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Competitiveness channel in Poland and Slovakia: a pre-EMU DSGE analysis

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Abstract

Once a country joins a monetary union, an efficient competitiveness channel is considered to be the main substitute for the abandoned autonomous monetary and exchange rate policy. This paper attempts to make an empirical assessment of how the price competitiveness of domestic producers stabilizes the Polish and Slovak economies against the background of potentially procyclical real interest rates in EMU. To address this issue, we use a small open economy DSGE model. We compare the FIML estimates and resulting IRFs for Poland and Slovakia, concluding that the latter country seems in general to be more capable of handling asymmetric shocks under the common monetary policy. Also, if there was a natural interest rate disparity of 1 percentage point in favour of a catching-up economy and agents expected a 30-year long period of closing this gap, our model would predict a terms of trade appreciation for both countries in question, whereby the required appreciation would be more pronounced for Poland than for Slovakia (5.1% and 3.4% respectively). In the context of Slovak revaluations in ERM II, this could be taken into consideration when setting the final conversion rate, along with its further pros and cons.

1 Introduction

Once a country joins a monetary union, its capacity of absorbing asymmetric shocks via policy tools is significantly reduced. Namely, it resigns from autonomous monetary policy and, to a large extent, nominal exchange rate volatility. The burden of adjustment shifts to market-based mechanisms, described in the literature as the competitiveness channel (de Grauwe, 2007; European Commission, 2008). After an asymmetric shock and in the absence of nominal exchange rate fluctuations, as well as fine-tuning possibilities with country-specific nominal interest rate, the speed of reversal to equilibrium is mainly conditional on the price dynamics, and thus on market flexibility, intrinsic inflation persistence and the process of forming expectations (see Torój, 2009).

In addition, this reversal could be hampered by procyclicality of country-specific real interest rates in a monetary union. When a positive demand shock raises the inflation rate and there is a close link between the inflation rate and inflation expectations on a country level, a real interest rate declines, which additionally fuels the boom (European Commission, 2006). A positive impulse for the economic activity can also stem from the fact that an economy with higher natural rate of interest joins a monetary union where lower interest rate level prevails.

There are dynamic interactions between the competitiveness channel and the real interest rate effect. The former is commonly believed to be an equilibrating force in the long run, whereby the latter – to boost output and inflation volatility via short run effects (Arnold and Kool, 2004; Roubini et al., 2007; European Commission, 2008; Wójcik, 2008). However, a joint consideration of both mechanisms seems to be the appropriate approach to model the adjustment process in the aftermath of an asymmetric shock (see e.g. Hoeller et al., 2004; European Commission, 2006; Torój, 2009), as it provides better insight into the adjustment dynamics in general.

This paper aims to contribute to the literature by performing an *ex ante* econometric assessment of the adjustment capacity in Poland. Using historical data for the 1997-2009 period, it assesses the relative sensitivity of the Polish economic activity to the real interest rate and real exchange rate changes. For comparative reasons perform analogous estimations for Slovakia – a new EU member state that adopted the euro in 2009.

The Slovak case is of particular interest because of its ERM II experience. In the course of ERM II participation, the Slovak Koruna was twice revalued. Despite the fact that its appreciation was probably to a large extent in line with equilibrium, this may have partly influenced the price competitiveness of Slovak goods on international markets. We attempt to establish a link between this permanent appreciation, followed by revaluations, and the permanent fall in Slovak nominal interest rates on their way to the euro area. In particular, we are interested whether these two shocks could be offseting each other as a positive and negative demand shock. We also attempt to perform a quantitative assessment of this offsetting power.

The issue of the relative impact of the real exchange and interest rates on economic activity in a small open economy has been heavily discussed at the turn of the centuries, when a concept of Monetary Conditions Index (MCI) was popular among policymakers (see Freedman, 1994, 1995, for the foundations of this concept). The main purpose of the MCI was to combine the interest rate with the exchange rate to provide a more adequate assessment of monetary policy stance for a small economy than the interest rate only. The so-called MCI-ratio (i.e. the impact of real interest rate divided by the impact of real exchange rate) was calibrated or calculated as a quotient of coefficients in a regression based mostly on IS-curve approach, i.e. by looking at the relative importance of both variables in controlling the output gap dynamics. In our study, we attempt to address this issue in a more complex manner by using a micro-founded New Keynesian DSGE model.

Having estimated its parameters for Poland and Slovakia, we perform simulations of the adjustment path after an asymmetric shock that hits both countries within the euro area. This can be seen as an econometric approach to an *ex ante* assessment of the competitiveness channel's efficiency in EMU, delivering some complementary information to institutional characteristics of both economies. Also, we argue that these ratios could provide some information on the required size of real appreciation that could potentially offset the positive demand pressure from the natural interest rate differential between the new member states and the euro area under common monetary policy with lower interest rates than in the NMS before the euro area entry.

The rest of the paper is organized as follows. Section 2 reviews some stylized facts about Slovakia's road to the euro, as well as the literature on estimating the relative impact of the real interest rate and the real exchange rate on the economic activity. In Section 3, a DSGE model is developed. Section 4 contains the estimation results for Poland and Slovakia. In Section 5, the adjustment capacity of both economies after an asymmetric shock is assessed on the basis of impulse-response functions. Section 6 investigates the mid-term consequences of natural interest rates misalignment and implications of central parity revaluation within the model developed in Section 3. Section 7 concludes.

2   Why does the MCI-ratio matter for the adjustment dynamics?

The history of euro area enlargements (Greece 2001, Slovenia 2007, Cyprus and Malta 2008, Slovakia 2009 and Estonia 2011) exhibits a remarkable pattern (see Figure 1). When we look at the GDP *per capita* on the eve of euro adoption in individual countries, we can see a downward-sloping curve (with the only notable exception of Cyprus in 2008). On January 1st 2009 Slovakia joined the monetary union at a record low of 42% of euro area GDP *per capita*. If Poland had done the same in 2009, the Slovak record would have been beaten by 9 percentage points. Estonia, scheduled to adopt the euro in 2011, fits into this trend with 37% of the euro area level.

Figure 1: GDP *per capita* in the year preceding euro area entry [% EA]
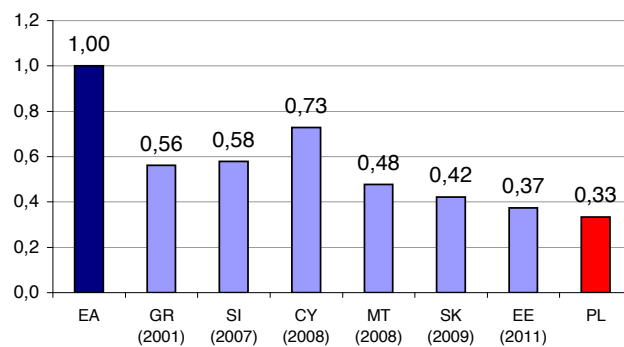
Source: Eurostat data.

One can think of two ways in which this discrepancy matters for the country under ECB's common monetary policy. Following the approach adopted by Flaig and Wollmershäuser (2007) or Calmfors (2007), we can differentiate between two possible sorts of common interest rate misalignment: the **cyclical** and the **structural** one.

Firstly, economies with low levels of income could possibly be characterized by different structures of production and/or consumption than high-income economies. Empirical research confirms that this is to some extent the case for Poland (Adamowicz et al., 2009). This makes such countries exposed to asymmetric shocks after joining a monetary union. Once such a shock occurs, it would most probably lead to a cyclical divergence in inflation. If a central bank in an autonomous monetary regime follows a Taylor-type rule (Taylor, 1993), e.g. $i_t = \gamma_\pi \pi_t + \gamma_y y_t$ (i − nominal interest rate, π − inflation rate, y − output gap), it would bring the economy back to the unique equilibrium as long as it complies with the Taylor principle, which requires a more than one-to-one response to inflation ($\gamma_\pi > 1$).

However, with $\gamma_\pi < 1$, sunspot equilibria may occur. In a monetary union the central bank is concerned with the aggregate price stability. This means that in an example 2-country monetary union, it responds to a regional weighted average, $\pi_t = w\pi_{1t} + (1 − w)\pi_{2t}$. Under

these circumstances, the response to the country-specific inflation rate, $\gamma_\pi w$, could be far below 1. This argument lied at the heart of "Walters critique" (Walters, 1994). However, the reasoning was incomplete because it left aside open economy considerations. When the domestic producers are involved in international trade and consumers could switch to imported products, an economy cannot afford a boom that deteriorates its competitiveness in the long run. It is the competitiveness channel that restores the long-term equilibrium.

Secondly, the New Keynesian literature (e.g. Clarida et al., 1999; Galí, 2008) establishes a widespread link between the natural interest rate (i.e. one that does not accelerate prices, in the spirit of Wicksell, 1907) and the level of an economy's technology. European Central Bank (2004) enumerates the level of an economy's development among the most essential factors behind the natural rate of interest. Countries with relatively lower income and productivity levels – like Poland and Slovakia – might therefore exhibit higher levels of natural interest rates.

Indeed, Bencik (2009) estimates the natural level of Slovak interest rates in the long term at around $2.5 - 3.0\%$. This seems to exceed the level estimated by the European Central Bank (2004) for the euro area in the interval of approximately 1% to 2%. The estimates by Brzoza-Brzezina (2003) suggest that the disparity can even be higher in the case of Poland, with Polish natural interest rate possibly even above 4%. A small economy that joins a monetary union whose level of interest rates is permanently lower could therefore be hit by a long-lasting postitive internal demand shock.

In both cases, the impact of cyclical or structural stress in monetary policy crucially depends on the sensitivity of economic activity to the discrepancy between the current real and the natural interest rate level. In the New Keynesian literature (see e.g. Clarida et al., 1999; Galí, 2008) it is usually modelled as an inverse function of intertemporal elasticity of substitution (σ).

Monetary policy stress was not the only demand disturbances on Slovakia's road to the euro. In the ERM II system, the Slovak koruna was twice revalued. The cumulative effect amounted to approximately 30% stronger currency over a of 2-year period (see Figure 2), and the final conversion rate was in line with the terminal value for the central parity.

A permanent fall in nominal interest rates and a policy-induced deterioration of country's competitiveness might be considered as complementary shocks. Intuitively, a positive shock hitting the economy via internal demand might be offset in terms of economic activity by a negative shock hitting the external demand and the domestic price competitiveness in general. One might be interested in the net effect of both disturbances and, consequently, whether the choice of the final conversion rate of the domestic currency into the euro could be considered as an adjustment policy tool on the way to the euro. According to Bencik (2009), "...after Slovakia's accession to the monetary union, there will be a discrepancy between the neutral interest rate in the euro area and in Slovakia. It will be probably necessary to offset the expansive influence of interest rates by other economic policy instruments."

