reported earlier. Finally, both variants with the financial sector imply procyclicality of the investment wedge component, slightly overestimating it relative to the data. This is a clear improvement over the NK model, where the investment wedge is countercyclical.

Summing up, the BCA exercise is consistent with the results derived earlier. First, it shows that financial frictions are not able to substantially improve the behaviour of labour in the NK framework. Second, it confirms the low internal propagation of the CC model. Finally, it confirms that the main distortionary impact of financial frictions embedded in the EFP and CC models shows up in the investment wedge, bringing them closer to the data.

### 7 Conclusions

In this paper we make a thorough comparison between two standard ways of introducing financial frictions into a standard New Keynesian model. To make this task possible, we tweak the models to make them comparable in all respects but the financial sector setup. We achieve this goal by using a careful calibration strategy. We next analyse the differences between the two alternative approaches with the following tools: moment comparison, impulse response analysis and business cycle accounting.

Both types of frictions clearly improve upon the standard NK framework. They bring the moments of most of the main macrovariables closer to the data. However, the CC model shows a number of features inconsistent with empirical observations. In particular, its internal propagation is weaker and the volatilities of some variables substantially higher than observed in the data.

The impulse response analysis reveals important differences in the propagation mechanisms between the CC and EFP variants. The former usually generates reaction functions with the deepest impact occurring in the first quarter of the shock, while those obtained from the latter are usually hump-shaped. Further, in the CC model all shocks exercise much stronger influence on the price of capital. It is worth noting that the impulse response functions obtained from the models with financial frictions are not always intuitive, like the initial decline of output after a positive productivity shock in the CC model.

Overall, we conclude that while both financial frictions models improve in many aspects the NK framework, some properties of the CC model seem undesirable. This is especially acute if the model is treated as a workhorse for policy-oriented macroprudential research. Notwithstanding the role of collateral constraints in the real world, their introduction through sharply binding constraints generates strong overreactions. We believe that this paper will support the process of developing macro-financial models that match the data and economic intuition. This should benefit both academic and policy-oriented research.

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# Tables and figures

Table 1: Structural parameters

Parameter	Values	Description
Households		
eta	0.995	discount rate
$\sigma_c$	2.0	inverse of intertemporal elasticity of substitution
$\sigma_n$	2.0	inverse of Frisch elasticity of labour supply
ξ	0.6	degree of external habit formation
$\phi_w$	1.2	labour markup
$ heta_w$	0.7	Calvo probability for wages
$\zeta_w$	0.58	indexation parameter for wages
Producers		
$\alpha$	0.33	product elasticity with respect to capital
$\phi$	1.1	product markup
$\delta$	0.025	depreciation rate
$\kappa$	5.74	investment adjustment cost
heta	0.66	Calvo probability for prices
$\zeta$	0.24	indexation parameter for prices
Taylor rule		
$\gamma_R$	0.8	interest rate smoothing
$\gamma_{\pi}$	1.5	response to inflation
$\gamma_y$	0.5	response to GDP
Financial se	ector - El	FP
$\mu$	0.10	monitoring costs
$\nu$	0.977	survival rate for entrepreneurs
$\sigma_{a_E}$	0.29	steady state st. dev. of idiosyncratic productivity
$T_E$	0.029	transfers to entrepreneurs (relative to steady state output)
Financial sector - C		C
$eta_E$	0.985	entrepreneurs discount factor
$\phi_L$	1.002	loan markup
$m_f$	0.52	steady state LTV
$ ilde{T_E}$	0.002	transfers to entrepreneurs (relative to steady state output)

Table 2: Stochastic processes

Parameter	Values	Description
Common sh	ocks - sa	me in both models
$ ho_A$	0.95	productivity shock
$\sigma_A$	0.007	productivity shock
$ ho_G$	0.95	government spending shock
$\sigma_G$	0.012	government spending shock
$\sigma_R$	0.001	monetary policy shock
Financial se	ector sho	cks - EFP
$\rho_{\nu}$ 0.84		net worth shock
$\sigma_{\nu}$ 0.006		net worth shock
$ ho_E$	0.83	entrepreneur riskiness shock
$\sigma_E$	0.012	entrepreneur riskiness shock
Financial se	ector sho	cks - CC
$\rho_{m}$ 0.96		LTV shock
$\sigma_m$	0.016	LTV shock
$ ho_{\phi_L}$	0.85	spread shock
$\sigma_{\phi_L}$	0.002	spread shock