Figure 2: Slovak koruna appreciation and revaluations in ERM II
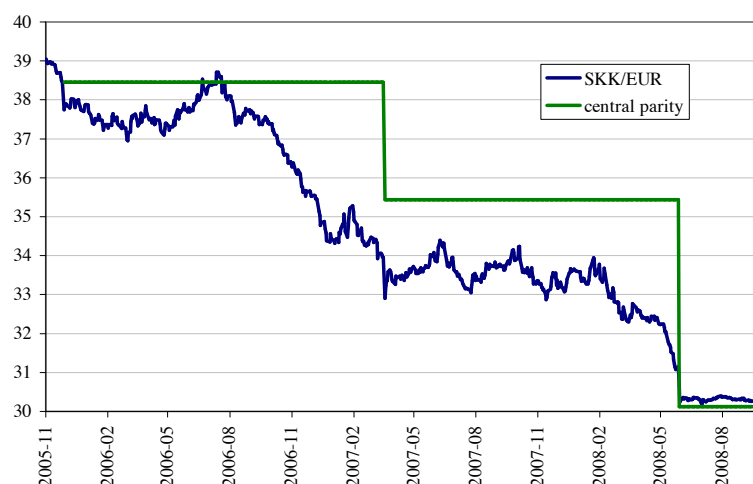
Source: Eurostat and ECB data.

This paper attempts to formalize this argument and outline an empirical strategy to get the necessary quantitative information, which in turn would make this argument operational. In particular, in order to know how to offset such a set of shocks against each other, one needs to be aware of the relative impact of real interest rate and real exchange rate on the domestic economy. In mid- to late 1990s, this question was heavily discussed in the literature as the problem of estimating the "MCI-ratio".

The MCI (Monetary Conditions Index) is a weighted sum of real interest rate and real exchange rate. This index is supposed to reflect the joint impact of both factors on the domestic economic activity and evaluate the current monetary policy stance (Freedman, 1994, 1995). The econometric problem, associated with the construction of the index, is related to the difficulties with the estimation of weights for both variables. Their ratio is the parameter in question. A value less than one suggested that the impact of the real exchange rate is stronger compared to the impact of real interest rate. The weights should be chosen as multipliers of some target variable with respect to both real interest and exchange rate. It was customary in the literature to select some measure of economic activity, e.g. the output gap (Freedman, 1994), although some authors (Kokoszczyński, 2004) argued that the inflation rate would be a preferable choice.

Specifying the index as a weighted average, we implicitly assume that both channels are perfectly substitutable and work independently (Bofinger, 2001). As a consequence, econometric studies in this field mostly involve an estimation of an IS curve, with a measure of an output gap linearly explained by real interest rate or real exchange rate (see e.g. Frochen, 1996; Mayes and Viren, 2000, 2002; Hyder and Khan, 2007). This assumption has been heavily criticized, e.g. by Stevens (1998). Batini and Turnbull (2002) attempted to amend this problem by implementing some additional interest rate and exchange rate

dynamics in the model, so as to measure the short to mid-term impact of monetary conditions on the real economy.

Mayes and Viren (2002) argued that the MCI might be useful in the macroeconomic analysis of monetary integration in Europe. However, due to a number of policy errors associated with monetary policy conduct using MCI, especially in New Zealand (Drew, 2001; Mishkin and Schmidt-Hebbel, 2007; Svensson, 2001), the estimation of MCI-ratio was abandoned. It happened before the New Open Macro progress came into the central banks' modelling practice in the form of dynamic stochastic general equilibrium models (DSGE). However, using this framework could help to better address the deep econometric critique that Eika et al. (1996) formulated in response to models attempting to capture the MCI-ratio, i.e.:

- **dynamic specification**. A rigorous derivation allows applying empirically verified specification and the sources of model dynamics are explicit. Moreover, rational expectations additionally fine-tune this dynamics.

- **parameter stability.** DSGEs are commonly considered to be immune to the Lucas (1976) critique. Even if it was not fully the case for the economies here in question, they place a number of restrictions on a comparable VAR specification that allows us to pin down a relatively small number of parameters in an economically sensible way. This is especially valuable when the time series are short. Also, the presense of expectational terms allows us to apply the model for counterfactual analyses of scenarios after the euro adoption.

- **endogeneity.** In a log-linearized New Keynesian system, there are at least 2 more equations in addition to the IS curve: the Phillips curve and the nominal interest rate equation (Taylor rule). The absence of the latter was particularly stressed by many authors as a potential source of endogeneity bias (Eika et al., 1996; Gottschalk, 2001). The presence of the former allows us to specify the real interest rate correctly, i.e. in an *ex ante* manner.

In this paper, we take a more complex perspective than the MCI literature. We specify a New Keynesian small open economy DSGE model. It contains the features of interest: competitiveness channel, separate inflation processes in the tradable and nontradable sector, Euler equation establishing a dynamic link between present and future consumption via the *ex ante* real interest rate, as well as common monetary policy. It turns out that an IS curve specification as a linear equation with the real interest rate and the real exchange rate (in level; such as e.g. in Kot, 2003 or Torój, 2008) is dynamically inconsistent with the inter- and intratemporal optimizing setup.

This is why we adopt a micro-founded approach and investigate in detail (i) the interest rate parameter in the Euler equation and (ii) the terms of trade parameter in the static equilibrium condition to evaluate the impact of real exchange and interest rate developments within a monetary union. We simulate the dynamic responses of the Polish and Slovak economies to asymmetric disturbances within a monetary union, as well as to the permanent fall in interest rates. This brings us to the answer whether – and to what extent –

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procyclical real interest rates are dampened by the competitiveness channel, and how the fall in nominal interest rates changes the relative prices between the tradable sector and the rest of the monetary union.

2

3   DSGE model setup

The DSGE model developed here builds strongly upon the multi-region currency union models with possible heterogeneity, such as e.g. ones considered in the works by Benigno (2004); Lombardo (2006); Brissimis and Skotida (2008). The currency union consists of 2 regions. The whole economy of the monetary union, in line with a conventional treatment in the DSGE literature[1], is represented by the interval $\langle 0; 1 \rangle$, whereby the first region (say, home economy) is indexed over $\langle 0; w \rangle$ (relative size of the region: w), and the second (foreign economy) is indexed over $\langle w; 1 \rangle$.

As the behaviour of the nontradable sector is considered to be a crucial element of adjustment dynamics (see e.g. European Commission, 2008, 2009), both economies consist of two sectors. Each of them is characterized by price rigidities, modelled with Calvo (1983) mechanism. Conventionally, consumers in each region maximize their utility and producers in each sector – their present and discounted future profits. International exchange of goods incorporates the competitiveness channel of adjustment into the model and ensures that in the long run both economies return to their equilibrium after a shock. This is also true for a small economy that does not have an autonomous monetary policy, which is modelled for the entire currency union via a simple Taylor rule with smoothing.

The model incorporates a number of standard New Keynesian rigidities, such as price stickiness modelled with the Calvo mechanism, wage stickiness, price and wage indexation or consumption habits. While monetary policy is always symmetric (with a possibly asymmetric transmission mechanism though), there are four other shocks in the model that can be asymmetric (region-specific): demand shocks (to consumption) and supply shocks, both in the tradable and non-tradable sector, as well as labour supply shocks.

Henceforth, parameters of the foreign economy are denoted analogously to home economy and marked with an asterisk, e.g. σ and σ^*. Lowercase letters denote the log-deviations of their uppercase counterparts from the steady-state values.

3.1   Household decisions

3.1.1   Intratemporal allocation of consumption

Households get utility from consumption and disutility from hours worked. In addition, utility from consumption depends on consumption habits formed in the previous period (see Smets and Wouters, 2003; Kolasa, 2009). The constant relative returns to scale utility function takes the form (as in Galí, 2008):

$$U_t\left(C_t, N_t, H_t\right) = \varepsilon_{d,t} \frac{\left(C_t - H_t\right)^{1-\sigma}}{1-\sigma} - \varepsilon_{l,t} \frac{N_t^{1+\phi}}{1+\phi} \tag{1}$$

where C_t – consumption at t, H_t – stock of consumption habits at t, N_t – hours worked

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[1] See Benigno (2004); Blessing (2008); Kolasa (2009).

at t, $\sigma > 0$ and $\phi > 0$. Consumption habits are assumed to be proportional to consumption at $t-1$ (see Fuhrer, 2000; Smets and Wouters, 2003):

$$H_t = hC_{t-1} \tag{2}$$

with $h \in [0;1)$ The overall consumption index aggregates the tradable and nontradable consumption bundles:

$$C_t \equiv \left[(1-\kappa)^{\frac{1}{\delta}} C_{T,t}^{\frac{\delta-1}{\delta}} + \kappa^{\frac{1}{\delta}} C_{N,t}^{\frac{\delta-1}{\delta}}\right]^{\frac{\delta}{\delta-1}} \tag{3}$$

where $\kappa \in (0;1)$ characterizes the share of nontradables in the home economy and $\delta > 0$ is elasticity of consumption substitution between the goods produced in both sectors.

The domestic consumption of tradables at t consists of goods produced at home, $C_{H,t}$, and abroad, $C_{F,t}$:

$$C_{T,t} \equiv \left[(1-\alpha)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \tag{4}$$

An analogous relationship holds for the foreign economy. Given this, α is an intuitive measure of degree of openness and $1 - \alpha$ − home bias in consumption. $\eta > 0$ is the elasticity of substitution between home and foreign tradables.

A single type of good is indexed as k and belongs to good variety indexed over the interval $\langle 0;1 \rangle$.

The consumption of domestic tradable goods in the home economy ($C_{H,t}$) and in the foreign one ($C_{H,t}^*$) is defined as:

$$C_{H,t} \equiv \left[\left(\frac{1}{w}\right)^{\frac{1}{\varepsilon_T}} \int_0^1 \left(\int_0^w C_{H,t,k}^j dj\right)^{\frac{\varepsilon_T-1}{\varepsilon_T}} dk\right]^{\frac{\varepsilon_T}{\varepsilon_T-1}} \qquad C_{H,t}^* \equiv \left[\left(\frac{1}{w}\right)^{\frac{1}{\varepsilon_T}} \int_0^1 \left(\int_0^w C_{H,t,k}^{j*} dj\right)^{\frac{\varepsilon_T-1}{\varepsilon_T}} dk\right]^{\frac{\varepsilon_T}{\varepsilon_T-1}} \tag{5}$$

The parameter $\varepsilon_T > 1$ measures the elasticity of substitution between various types of goods in international trade, k indexes the variety of goods, and j − the households (integral over j reflects the difference in both economies' size).

We define in an analogous way the domestic and foreign consumption of goods produced abroad, $C_{F,t}$ and $C_{F,t}^*$:

$$C_{F,t} \equiv \left[\left(\frac{1}{1-w}\right)^{\frac{1}{\varepsilon_T}} \int_0^1 \left(\int_w^1 C_{F,t,k}^j dj\right)^{\frac{\varepsilon_T-1}{\varepsilon_T}} dk\right]^{\frac{\varepsilon_T}{\varepsilon_T-1}} \qquad C_{F,t}^* \equiv \left[\left(\frac{1}{1-w}\right)^{\frac{1}{\varepsilon_T}} \int_0^1 \left(\int_w^1 C_{F,t,k}^{j*} dj\right)^{\frac{\varepsilon_T-1}{\varepsilon_T}} dk\right]^{\frac{\varepsilon_T}{\varepsilon_T-1}} \tag{6}$$

For both tradable consumption baskets (i.e. H and F), we define equal elasticity of substitution between various types of goods, ε^T, both at home and abroad.

The nontradable consumption bundles, domestic ($C_{N,t}$) and foreign ($C_{N*,t}$), are characterized in a similar fashion as:

$$C_{N,t} \equiv \left[\left(\frac{1}{w} \right)^{\frac{1}{\varepsilon_N}} \int_0^1 \left(\int_0^w C_{N,t,k}^j dj \right)^{\frac{\varepsilon_N-1}{\varepsilon_N}} dk \right]^{\frac{\varepsilon_N}{\varepsilon_N-1}} \qquad C_{N,t}^* \equiv \left[\left(\frac{1}{1-w} \right)^{\frac{1}{\varepsilon_N^*}} \int_0^1 \left(\int_w^1 C_{N*,t,k}^{j*} dj \right)^{\frac{\varepsilon_N^*-1}{\varepsilon_N^*}} dk \right]^{\frac{\varepsilon_N^*}{\varepsilon_N^*-1}}$$

Consequently, ε^N and ε^{N*} is defined as elasticity of substitution between various types of nontradable goods.

Households maximize at t the discounted flow of future utilities:

$$E_t \sum_t^\infty \beta^t U\left(C_t, N_t, H_t \right) \to \max_{C,N} \tag{7}$$

where $\beta \in (0,1)$ is households' discount factor. Maximization of (7) is subject to a sequence of current and future budget constraints faced by a representative household:

$$\forall_t \quad \int_0^1 \int_0^w P_{H,t,k}^j C_{H,t,k}^j dj\, dk + \int_0^1 \int_w^1 P_{F,t,k}^j C_{F,t,k}^j dj\, dk + \\ + \int_0^1 \int_0^w P_{N,t,k}^j C_{N,t,k}^j dj\, dk + E_t \left\{ Q_{t,t+1} D_{t+1} \right\} \leq D_t + W_t N_t \tag{8}$$

The right-hand side is a household's budget at t. Its income consists of payoffs of securities acquired in the previous periods (D_t), labour incomes (W_t – nominal wage for hours worked at t) and government transfers (T_t). The left-hand side of the inequality sums the consumption spendings of households (where P denotes a price of a particular consumption bundle, indexed in line with these bundles) and acquisition of securities. $Q_{t,t+1}$ is a stochastic discount factor for the payoffs at $t+1$, faced by the households.

Maximizing (7) subject to (8) leads to the following first order conditions:

- demand equations (home and foreign) for individual goods k produced at home:

$$C_{H,t,k} = \frac{1}{w} \left(\frac{P_{H,t,k}}{P_{H,t}} \right)^{-\varepsilon_T} C_{H,t} \qquad C_{H,t,k}^* = \frac{1}{w} \left(\frac{P_{H,t,k}}{P_{H,t}} \right)^{-\varepsilon_T} C_{H,t}^* \tag{9}$$

- demand equations (home and foreign) for individual goods k produced abroad:

$$C_{F,t,k} = \frac{1}{1-w} \left(\frac{P_{F,t,k}}{P_{F,t}} \right)^{-\varepsilon_T} C_{F,t} \qquad C_{F,t,k}^* = \frac{1}{1-w} \left(\frac{P_{F,t,k}}{P_{F,t}} \right)^{-\varepsilon_T} C_{F,t}^* \tag{10}$$

- demand equations (home and foreign) for individual nontradable goods:

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$$C_{N,t,k} = \frac{1}{w}\left(\frac{P_{N,k}}{P_N}\right)^{-\varepsilon_N} C_N \qquad C_{N,t,k}^* = \frac{1}{1-w}\left(\frac{P_{N,k}^*}{P_N^*}\right)^{-\varepsilon_N*} C_{N,t}^* \qquad (11)$$

- demand equations (home and foreign) for domestic tradable goods:

$$C_{H,t} = (1-\alpha)\left(\frac{P_{H,t}}{P_{T,t}}\right)^{-\eta} C_{T,t} \qquad C_{H,t}^* = \alpha^*\left(\frac{P_{H,t}}{P_{T,t}^*}\right)^{-\eta^*} C_{T,t}^* \qquad (12)$$

- demand equations (home and foreign) for foreign tradable goods:

$$C_{F,t} = \alpha\left(\frac{P_{F,t}}{P_{T,t}}\right)^{-\eta} C_{T,t} \qquad C_{F,t}^* = (1-\alpha^*)\left(\frac{P_{F,t}}{P_{T,t}^*}\right)^{-\eta^*} C_{T,t}^* \qquad (13)$$

- home and foreign demand equations for all tradable goods:

$$C_{T,t} = (1-\kappa)\left(\frac{P_{T,t}}{P_t}\right)^{-\delta} C_t \qquad C_{T,t}^* = (1-\kappa^*)\left(\frac{P_{T,t}^*}{P_t^*}\right)^{-\delta^*} C_t^* \qquad (14)$$

- home and foreign demand equations for all nontradable goods:

$$C_{N,t} = \kappa\left(\frac{P_{N,t}}{P_t}\right)^{-\delta} C_t \qquad C_{N^*,t} = \kappa^*\left(\frac{P_{N,t}^*}{P_t^*}\right)^{-\delta^*} C_t^* \qquad (15)$$

- home and foreign labour supply equations:

$$(C_t - H_t)^{\sigma} N_t^{\varphi} \frac{\varepsilon_t^l}{\varepsilon_t^d} = \frac{W_t}{P_t} \qquad (C_t^* - H_t^*)^{\sigma^*} N_t^{*\varphi^*} \frac{\varepsilon_t^{l*}}{\varepsilon_t^{d*}} = \frac{W_t^*}{P_t^*} \qquad (16)$$

The respective price indices are defined in the following way:

$$P_{H,t} \equiv \left[\frac{1}{w}\int_0^1\left(\int_0^w P_{H,t,k}^j dj\right)^{1-\varepsilon_T} dk\right]^{\frac{1}{1-\varepsilon_T}} \qquad P_{H,t}^* \equiv \left[\frac{1}{w}\int_0^1\left(\int_0^w P_{H,t,k}^{j*} dj\right)^{1-\varepsilon_T} dk\right]^{\frac{1}{1-\varepsilon_T}}$$
$$(17)$$

$$P_{F,t} \equiv \left[\frac{1}{1-w}\int_0^1\left(\int_w^1 P_{F,t,k}^j dj\right)^{1-\varepsilon_T} dk\right]^{\frac{1}{1-\varepsilon_T}} \qquad P_{F,t}^* \equiv \left[\frac{1}{1-w}\int_0^1\left(\int_w^1 P_{F,t,k}^{j*} dj\right)^{1-\varepsilon_T} dk\right]^{\frac{1}{1-\varepsilon_T}}$$
$$(18)$$

$$P_{T,t} \equiv \left[(1-\alpha)P_{H,t}^{1-\eta} + \alpha P_{F,t}^{1-\eta}\right]^{\frac{1}{1-\eta}} \qquad P_{T,t}^* \equiv \left[(1-\alpha^*)P_{F,t}^{1-\eta} + \alpha^* P_{H,t}^{1-\eta}\right]^{\frac{1}{1-\eta}} \qquad (19)$$

$$P_{N,t} \equiv \left(\frac{1}{w} \int_0^1 \left(\int_0^w P_{N,t,k} dj \right)^{1-\varepsilon_N} dk \right)^{\frac{1}{1-\varepsilon_N}} \qquad P_{N,t}^* \equiv \left(\frac{1}{1-w} \int_0^1 \left(\int_w^1 P_{N,t,k}^* dj \right)^{1-\varepsilon_{N*}} dk \right)^{\frac{1}{1-\varepsilon_{N*}}}$$

$$\text{(20)}$$

$$P_t \equiv \left[(1-\kappa) P_{T,t}^{1-\delta} + \kappa P_{N,t}^{1-\delta} \right]^{\frac{1}{1-\delta}} \qquad P_t^* \equiv \left[(1-\kappa^*) P_{T,t}^{*1-\delta^*} + \kappa^* P_{N,t}^{*1-\delta^*} \right]^{\frac{1}{1-\delta^*}} \qquad \text{(21)}$$

Log-linearization and differencing the formulas (19) and (21) lead to the following dependencies:

$$\pi_{T,t} = (1-\alpha) \pi_{H,t} + \alpha \pi_{F,t} \qquad \pi_{T,t}^* = (1-\alpha^*) \pi_{F,t} + \alpha^* \pi_{H,t} \qquad \text{(22)}$$

$$\pi_t = (1-\kappa) \pi_{T,t} + \kappa \pi_{N,t} \qquad \pi_t^* = (1-\kappa^*) \pi_{T,t}^* + \alpha^* \pi_{N,t}^* \qquad \text{(23)}$$

Using the above equations, we derive domestic demand functions for the domestic tradable, foreign tradable and nontradable goods:

$$C_{H,t,k} = \frac{1}{w} (1-\alpha)(1-\kappa) \left(\frac{P_{H,t,k}}{P_{H,t}} \right)^{-\varepsilon_T} \left(\frac{P_{H,t}}{P_{T,t}} \right)^{-\eta} \left(\frac{P_{T,t}}{P_t} \right)^{-\delta} C_t \qquad \text{(24)}$$

$$C_{F,t,k} = \frac{1}{1-w} \alpha (1-\kappa) \left(\frac{P_{F,t,k}}{P_{F,t}} \right)^{-\varepsilon_T} \left(\frac{P_{F,t}}{P_{T,t}} \right)^{-\eta} \left(\frac{P_{T,t}}{P_t} \right)^{-\delta} C_t \qquad \text{(25)}$$

$$C_{N,t,k} = \frac{1}{w} \kappa \left(\frac{P_{N,t,k}}{P_{N,t}} \right)^{-\varepsilon_N} \left(\frac{P_{N,t}}{P_t} \right)^{-\delta} C_t \qquad \text{(26)}$$

Analogous equations hold for the foreign economy.

3.1.2 Intertemporal allocation of consumption

We define the stochastic discount factor as:

$$Q_{t,t+1} \equiv \frac{V_{t,t+1}}{\xi_{t,t+1}} \qquad \text{(27)}$$

where $V_{t,t+1}$ is the price at t of an Arrow security, i.e. a one-period security paying 1 at $t+1$ when a specific state of nature occurs and 0 otherwise. $\xi_{t,t+1}$ is the probability that the state of nature in which 1 is paid materializes, conditional on the state of nature at t. Having the access to such a security market, households can transfer utility between periods, maximizing its discounted flow (see Galí and Monacelli, 2005).

The optimality of decisions requires that the marginal loss in utility due to buying the security at t instead of allocating this money to consumption must equal the discounted payoff at $t+1$, also expressed in terms of marginal growth of future utility:

$$\frac{V_{t,t+1}}{P_t}\varepsilon_{d,t}\left(C_t - H_t\right)^{-\sigma} = \xi_{t,t+1}\beta\varepsilon_{d,t+1}\left(C_{t+1} - H_{t+1}\right)^{-\sigma}\frac{1}{P_{t+1}} \tag{28}$$

whereby C_{t+1} and P_{t+1} in the above equation should be interpreted as conditional expected values given the state of nature when the payoff is nonzero.