Table 3: Steady state ratios

Variable	Values
Consumption share in output	0.63
Government expenditure share in output	0.16
Investment share in output	0.21
Steady state inflation rate	1.006
Spread	0.002
Capital to debt ratio	2.0
Monitoring costs (EFP) / entrepreneurs' consumption (CC) share in output	0.005

Table 4: Variance decomposition: NK version

	Productivity	Government	Monetary
GDP	98.5	0.9	0.5
Labour	95.6	3.4	1.0
Consumption	88.8	10.6	0.6
Investment	93.7	5.9	0.3
Inflation	99.8	0.2	0.0
Interest rate	99.0	0.5	0.6
Real price of capital	78.6	7.5	13.9
Real return on capital	91.3	1.1	7.6

Table 5: Variance decomposition: EFP version

	Productivity	Government	Monetary	Net worth	Riskiness
GDP	46.1	0.9	0.8	49.0	3.3
Labour	70.0	2.8	0.8	21.5	5.0
Consumption	47.8	5.8	0.3	42.2	3.8
Investment	0.9	0.8	0.6	80.7	17.0
Loans	26.4	0.5	0.4	56.9	15.8
Inflation	63.6	0.0	0.2	35.7	0.5
Interest rate	65.0	0.1	0.9	33.7	0.3
Spread	3.3	0.0	0.2	17.0	79.5
Real price of capital	7.9	0.9	2.4	13.5	75.3
Real return on capital	31.7	0.6	3.1	4.8	59.9

Table 6: Variance decomposition: CC version

	Productivity	Government	Monetary	LTV	Spread
GDP	59.1	0.7	1.4	31.5	7.4
Labour	77.0	1.6	1.4	15.6	4.3
Consumption	70.0	8.3	0.3	11.7	9.6
Investment	23.6	1.3	1.5	50.1	23.5
Loans	5.0	0.4	0.1	91.1	3.5
Inflation	74.7	0.1	0.1	22.2	3.0
Interest rate	72.8	0.2	0.4	24.8	1.8
Spread	0.0	0.0	0.0	0.0	100.0
Real price of capital	29.1	0.2	1.9	58.2	10.6
Real return on capital	29.0	0.2	1.7	59.7	9.4

Table 7: Moments of the model generated variables against the data

	Data	NK	EFP	CC
Standard deviations				
GDP	2.30	1.79	1.87	1.90
Labour	2.41	1.84	2.15	2.59
Consumption	2.34	1.48	2.13	1.80
Investment	9.26	4.21	9.59	6.92
Loans	7.80	-	7.80	7.80
Inflation	0.80	1.70	1.68	1.73
Interest rate	0.81	1.50	1.45	1.64
Spread	0.42	-	0.42	0.42
Real price of capital	4.28	0.86	2.74	22.27
Real return on capital	1.25	0.52	0.92	22.89
Autocorrelations				
GDP	0.93	0.99	0.98	0.92
Employment	0.96	0.82	0.86	0.75
Consumption	0.96	0.99	0.99	0.99
Investment	0.97	0.99	0.99	0.88
Loans	0.99	-	0.99	0.99
Inflation	0.74	0.94	0.95	0.94
Interest rate	0.95	0.99	0.99	0.98
Spread	0.85	-	0.84	0.84
Real price of capital	0.98	0.61	0.74	-0.02
Real return on capital	0.92	0.51	0.73	0.00
Correlations with GDP				
GDP	1.00	1.00	1.00	1.00
Employment	0.69	-0.61	-0.40	-0.04
Consumption	0.86	0.89	0.54	0.57
Investment	0.79	0.93	0.61	0.81
Loans	0.41	-	0.07	-0.05
Inflation	0.23	-0.86	-0.88	-0.69
Interest rate	0.04	-0.96	-0.93	-0.77
Spread	-0.45	-	-0.17	-0.23
Real price of capital	-0.24	0.35	0.08	0.32
Real return on capital	0.47	-0.11	-0.11	-0.32