Applying the definition of $Q_{t,t+1}$ (27) and (2), the equation (28) can be written as:

$$\beta\frac{\varepsilon_{d,t+1}}{\varepsilon_{d,t}}\left(\frac{C_{t+1} - hC_t}{C_t - hC_{t-1}}\right)^{-\sigma}\left(\frac{P_t}{P_{t+1}}\right) = Q_{t,t+1} \tag{29}$$

We calculate the conditional expected value of both sides, which – along with $\Im_t \equiv E_t\left(Q_{t,t+1}\right)$ – leads to the Euler equation for consumption:

$$\Im_t = \beta E_t\left[\frac{\varepsilon_{d,t+1}}{\varepsilon_{d,t}}\left(\frac{C_{t+1} - hC_t}{C_t - hC_{t-1}}\right)^{-\sigma}\left(\frac{P_t}{P_{t+1}}\right)\right] \tag{30}$$

Log-linearization of (30) around the steady state allows us to write the following dependence:

$$c_t - hc_{t-1} = E_t\left(c_{t+1} - hc_t\right) - \frac{1-h}{\sigma}\left[i_t - \left(E_t p_{t+1} - p_t\right) + ln\beta\right] + \frac{1-h}{\sigma}\left(\varepsilon_{d,t} - E_t\varepsilon_{d,t+1}\right) \tag{31}$$

where lowercase variables are percentage deviations from the steady state for their uppercase counterparts. After basic simplifications, we obtain (see Smets and Wouters, 2003):

$$c_t = \frac{h}{1+h}c_{t-1} + \frac{1}{1+h}E_t c_{t+1} - \frac{1-h}{\left(1+h\right)\sigma}\left(i_t - E_t\pi_{t+1} - \rho\right) + \frac{1-h}{\left(1+h\right)\sigma}\left(\varepsilon_{d,t} - E_t\varepsilon_{d,t+1}\right) \tag{32}$$

where $i_t \equiv -ln\Im_t$ denotes short-term nominal interest rate at t, $E_t\pi_{t+1} = E_t p_{t+1} - p_t$ – expected domestic consumer price growth, $\rho = -ln\beta$ – natural interest rate corresponding to the households' discount factor β.

3.2 International prices

Define bilateral *terms of trade* between the home and foreign economy as:

$$S_t \equiv \frac{P_{H,t}}{P_{F,t}} \tag{33}$$

Log-linearizing (33) around a symmetric steady state $S_t = 1$ – the law of one price in the tradable sector – leads to the following relationship:

$$s_t = p_{H,t} - p_{F,t} \tag{34}$$

Also, define internal *terms of trade* as price ratio between tradables and nontradables:

$$X_t \equiv \frac{P_{T,t}}{P_{N,t}} \tag{35}$$

An analogous approximation allows us to write:

$$x_t = p_{T,t} - p_{N,t} \tag{36}$$

Using (33) and (23) we can write:

$$p_{T,t} = p_{H,t} - \alpha s_t \tag{37}$$

$$p_t = p_{T,t} - \kappa x_t = p_{N,t} + (1 - \kappa) x_t \tag{38}$$

The real exchange rate Q_t (q_t for log-deviation from the steady state) versus the rest of the monetary union takes the form:

$$q_t = p_t - p_t^* = (1 - \alpha - \alpha^*) s_t - \kappa x_t + \kappa^* x_t^* \tag{39}$$

Real exchange rate Q_t (q_t in log-deviations from the steady state) appreciation is then linked to the appreciation of external *terms of trade*, depreciation of domestic internal terms of trade (defined as in (36)) and appreciation of foreign internal terms of trade.

3.3   International risk sharing

Household can smooth their consumption not only in time, but also in international financial markets (Blessing, 2008; Galí, 2008; Kolasa, 2009; Lipińska, 2008). Under complete markets, equation (28) holds for both home and foreign economy (see Galí and Monacelli, 2005 for derivation of a more general version):

$$\frac{V_{t,t+1}}{P_t^*} \varepsilon_{d,t}^* \left(C_t^* - H_t^* \right)^{-\sigma} = \xi_{t,t+1} \beta^* \varepsilon_{d,t+1}^* \left(C_{t+1}^* - H_{t+1}^* \right)^{-\sigma} \frac{1}{P_{t+1}^*} \tag{40}$$

Access to common, integrated financial market, allows to write an equation analogous to (41), derived from (40), with a common stochastic discount factor:

$$\beta^* \frac{\varepsilon_{d,t+1}^*}{\varepsilon_{d,t}^*} \left(\frac{C_{t+1}^* - h^* C_t^*}{C_t^* - h^* C_{t-1}^*} \right)^{-\sigma} \left(\frac{P_t^*}{P_{t+1}^*} \right) = Q_{t,t+1} \tag{41}$$

Combining (29) and (41), we obtain:

$$\varepsilon_{d,t}\left(C_t - hC_{t-1}\right)^{-\sigma} = \vartheta^*\varepsilon_{d,t}^*\left(C_t^* - h^*C_{t-1}^*\right)^{-\sigma^*} Q_t \tag{42}$$

Following Galí and Monacelli (2005) we assume that $\vartheta^* = \vartheta = 1$. This does not affect the generality, except for restricting the initial conditions on the stock of net foreign assets and states of nature. Log-linearizing equation (42) around a steady-state allows to derive a relation between home and foreign consumption and the real exchange rate (see also Chari et al., 2002):

$$\frac{\sigma}{1-h}\left(c_t - hc_{t-1}\right) - \varepsilon_{d,t} = \frac{\sigma^*}{1-h^*}\left(c_t^* - h^*c_{t-1}^*\right) - \varepsilon_{d,t}^* - q_t \tag{43}$$

3.4 Producers

3.4.1 Real marginal costs

The producers of variety k in the tradable or nontradable bundle face the following production function (see Galí, 2008):

$$Y_{t,k}^H = A_t^H N_{t,k}^H \varepsilon_t^H \tag{44}$$

$$Y_{t,k}^N = A_t^N N_{t,k}^N \varepsilon_t^N \tag{45}$$

whereby $lnA_t^H \equiv a_t^H$ is an exogenous technological process (analogously for the nontradable sector N). Following Clarida et al. (1999), we assume away the price deviations of individual varieties within a sector as of second-order importance in the proximity of the steady state. This allows us to integrate the formulas (44) and (45) into sectoral production functions with supply shocks denoted ε_t^H and ε_t^N respectively (henceforth as recycling notation for the logs).

The real marginal cost (as log-deviation from the steady-state) is calculated as a difference between the wage level in the region (w) and the sectoral producer price log-level plus the log of marginal labour product (mpn) (por. Galí and Monacelli, 2005):

$$mc_t^H \equiv w_t - p_t^H - mpn_t^H \qquad mc_t^N \equiv w_t - p_t^N - mpn_t^N \tag{46}$$

The real marginal product is equal across producers in a given sector. After substituting into (46) the derivatives of both functions with respect to N_t, we obtain:

$$mc_t^H = w_t - p_{H,t} - \left(a_t^H + \varepsilon_t^H\right) \tag{47}$$

$$mc_t^N = w_t - p_{N,t} - \left(a_t^N + \varepsilon_t^N\right) \tag{48}$$

Using equations (37)-(38) and the labour supply equation (16) leads to:

$$
\begin{aligned}
mc_t^H &= (w_t - p_t) + (p_t - p_{T,t}) + (p_{T,t} - p_{H,t}) - \left(a_t^H + \varepsilon_t^H\right) = \\
&= (w_t - p_t) - \alpha s_t - \kappa x_t - \left(a_t^H + \varepsilon_t^H\right)
\end{aligned}
\tag{49}
$$

$$
\begin{aligned}
mc_t^N &= (w_t - p_t) + (p_t - p_{N,t}) - \left(a_t^N + \varepsilon_t^N\right) = \\
&= (w_t - p_t) + (1 - \kappa)\, x_t - \left(a_t^N + \varepsilon_t^N\right)
\end{aligned}
\tag{50}
$$

3.4.2 Pricing decisions

There are price rigidities in the economy. Following the usual approach in the New Keynesian literature, we model them by means of the Calvo (1983) scheme. In a given period, a fraction θ of producers are not allowed to reoptimize their prices in reaction to economic innovations and must sell at the price from the previous period. The probability of being allowed to reoptimize the price is equal across producers: $1 - \theta$ in each period, independently of the amount of time elapsed since the last price change.

Some of the producers allowed to change their price do not really reoptimize. Following Galí and Gertler (1999) we assume that the change in price is partly implemented as an indexation to past inflation. This mechanism leads to a hybrid Phillips curve (see Galí and Gertler (1999); Galí et al. (2001)), empirically outperforming the purely forward-looking specifications in terms of goodness-of-fit. Following Kolasa (2009), inflation is modelled separately in the tradable and nontradable sector.

As Galí and Gertler (1999) we assume that a fraction $1 - \theta$ of producers are able to change their price in t in each sector, which implies the following dependence between the price levels at $t - 1$ and t:

$$p_t^H = \theta^H p_{t-1}^H + \left(1 - \theta^H\right) \bar{p}_t^H \qquad p_t^N = \theta^N p_{t-1}^N + \left(1 - \theta^N\right) \bar{p}_t^N \tag{51}$$

where $\bar{p}_t^H$ and $\bar{p}_t^N$ denote the prices set newly at t by the $1 - \theta$ fraction of producers. Among the producers who reoptimize prices there is a fraction of $1 - \omega$ producers reoptimizing in an anticipatory manner as in Calvo (1983). They maximize the discounted flow of future profits, using all information available at the time of decision and taking into account future constraints. The rest of producers (ω) reset their prices, according to past price dynamics:

$$\bar{p}_t^H = \omega^H p_{b,t}^H + \left(1 - \omega^H\right) p_{f,t}^H \qquad \bar{p}_t^N = \omega^N p_{b,t}^N + \left(1 - \omega^N\right) p_{f,t}^N \tag{52}$$

Prices set by the latter group of producers are modelled, as in Galí and Gertler (1999), as reoptimized prices from the previous period, indexed to past inflation:

$$p_{b,t}^H = \bar{p}_{t-1}^H + \pi_{t-1}^H \qquad p_{b,t}^N = \bar{p}_{t-1}^N + \pi_{t-1}^N \tag{53}$$

One can show (see Galí and Gertler, 1999; Galí et al., 2001; Galí, 2008 for details) that the reoptimized prices satisfy the following conditions:

$$p_{f,t}^H = \mu^H + \left(1 - \beta\theta^H\right) \sum_{s=0}^{\infty} \left(\beta\theta^H\right)^s E_t \left(mc_{t+s}^H + p_{H,t+k}\right) \tag{54}$$

$$p_{f,t}^N = \mu^N + \left(1 - \beta\theta^N\right) \sum_{s=0}^{\infty} \left(\beta\theta^N\right)^s E_t \left(mc_{t+s}^N + p_{N,t+k}\right) \tag{55}$$

where $\mu^T \equiv -ln\frac{\varepsilon^T}{\varepsilon^T - 1}$ and $\mu^N \equiv -ln\frac{\varepsilon^N}{\varepsilon^N - 1}$ are log-markups in the steady state (or markups in an economy without price rigidities), mc_t – real marginal cost at t.

Combined relationships (51)-(55) lead to the following hybrid Phillips curves in both domestic sectors:

$$\begin{aligned}
\pi_t^H &= \tfrac{\omega^H}{\theta^H + \omega^H[1 - \theta^H(1-\beta)]}\pi_{t-1}^H + \tfrac{\beta\theta^H}{\theta^H + \omega^H[1 - \theta^H(1-\beta)]}E_t\pi_{t+1}^H + \\
&\quad + \tfrac{\left(1 - \omega^H\right)\left(1 - \theta^H\right)\left(1 - \beta\theta^H\right)}{\theta^H + \omega^H[1 - \theta^H(1-\beta)]}mc_t^H
\end{aligned} \tag{56}$$

$$\begin{aligned}
\pi_t^N &= \tfrac{\omega^N}{\theta^N + \omega^N[1 - \theta^N(1-\beta)]}\pi_{t-1}^N + \tfrac{\beta\theta^N}{\theta^N + \omega^N[1 - \theta^N(1-\beta)]}E_t\pi_{t+1}^N + \\
&\quad + \tfrac{\left(1 - \omega^N\right)\left(1 - \theta^N\right)\left(1 - \beta\theta^N\right)}{\theta^N + \omega^N[1 - \theta^N(1-\beta)]}mc_t^N
\end{aligned} \tag{57}$$

where mc_t now denote the deviation of real marginal cost from its long-run value in the respective sector (analogously for the foreign economy).

3.5 Market clearing conditions

Equilibrium on the world markets of individual goods requires equality of overall production and consumption of every variety k in the basket of domestically produced tradables:

$$\begin{aligned}
\int_0^w Y_{H,t,k}^j dj &= \int_0^w C_{H,t,k}^j dj + \int_w^1 C_{H,t,k}^{j*} dj = \\
&= C_{H,t,k} + C_{H,t,k}^* = \\
&= \tfrac{1}{w}\left(1 - \alpha\right)\left(1 - \kappa\right)\left(\tfrac{P_{H,t,k}}{P_{H,t}}\right)^{-\varepsilon_T}\left(\tfrac{P_{H,t}}{P_{T,t}}\right)^{-\eta}\left(\tfrac{P_{T,t}}{P_t}\right)^{-\delta}C_t + \\
&\quad + \tfrac{1}{w}\alpha^*\left(1 - \kappa^*\right)\left(\tfrac{P_{H,t,k}}{P_{H,t}}\right)^{-\varepsilon_T}\left(\tfrac{P_{H,t}}{P_{T,t}^*}\right)^{-\eta^*}\left(\tfrac{P_{T,t}^*}{P_t^*}\right)^{-\delta^*}C_t^* = \\
&= \tfrac{1}{w}\left(\tfrac{P_{H,t,k}}{P_{H,t}}\right)^{-\varepsilon_T}[\left(1 - \alpha\right)\left(1 - \kappa\right)\left(\tfrac{P_{H,t}}{P_{T,t}}\right)^{-\eta}\left(\tfrac{P_{T,t}}{P_t}\right)^{-\delta}C_t + \\
&\quad + \alpha^*\left(1 - \kappa^*\right)\left(\tfrac{P_{H,t}}{P_{T,t}^*}\right)^{-\eta^*}\left(\tfrac{P_{T,t}^*}{P_t^*}\right)^{-\delta^*}C_t^*]
\end{aligned} \tag{58}$$

Plugging the above expression into the definition of aggregate domestic tradable product,

$$Y_t^H \equiv \left[\int_0^1 \left(\int_0^w Y_{H,t,k}^j dj \right)^{\frac{\varepsilon^T - 1}{\varepsilon^T}} dk \right]^{\frac{\varepsilon^T}{\varepsilon^T - 1}} \tag{59}$$

yields:

$$
\begin{aligned}
Y_t^H &= (1-\alpha)(1-\kappa) \left(\frac{P_{H,t}}{P_{T,t}} \right)^{-\eta} \left(\frac{P_{T,t}}{P_t} \right)^{-\delta} C_t + \alpha^* (1-\kappa^*) \left(\frac{P_{H,t}}{P_{T,t}^*} \right)^{-\eta^*} \left(\frac{P_{T,t}^*}{P_t^*} \right)^{-\delta^*} C_t^* = \\
&= (1-\alpha)(1-\kappa) S_t^{-\alpha\eta} X_t^{-\kappa\delta} C_t + \alpha^* (1-\kappa^*) S_t^{-(1-\alpha^*)\eta^*} X_t^{*-\kappa^*\delta^*} C_t^*
\end{aligned}
\tag{60}
$$

Analogous expression can be written for the sector of foreign tradables (F):

$$
\begin{aligned}
Y_t^F &= \alpha(1-\kappa) \left(\frac{P_{F,t}}{P_{T,t}} \right)^{-\eta} \left(\frac{P_{T,t}}{P_t} \right)^{-\delta} C_t + (1-\alpha^*)(1-\kappa^*) \left(\frac{P_{F,t}}{P_{T,t}^*} \right)^{-\eta^*} \left(\frac{P_{T,t}^*}{P_t^*} \right)^{-\delta^*} C_t^* = \\
&= \alpha(1-\kappa) S_t^{-(1-\alpha)\eta} X_t^{-\kappa\delta} C_t + (1-\alpha^*)(1-\kappa^*) S_t^{-\alpha^*\eta^*} X_t^{*-\kappa^*\delta^*} C_t^*
\end{aligned}
\tag{61}
$$

Log-linearizing around the steady-state, in which ration of consumption in both economies is proportional to their relative size ($\frac{C}{C^*} = \frac{w}{1-w}$) leads to the following conditions:

$$y_t^H = \tilde{w} c_t + (1-\tilde{w}) c_t^* - [\tilde{w}\alpha\eta + (1-\tilde{w})(1-\alpha^*)\eta^*] s_t - \tilde{w}\kappa\delta x_t - (1-\tilde{w})\kappa^*\delta^* x_t^* \tag{62}$$

$$y_t^{F*} = \tilde{w}^* c_t + (1-\tilde{w}^*) c_t^* + [\tilde{w}^*(1-\alpha)\eta + (1-\tilde{w}^*)\alpha^*\eta^*] s_t - \tilde{w}^*\kappa\delta x_t - (1-\tilde{w}^*)\kappa^*\delta^* x_t^* \tag{63}$$

whereby:

$$\tilde{w} = \frac{w(1-\alpha)(1-\kappa)}{w(1-\alpha)(1-\kappa) + (1-w)\alpha^*(1-\kappa^*)} \qquad \tilde{w}^* = \frac{w\alpha(1-\kappa)}{w\alpha(1-\kappa) + (1-w)(1-\alpha^*)(1-\kappa^*)} \tag{64}$$

Market clearing conditions for the nontradable sector can be written using (11) as:

$$Y_{N,t} = C_{N,t} = \kappa \left(\frac{P_{N,t}}{P_t} \right)^{-\delta} C_t \qquad Y_{N,t}^* = C_{N,t}^* = \kappa^* \left(\frac{P_{N,t}^*}{P_t^*} \right)^{-\delta^*} C_t^* \tag{65}$$

Using the definition of internal terms of trade, (35), we get:

$$Y_{N,t} = \kappa X_t^{(1-\kappa)\delta} C_t \qquad Y_{N,t}^* = \kappa^* (X_t^*)^{(1-\kappa^*)\delta^*} C_t^* \tag{66}$$

Log-linearizing (66) around the steady state leads to the following equilibrium conditions:

$$y_t^N = (1 - \kappa) \delta x_t + c_t \quad y_t^{N*} = (1 - \kappa^*) \delta^* x_t^* + c_t^* \tag{67}$$

In further analyses, we treat all the log-linearized variables as deviations from a "natural" state of economy, driven by the exogenous technological processes a_t^T and a_t^N and undistorted by price relations. Henceforth we drop a_t^T and a_t^N and treat y_t^T and y_t^N as output gaps in each sector.

3.6 Monetary policy

The central bank's monetary policy is described by a Taylor (1993) rule with smoothing, which is a commonly applied description in the literature and empirically tested as an adequate tool for both the euro area (Sauer and Sturm, 2003) and Poland (see i.a. Kolasa, 2009; Gradzewicz and Makarski, 2009). The common nominal interest rate is set according to the equation:

$$i_t = \rho + (1 - \gamma_\rho) (\gamma_\pi \tilde{\pi}_t + \gamma_y \tilde{y}_t) + \gamma_\rho i_{t-1} \tag{68}$$

where i_t – central bank policy rate at t, $\tilde{y}_t$ – the output gap in a currency union, $\tilde{\pi}_t$ – inflation rate in a currency union, $\gamma_\rho \in (0; 1)$ – smoothing parameter, $\gamma_\pi > 1$, $\gamma_y > 0$ – parameters of central bank's response to deviations of inflation and output from the equilibrium levels. The condition $\gamma_\pi > 1$ is necessary to satisfy the Taylor rule (Taylor, 1993), leading to a unique equilibrium.

The output gap and inflation rate for the currency union aggregate the values for individual regions, according to their size:

$$\tilde{\pi}_t = \int_0^1 \pi_t^j dj = w\pi_t + (1 - w) \pi_t^* \tag{69}$$

$$\tilde{y}_t = \int_0^1 \tilde{y}_t^j dj = w\tilde{y}_t + (1 - w) \tilde{y}_t^* \tag{70}$$

3.7 Labour market

Equation (16) implies a perfect labour market flexibility. According to Walsh (2010), however, this would lead to a poor empirical fit of the model. We therefore apply a simplified version of a mechanism described by Erzeg et al. (2000) and used i.a. by Kolasa (2009). It allows the marginal rate of substitution between consumption and leisure, mrs_t, to equal the real wage, $w_t - p_t$, but only in the long run. Define mrs_t as:

$$mrs_t \equiv \frac{-\frac{\partial U(c_t, n_t)}{\partial n_t}}{\frac{\partial U(c_t, n_t)}{\partial_{\cdot}}} = \frac{\sigma}{1 - h} (c_t - hc_{t-1}) + \phi n_t + \varepsilon_t^l - \varepsilon_t^d \tag{71}$$

Sectoral production functions imply:

$$
\begin{aligned}
n_t &= \tfrac{N^N}{N} n_t^N + \tfrac{N^H}{N} n_t^H = \frac{\frac{Y^N}{A^N}}{\frac{Y^N}{A^N}+\frac{Y^T}{A^T}} n_t^N + \frac{\frac{Y^T}{A^T}}{\frac{Y^N}{A^N}+\frac{Y^T}{A^T}} n_t^H \approx \\
&\approx \kappa n_t^N + (1-\kappa) n_t^H = \kappa y_t^N + (1-\kappa) y_t^H - \kappa a_t^N - (1-\kappa) a_t^H - \kappa \varepsilon_t^N - (1-\kappa) \varepsilon^H
\end{aligned}
\tag{72}
$$

whereby the approximation assumes a long-term technological symmetry across sectors. The above equation can be used to replace employment in equation (71) by production and current values of supply shocks.