Sample for the US data: 1q1970-4q2008. Loans are defined as credit market instruments liabilities of nonfarm nonfinancial businesses (source: FRB) deflated with the GDP deflator. Inflation is the consumer price index. Interest rate is the federal funds rate. Spread is the difference between the industrial BBB corporate bond yield, backcasted using BAA corporate bond yields (source: Bloomberg), and the federal funds rate. Real price of capital is calculated as price of nonresidential fixed capital (source: BEA) deflated with the GDP deflator. Real return on capital is based on estimates provided by Mulligan and Threinen (2010).}

Table 8: Output components from the business cycle accounting exercise

	D .	NIT 7	DDD	- 00
	Data	NK	EFP	CC
Standard Dev	viations			
All wedges	2.32	1.81	1.87	1.90
Efficiency	1.21	2.62	2.62	2.62
Labour	1.72	0.89	0.91	0.97
Government	0.26	0.21	0.37	0.19
Investment	1.16	0.11	1.74	1.26
Autocorrelati	ons			
All wedges	0.95	0.99	0.98	0.93
Efficiency	0.95	0.99	0.99	0.99
Labour	0.98	0.92	0.93	0.89
Government	0.93	0.83	0.73	0.80
Investment	0.98	0.99	0.99	0.93
Correlations	with ou	tput		
All wedges	1.00	1.00	1.00	1.00
Efficiency	0.85	0.97	0.66	0.69
Labour	0.59	-0.73	-0.68	-0.36
Government	0.10	-0.10	0.18	0.02
Investment	0.22	-0.81	0.39	0.33

Figure 1: Productivity shock IRFs

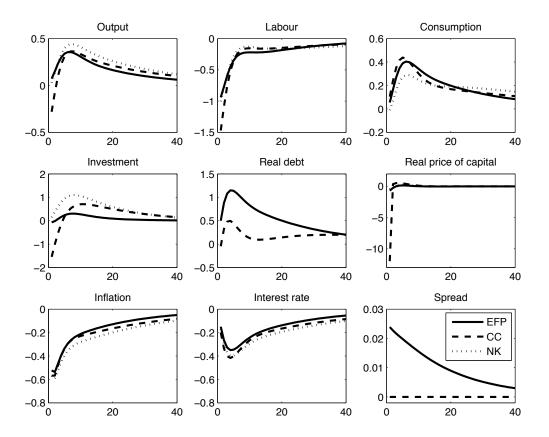
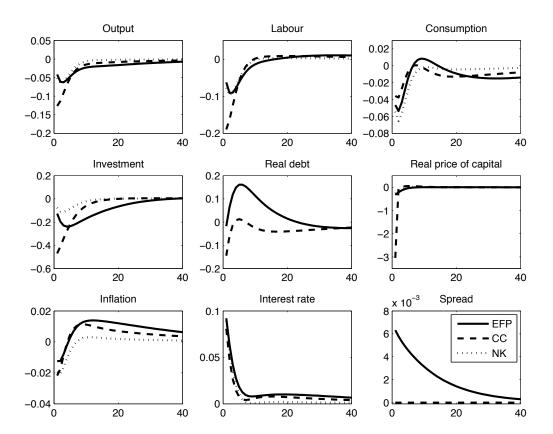


Figure 2: Monetary shock IRFs



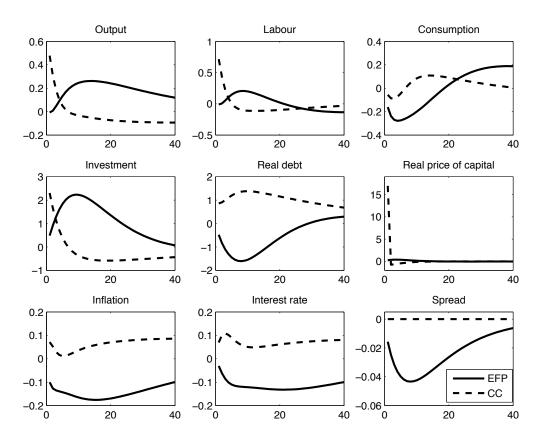


Figure 3: Net worth (EFP) and LTV (CC) shock IRFs

Figure 4: Entrepreneur riskiness (EFP) and spread (CC) shock IRFs