In the short run, let nominal wages be sticky and behave according to the Calvo scheme. Under monopolistic competition in the labour market, individual domestic and foreign households supply differentated types of labour services, N_j, with elasticity of substitution ε_w. Total labour supply at t, N_t, can be aggregated as:

$$
N_t \equiv \left[\left(\frac{1}{w} \right)^{\frac{1}{\varepsilon_w}} \left(\int_0^w N_{j,t}^{\frac{\varepsilon_w-1}{\varepsilon_w}} dj \right) \right]^{\frac{\varepsilon_w}{\varepsilon_w-1}}
\tag{73}
$$

The wage index is defined similarly as:

$$
W_t \equiv \left[\frac{1}{w} \int_0^w W_{j,t}^{1-\varepsilon_w} dj \right]^{\frac{1}{1-\varepsilon_w}}
\tag{74}
$$

Only a fraction of households, $1 - \theta^w \in (0;1)$, can renegotiate their wages at every period. This fraction remains constant and households allowed to reoptimize are selected at random. In particular, the probability of being allowed to renegotiate the wage does not depend on the period elapsed since the last change. Other households partly index their their wages to past consumer inflation. Their fraction is represented by the parameter $\omega^w \in (0;1)$.

Households able to renegotiate their nominal wage maximize the present and the discounted future utilities subject to constraints resulting from expected future labour demand and the fact that the wage level might remain unchainged for a number of periods. Solving this problem leads to the following wage dynamics equation:

$$
\pi_t^w = \beta E_t \pi_{t+1}^w + \frac{(1-\theta^w)(1-\beta\theta^w)}{\theta^w [1+\phi\varepsilon_w]} [mrs_t - (w_t - p_t)] - \omega^w (\beta\pi_t - \pi_{t-1})
\tag{75}
$$

An analogous solution holds for the foreign economy.

3.8   Model equations

The log-linearized dynamic model is composed of the Euler equation for consumption (32), sectoral Phillips curves (56) and (57), wage equation (75), real marginal cost definitions (49) and (50) along with their foreign counterparts, equilibrium conditions (62), (63) and (67), equation of common monetary policy (68) and a set of identities defining the aggregate

values for the monetary union (69) and (70), aggregate price dynamics and deflators. Model equations are explicitly listed in the Appendix.

The list of the random disturbances includes region-specific demand (ε_t^D), supply (ε_t^T, ε_t^N) and labour supply (ε_t^l) shocks, as well as monetary policy (ε_t^i) shock. In the estimation, shocks of the same type are allowed to be correlated across regions, but are assumed to be independent across types. They also follow a first-order autoregressive process.

3

4 Estimation

The presence of expectational components in the model requires using specific estimation techniques. As shown in Torój (2009), careful estimation of forward- and backward-looking parameters in the Euler and Phillips curves is critical for accurate modelling of adjustment dynamics after asymmetric shocks.

In the literature, there are two standard manners of handling this problem. Firstly, following a seminal paper of Galí and Gertler (1999), the system can be estimated equation-by-equation using the generalized method of moments (GMM, see Hansen, 1982). Secondly, one can specify a closed system and solve out the forward-looking components using standard algorithms (Blanchard and Kahn, 1980; Klein, 2000; Sims, 2001) and estimate the log-linearized system as a structural VAR, either with classical or Bayesian methods. The classical approach, used in this study, is based on full information maximum likelihood (FIML) estimation as proposed by Ireland (2004).[2]

The log-linearized model can be summarized in a matrix form as:

$$\mathbf{A} E_t \mathbf{x}_{t+1} = \mathbf{B} \mathbf{x}_t + \mathbf{C} \boldsymbol{\varepsilon}_t \tag{76}$$

The model is solved under the assumption of expectations rationality into the following form (Blanchard and Kahn, 1980; Klein, 2000):

$$\mathbf{x}_t = \mathbf{M} \mathbf{x}_{t-1} + \mathbf{N} \boldsymbol{\varepsilon}_t \tag{77}$$

where $\mathbf{M}_{n_1 \times n_1}(\mathbf{A}, \mathbf{B}, \mathbf{C})$ and $\mathbf{N}_{n_1 \times n_1}(\mathbf{A}, \mathbf{B}, \mathbf{C})$, n_1 – number of endogenous variables in the reduced system.

In rational expectations models, it is common to assume AR(1) residuals by construction (see e.g. Mavroeidis, 2005, for motivation):

$$\boldsymbol{\varepsilon}_t = \boldsymbol{\Phi} \boldsymbol{\varepsilon}_{t-1} + \boldsymbol{\xi}_t \tag{78}$$

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[2]It is commonly argued that the full information approach outperforms the former in a number of aspects (i.e. arbitrary choice or non-orthogonality of instruments, non-identifiable parameters or multiple expectational terms to be instrumentalized; see e.g. Staiger and Stock, 1997; Shea, 1997; Stock and Yogo, 2003; Mavroeidis, 2005; Fuhrer and Rudebusch, 2004; Nason and Smith, 2008; Lindé, 2005). Although Monte Carlo studies of Fuhrer and Rudebusch (2004) and Lindé (2005) confirmed the superiority of FIML over GMM in relatively basic New Keynesian closed-economy trinity model, the FIML method is not free of drawbacks. It is based on the assumption of normally distributed disturbances. Also, it requires a careful selection of starting values for parameters in the maximization procedure, possibly close to the global maximum. Moreover, both methods require relatively large samples and results obtained with usually available length of macroeconomic time series should be interpreted with caution. Nonetheless, due to severity of potential problems with GMM estimation – especially with remarkably short samples for Poland and Slovakia – we apply the FIML approach in this study.

Finally, it should be borne in mind that FIML does not fully resolve a number of issues associated with estimation of DSGE models, such as non-identification of certain parameters or the treatment of latent variables. One might argue that Bayesian methods could be preferable from this point of view by – respectively – allowing to introduce informative priors and an alternative way of accounting for the unobservable components in the likelihood function. We leave this interesting extension for future research.

where $\mathbf{\Phi}$ has nonexplosive eigenvalues and $\boldsymbol{\varepsilon} \sim N(0, \mathbf{D})$.

Combine equations (77) and (78) into a single matrix equation and denote $\mathbf{T}$ and $\mathbf{R}$ for unknown matrices, being functions of model parameters (notation follows Durbin and Koopman, 2001):

$$\begin{bmatrix} \mathbf{x}_{1,t} \\ \mathbf{f}_t \end{bmatrix} = \underbrace{\begin{bmatrix} \mathbf{I} & -\mathbf{N} \\ \mathbf{0} & \mathbf{I} \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{M} & \mathbf{0} \\ \mathbf{0} & \mathbf{\Phi} \end{bmatrix}}_{\mathbf{T}} \begin{bmatrix} \mathbf{x}_{1,t-1} \\ \mathbf{f}_{t-1} \end{bmatrix} + \underbrace{\begin{bmatrix} \mathbf{I} & -\mathbf{N} \\ \mathbf{0} & \mathbf{I} \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{0} \\ \mathbf{I} \end{bmatrix}}_{\mathbf{R}} \boldsymbol{\varepsilon}_t \qquad (79)$$

This is the state equation in the Kalman filter, accompanied by the following measurement equation:

$$\mathbf{y}_t = \mathbf{Z}\mathbf{x}_{1,t} + \mathbf{d} + \boldsymbol{\epsilon}_t \qquad (80)$$

with $\boldsymbol{\epsilon}_t \sim N(\mathbf{0}, \mathbf{H})$ is a vector of measurement errors, $\mathbf{d}$ – vector of constants, and $\mathbf{Z}$ is a known matrix linking the measurement vector ($\mathbf{y}$) with its state counterparts ($\mathbf{x}_{1,t}$). The matrix of observable variables contains the following series (compare table 2): y^H, y^N, y^F, y^{N*}, c, c^*, π^H, π^N, π^F, π^{N*}, i, Δw, Δw^*.

The Kalman filter consists of the following equations:

$$\mathbf{a}_{t|t-1} = \mathbf{T}\mathbf{a}_{t-1} \qquad (81)$$

$$\mathbf{P}_{t|t-1} = \mathbf{T}\mathbf{P}_{t-1}\mathbf{T}^T + \mathbf{R}\mathbf{D}\mathbf{R}^T \qquad (82)$$

$$\mathbf{y}_{t|t-1} = \mathbf{Z}\mathbf{a}_{t|t-1} + \mathbf{d} \qquad (83)$$

$$\boldsymbol{v}_t = \mathbf{y}_t - \mathbf{y}_{t|t-1} \qquad (84)$$

$$\mathbf{F}_t = \mathbf{Z}\mathbf{P}_{t|t-1}\mathbf{Z}^T + \mathbf{H} \qquad (85)$$

Following Hamilton (1994, p. 248), using the above definitions and assumptions about normal distributions of $\boldsymbol{\epsilon}_t$ and $\boldsymbol{\varepsilon}_t$, we derive the following conditional log-likelihood function:

$$\mathcal{L}\left[\mathbf{y}, \mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{\Phi}, \mathbf{D}, \mathbf{H}, \mathbf{d}\right] = -\frac{Tn}{2}ln(2\pi) - \frac{1}{2}ln|\mathbf{D}| + \frac{T}{2}\sum_{t=1}^{T}ln|\mathbf{F}_t| - \frac{1}{2}\sum_{t=1}^{T}\boldsymbol{v}_t^T\mathbf{F}_t^{-1}\boldsymbol{v}_t \quad (86)$$

In order to estimate the parameters of model (76), function (86) was maximized with respect to unknown parameters in matrices $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$, $\mathbf{\Phi}$ and $\mathbf{D}$ as well as the parameters of measurement equation: $\mathbf{d}$ and $\mathbf{H}$. Standard errors of the estimation are based on the variance-covariance matrix proposed by Calzolari and Panattoni (1988):

$$D\left(\hat{\boldsymbol{\theta}}\right) = \left(-\frac{\partial^2 \mathcal{L}}{\partial\left[\boldsymbol{\theta}\right]^2}\right)^{-1} \tag{87}$$

where $\hat{\boldsymbol{\theta}}$ denotes the vector of all the unknown parameters.

The vector of starting values for the iterative procedure has been calibrated in accordance with the estimation results for individual equations obtained via generalized method of moments (see Table 1). Due to well-known issues with identification, orthogonality, arbitrary selection and low power of instrumental variables (Mavroeidis, 2005; Fuhrer and Rudebusch, 2004; Nason and Smith, 2008), the reported estimates should be interpreted with caution. Indeed, most of the estimates were imprecise and some of them have an economically unacceptable sign. Also, taking into account the sensitivity of the method to nonlinearities and limited numerical stability, only reduced-form estimates were calculated. Should the starting values for structural parameters exceed an economically acceptable range (and hence violate the Blanchard-Kahn conditions necessary for FIML estimation), a number close to the bound was selected.

The results in Table 1 confirm the existence of significant backward-looking components, at least in the IS curve and the Phillips curves for the tradable sector. The terms of trade parameter in the dynamic IS equation estimated for Slovakia clearly exceeds the analogous estimate for Poland, confirming a higher sensitivity of the Slovak economy to the exchange rate developments. In line with a large battery of empirical literature on IS and Phillips curves (see e.g. Galí and Gertler, 1999; Goodhart and Hofmann, 2005; Rumler, 2007; Brissimis and Skotida, 2008), GMM provided significant and correctly signed estimates neither for the interest rate parameter in the IS equation nor for the driving variable's parameter in the Phillips curves (the only exceptions being the Phillips curve for nontradables and the wage equation in Poland). These results should be seen as unsatisfactory, regardless of the non-rejection of null hypothesis that the orthogonality conditions hold for all the estimated equations.

The entire model was then estimated with FIML in two region pairs:

- euro area and Poland;

- euro area and Slovakia.

In order to fit it to the data over the period 1997q1-2009q4, when Poland and (most of the time) Slovakia were not the euro area members, model structure needed some adjustments:

- Every region leads an autonomous monetary policy. The smaller region (Poland or Slovakia) has therefore its own Taylor rule.

- As a consequence, the driving variable in country-specific Euler equations is the country-specific nominal interest rate.

- There are nominal exchange rate fluctuations between the two regions. Expected increments of the nominal exchange rate are determined by the uncovered interest parity equation, resulting from the risk sharing condition.

- Therefore, the logarithmic increment in terms of trade is now additionally composed of a difference in nominal exchange rate, Δe_t (on top of tradable inflation differential).

The model was fitted using data on volumes and deflators of real value added, volumes of consumption, nominal interest rates, nominal exchange rate and wage indices (see Table 2).

The data needed some additional adjustment due to disinflationary effects in Poland and Slovakia over the sample period that make the series nonstationary.

Table 1: GMM results

Equation	Parameter (*p-value*)	PL		SK	
IS	Γ_h	**0.362**	*(0.000)*	**0.379**	*(0.021)*
	Γ_σ	0.041	*(0.623)*	0.336	*(0.460)*
	Γ_s	0.040	*(0.619)*	**1.283**	*(0.084)*
	Γ_x	**-0.552**	*(0.003)*	-0.628	*(0.121)*
	Γ_x^*	1.428	*(0.109)*	-1.106	*(0.748)*
	J	*6.525*	*(0.769)*	*8.115*	*(0.618)*
Phillips (T)	Γ_ω	**0.196**	*(0.006)*	**0.265**	*(0.018)*
	Γ_θ	-0.037	*(0.207)*	0.154	*(0.265)*
	J	*10.95*	*(0.361)*	*8.063*	*(0.623)*
Phillips (NT)	Γ_ω	0.046	*(0.827)*	-0.204	*(0.169)*
	Γ_θ	**0.090**	*(0.006)*	0.328	*(0.203)*
	J	*5.631*	*(0.583)*	*7.199*	*(0.408)*
wage	Γ_ω	-0.236	*(0.724)*	0.473	*(0.460)*
	Γ_θ	**1.498**	*(0.001)*	1.333	*(0.221)*
	Γ_ϕ	0.344	*(0.344)*	0.203	*(0.278)*
	J	*2.808*	*(0.590)*	*2.105*	*(0.717)*

The dynamic IS equation was obtained by substituting the equilibrium condition (62) into the Euler equation (31) to obtain:

$$\hat{y}_t^H = \Gamma_h y_{t-1}^H + (1 - \Gamma_h) y_{t+1}^H - \Gamma_i (i_t - E_t \pi_{t+1} - \rho) + (1 - \tilde{w}) \left(c_t^* - \frac{h}{1+h} c_{t-1}^* - \frac{1}{1+h} E_t c_{t+1}^* \right) +$$
$$-\Gamma_s \left(s_t - \frac{h}{1+h} s_{t-1} - \frac{1}{1+h} E_t s_{t+1} \right) - \Gamma_x \left(x_t - \frac{h}{1+h} x_{t-1} - \frac{1}{1+h} E_t x_{t+1} \right) - \Gamma_x^* \left(x_t^* - \frac{h}{1+h} x_{t-1}^* - \frac{1}{1+h} E_t x_{t+1}^* \right)$$

The Phillips curve in sector s was estimated as $\hat{\pi}_t^s = \Gamma_s^\omega \pi_{t-1}^H + \frac{\theta^s}{\theta^s + \omega^s} \pi_{t+1}^s + \Gamma_s^\theta mc_t^s$, whereby real unit labour costs detrended with HP-filter were used as a proxy variable for the real marginal cost. The dynamic wage equation was estimated

as $\hat{\pi}_t^W = E_t \pi_{t+1}^W - \omega^W (\pi_t^C - \pi_{t-1}^C) + \Gamma_W^\theta \left[\frac{\hat{\sigma}}{1-\hat{h}} \left(c_t - \hat{h} c_{t-1} \right) - (w_t - p_t) \right] + \Gamma_W^\phi y_t$ using the estimation results from the IS equation. The instrument sets included:

- IS curve: output in T (-1 to -3), *ex post* real interest rate (-1 to -2), terms of trade increments (-1 to -2), home and foreign consumption (-1 to -2).

- Phillips curve in T: lags of the T inflation (-1 to -4), unit labour costs (-1 to -4), consumption (-1 to -2), foreign consumption (-1 to -2);

- Phillips curve in NT: lags of the NT inflation (-1 to -3), unit labour costs (-1 to -4), consumption (-1 to -2);

- wage equation: lags of wage growth rate and log-differenced consumption deflator (-1 to -4).

Source: author.

- In the case of Poland, the interest rate series were detrended using the National Bank of Poland's data on inflation target. This data is not continuous in quarterly terms, and it was smoothed using the Hodrick-Prescott filter.

Table 2: Data sample (1997q1-2009q4)

Variable	Description	Data and transformations
y^H	output in the tradable sector, log-deviation from the steady state	value added in sectors A-E (NACE) in Mio EUR, deflated for individual sectors and added for AB and CDE; Eurostat; dlog
y^N	output in the nontradable sector, log-deviation from the steady state	value added in sectors F-P (NACE) in Mio EUR, deflated for individual sectors and added for F, GHI, JK and LMNOP; Eurostat; dlog
c	consumption, log-deviation from the steady state	consumption expenditure, National Accounts data in constant prices from the previous year, reference year 2000, dlog
π^H	quarterly price dynamics in the tradable sector	value added deflator in sectors A-E (NACE), geometric average for sectors AB and CDE with shares in A-E aggregate as weights (sample average); dlog
π^N	quarterly price dynamics in the nontradable sector	value added deflator in sectors F-P (NACE), geometric average for sectors F, GHI, JK and LMNOP with shares in F-P aggregate as weights (sample average); dlog
π^W	quarterly wage dynamics	wage index in national economy, auxiliary data for national accounts, Eurostat, dlog
i	one-period nominal interest rate	3M money market interest rate, Eurostat
Δe	nominal exchange rate, log-deviation from the steady state, difference	average quarterly PLN/EUR and SKK/EUR exchange rates, Eurostat, dlog

Source: author.

- No such data is available for Slovakia, as there was no explicit inflation targeting strategy until early 2005 (NBS, 2004). Instead, the main monetary policy objective was defined as a low inflation rate that would allow the fulfilment of the Maastricht criterion. This is why we interpret the Slovak disinflation as an element of euro adoption strategy. In consequence, we disentangle the nominal interest rate on the Slovak money market into an element due to convergence to the euro area and an the residual component of regular monetary policy and policy shocks. We do this by estimating the following equation of Slovak interest rate convergence to the euro area:

$$\triangle i_t = \rho^{\hat{S}K} (i_t - i_t^*) + \hat{\triangle} i_t$$

We obtain an estimate of $\rho^{\hat{S}K} = -0.031$ (with a standard error 0.02). Using the values of $\hat{\triangle} i_t$ and the terminal value of $i_{2009Q1} = i_{2009Q1}^*$, we construct the "net of convergence" component of the nominal interest rate.

Tablica 3: Stationarity tests

ADF								
country	y^H	y^N	c	π^H	π^N	π^W	i	Δe
PL	0.005***	0.000***	0.000***	0.003***	0.000***	0.000***	0.153	0.000***
SK	0.000***	0.000***	0.000***	0.000***	0.000***	0.000***	0.091*	0.000***
EA	0.002***	0.015**	0.000***	0.000***	0.000***	0.000***	0.141	–
KPSS								
country	y^H	y^N	c	π^H	π^N	π^W	i	Δe
PL	0.120	0.161	0.121	0.191	0.449*	0.449*	0.660**	0.056
SK	0.245	0.074	0.116	0.592**	0.360*	0.476**	0.743***	0.535**
EA	0.332	0.326	0.538**	0.226	0.206	0.177	0.237	–
IPS								
panel	y^H	y^N	c	π^H	π^N	π^W	i	Δe
	0,000***	0,000***	0,000***	0,000***	0,000***	0,000***	0.029*	0.000***

Critical values for KPSS: 0,739 (significance level 0.01), 0,463 (sig. lev. 0.05) and 0,347 (sig. lev. 0.10).
Source: own calculations.

Both ADF and KPSS tests confirm that the series used to fit the model are stationary in most of the cases, for both Poland and Slovakia.

Finally, to ensure a more efficient estimation with relatively short time series, we calibrate 4 model parameters for each region. This is inevitable, given weakness of information contained in the data, presence of unidentified parameters, as well as selected estimation procedure. We set α (α^*) as a corresponding measure of economies' openness, i.e. the share of imports (exports) in a country's (the euro area) GDP. κ was calibrated to reflect the share of NACE branches F-P in the value added of every economy in question, in accordance with the construction of proxy variables. The calibration of β was implied by *ex post* real interest rates, calculated using the consumption deflator and averaged over the sample period. Also, the unidentified parameter ϕ^W is calibrated at 3.0 in line with Smets and Wouters (2003) and Kolasa (2009).

Table 4 contains the estimation results. The dynamic properties of the adjustment process hinge mainly on the strength of consumption habits (h), parameter of disutility from labour (ϕ), intertemporal elasticity of substitution ($1/\sigma$) and Phillips curve parametrisation (price indexation ω and Calvo probabilites θ). Also, the speed of adjustment is affected by other parameters that are linked to dynamic parameters via contemporaneous dependencies and rational expectations.

The parameter of habit persistence for the euro area, estimated at 0.66, remains slightly higher than the values reported in the previous literature (see e.g. Smets and Wouters, 2003). While Slovakia exhibits somewhat lower h (0.57 with a relatively small standard error of estimation), in Poland this parameter seems to exceed the euro area average at 0.84. On the one hand, this could result in a more sluggish adjustment of the real categories after asymmetric shocks. On the other hand, higher values of h immunize the economic activity to the shifts of nominal and real interest rates, as well as to demand shocks.

Against our expectations, the parameters of intrinsic inflation persistence in the tradable sector in Poland and Slovakia remain relatively high. The estimate for Poland amounts to approximately 0.62 and clearly exceeds the one for the Slovak economy (0.38), which suggests potentially more sluggish external competitive adjustments. However, both economies seem to remain below the euro area average at 0.742. Remarkably, in the nontradable sector both Slovakia and Poland exhibit less inflation persistence (0.01and 0.12 respectively). On the other hand, the estimate for the euro area remains in line with previous literature (0.82) and higher than the one for tradable sector. It needs perhaps to be stressed that both Poland and Slovakia were transition economies during at least part of the sample period. Consequently, a significant part of the price dynamics evolved on a non-market basis and could produce shifts in the data that result in a downward bias of the autoregressive coefficients. Also, the parameter of intrinsic inflation persistence is argued to be regime-dependent and not immune to Lucas (1976) critique. As shown by Benati (2008), this is particularly the case when it comes to the process of monetary integration in Europe. In terms of wage indexation parameter, Poland ranks best at 0.004, followed by Slovakia (0.198 with a small standard error) and much ahead of the euro area (0.691).

The estimated Calvo probabilities point to some meaningful differences between Poland and Slovakia in terms of product market rigidities. In both sectors, the Calvo parameter in Poland exceeds the one for Slovakia (0.58 vs 0.38 in tradables, 0.72 vs 0.48 in nontradables). This suggests that the real exchange rate would adjust more flexibly to possible cyclical divergences in Slovakia. Remarkably, while the estimated Calvo probabilities for Poland do not significantly differ from their euro area counterparts (0.51 for the tradables, 0.69 for the nontradables), Slovakia outperforms the rest of the monetary union when it comes to market flexibility. When it comes to the labour market rigidities, the estimation results exhibit a substantially higher labour market flexibility in Poland ($\theta^W = 0.28$), compared both with the euro area (0.62) and especially Slovakia (0.89). Although these parameters might be particularly sensitive to structural changes in transition economies due to product and labour market reforms, all the Calvo estimates in discussion are characterized with relatively small standard errors.

The estimated inverse of intertemporal elasticity of substitution implies that Poland should be less sensitive to interest rate fluctuations than Slovakia, with $\sigma = 3.90$ and 2.59 respectively. This result is rather surprising, given the difference in the structures and openness of both economies. It also implies that Poland would be a little less sensitive to the procyclicality of real interest rates inside the monetary union than this difference would allow to predict. Although a short sample and well-known issues of oversimplification

Table 4: FIML estimates of model parameters

Parameters \ Region	PL		SK		EA	
calibrated values						
openness of the home economy (α)	0.175		0.496		–	
openness of the foreign economy vs... (α^*)	0.023		0.009		–	
share of NT sector (κ)	0.702		0.615		0.76	
households' impatience (β)	0.9851		0.9909		0.9959	
estimated values (standard errors)						
elasticity of substitution H/F (η)	0.519	(0.154)	0.401	(0.052)	–	–
elasticity of substitution H/F – EA vs... (η^*)	1.733	(0.140)	1.442	(0.103)	–	–
elasticity of substitution T/NT (δ)	1.098	(0.242)	1.651	(0.053)	0.128	(0.091)
inverse elasticity of intertemporal substitution (σ)	3.901	(0.183)	2.593	(0.226)	0.996	(0.148)
inverse Frisch elasticity of labour supply (ϕ)	2.589	(0.165)	2.070	(0.103)	2.780	(0.056)
consumption habits (h)	0.841	(0.052)	0.574	(0.072)	0.661	(0.057)
price indexation in T (ω^T)	0.617	(0.082)	0.382	(0.064)	0.742	(0.083)
price indexation in NT (ω^N)	0.116	(0.007)	0.010	(0.211)	0.824	(0.067)
wage indexation (ω^W)	0.004	(0.143)	0.198	(0.005)	0.691	(0.099)
Calvo probability in T (θ^T)	0.575	(0.032)	0.382	(0.074)	0.507	(0.067)
Calvo probability in NT (θ^N)	0.715	(0.026)	0.484	(0.049)	0.688	(0.085)
Calvo probability for wages (θ^W)	0.283	(0.028)	0.892	(0.038)	0.616	(0.038)
CB response to inflation (γ_π)	1.597	(0.269)	2.212	(0.017)	2.622	(0.020)
CB response to output (γ_y)	1.005	(0.122)	3.624	(0.061)	3.412	(0.022)
CB smoothing (γ_ρ)	0.618	(0.050)	0.963	(0.008)	0.736	(0.044)
serial correlation of demand shock (ρ_c)	0.005	(0.239)	-0.004	(0.181)	0.701	(0.063)
serial correlation of T supply shock (ρ_T)	0.084	(0.024)	-0.225	(0.174)	-0.213	(0.116)
serial correlation of NT supply shock (ρ_{NT})	-0.172	(0.121)	-0.380	(0.148)	-0.198	(0.188)
serial correlation of labour supply shock (ρ_w)	0.009	(0.325)	-0.203	(0.112)	0.173	(0.193)
serial correlation of monetary policy shock (ρ_i)	0.866	(0.081)	-0.034	(0.272)	0.946	(0.012)
serial correlation of UIP shock (ρ_i)	0.877	(0.139)	0.981	(0.032)	–	–
variance of demand shock (σ_c^2)	49.008	(0.278)	24.281	(0.274)	0.328	(0.079)
variance of T supply shock ($\sigma_{\pi_T}^2$)	37.337	(0.122)	16.662	(0.116)	10.286	(0.189)
variance of NT supply shock ($\sigma_{\pi_{NT}}^2$)	1.374	(0.752)	25.672	(0.516)	8.797	(0.186)
variance of labour supply shock ($\sigma_{\pi_{NT}}^2$)	56.521	(0.166)	63.849	(0.054)	13.069	(0.109)
variance of monetary policy shock (σ_i^2)	0.067	(0.025)	0.026	(0.021)	0.017	(0.006)
variance of UIP shock (σ_e^2)	0.004	(0.009)	0.001	(0.0002)	–	–
correlation of demand shocks against EA	0.640	(0.271)	0.805	(0.194)	–	–
correlation of T supply shocks against EA	0.772	(0.225)	0.854	(0.150)	–	–
correlation of NT supply shocks against EA	0.105	(0.186)	0.173	(0.173)	–	–
correlation of labour supply shocks against EA	-0.647	(0.122)	0.084	(0.199)	–	–
correlation of monetary policy shocks against EA	0.511	(0.163)	0.079	(0.233)		

Source: author.

and dynamic mis-specification of the standard New Keynesian setup could be seen as an explanation, this difference should not be seen as particularly meaningful for the adjustment process. It should be noted that the openness and size of the economy are explicitly accounted for in the calibration. The other parameter of the utility function – ϕ as a measure of disutility from labour – was estimated at comparable levels for all 3 regions. A higher value for Poland may have been produced due to the fact of higher labour market flexibility and hence stronger transmission of the demand conditions into real marginal costs and hence real wages.

The estimated elasticities of substitution between domestic and foreign tradables have turned out to be somewhat higher for Poland ($\eta = 0.52$ and $\eta^* = 1.73$) than for Slovakia ($\eta =$

0.40 and $\eta^* = 1.44$). This can be explained in two ways. Firstly, the sensitivity of home tradable production to the shifts in terms of trade is not only a function of the elasticities in question, but also the calibrated country weight and degree of openness. If the estimated elasticities were equal, Slovak production would be characterized by a high (reduced-form) parameter of sensitivity to the terms of trade compared to Poland due to these characteristics and estimated low values can be regarded as offsetting this. As the sample information on the international elasticities of substitution is generally considered to be weak, authors normally assume the aggregators of tradable and nontradable consumption that implicitly calibrate these values at 1 (see e.g. Kolasa, 2009). However, this parameter plays a critical role in the functioning of competitiveness channel and hence its calibration in this study was intentionally avoided. Secondly, Slovakia experienced a noticeable real exchange rate appreciation during the sample period, which – however – did not undermine the country's competitiveness. This fact is directly reflected in the estimation results.

Finally, both Slovakia and Poland outperform the euro area in terms of elasticity of substitution between the tradable and nontradable goods. As a result, Slovak nontradable output is the most sensitive to changes in internal terms of trade with δ at 1.65 (1.10 in Poland. This should additionally support the cross-sectoral adjustments after asymmetric shocks in the Slovak economy.

The estimated, micro-founded model does not correspond to the MCI-ratio analyses based on reduced-form IS curves, incorporating measures of both real interest rate and real exchange rate on the right-hand side and a measure of output gap on the left-hand side of the equation (see e.g. Kot, 2003; Torój, 2008). However, we could attempt to perform a similar comparison by tracing the following reduced-form parameter of the model:

- the interest rate parameter in the Euler equation (31), $\frac{1-h}{\sigma}$;

- the terms of trade parameter in the equilibrium condition on the market of domestic tradable goods (62), $\tilde{w}\alpha\eta + (1 - \tilde{w})(1 - \alpha^*)\eta^*$.

These parameters (along with their initial GMM counterparts and confidence intervals[3]) have been calculated in Table 5. It follows that the sensitivity of the Slovak economic activity both to the real interest rate and the terms of trade clearly exceeds the analogous sensitivity in Poland. Albeit this conclusion holds for both the GMM and FIML results, both parameter sets imply a different magnitude of the effect. However, the range resulting from a more precise FIML estimation can be regarded as more plausible. A higher value of the terms-of-trade parameter suggest that the mid-term dominance of the competitiveness channel over the real interest rate effect could emerge more effectively in Slovakia than in Poland (although the confidence intervals are partly overlapping in this case). Also, a ratio of both parameters in each country is provided (along with a respective confidence interval), although the DSGE model can by no means be compared with simple reduced form models and no direct comparison to the MCI ratio applies.

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[3]Confidence intervals were simulated from normal distribution using the point estimates of the structural parameters as means and appropriate portions of the coefficients' variance-covariance matrix.

Table 5: Estimated terms of trade and real interest rate parameters for Poland and Slovakia

	GMM		FIML (90% C.I.) (95% C.I.)	
	PL	SK	PL	SK
interest rate parameter in Euler curve	0.041	0.336	0.041 (0.017; 0.064) (0.012; 0.068)	0.164 (0.080; 0.064) (0.058; 0.233)
terms of trade parameter in equilibrium condition	0.039	1.283	0.823 (0.497; 1.150) (0.435; 1.212)	1.011 (0.747; 1.278) (0.697; 1.332)
ratio	**1.051**	**0.262**	**0.050** (0.022; 0.082) (0.016; 0.092)	**0.163** (0.087; 0.225) (0.067; 0.240)

Source: author.

The values of stochastic parameters confirm that there is some serial correlation in shocks, although its coefficients were estimated rather imprecisely. According to the results, Polish and Slovak economy were far more volatile than the euro area (the only exception being the nontradable supply shocks in Poland with a lower variance). The demand and tradable supply shocks exhibited a relatively high degree of correlation against EA both in Poland and – especially – in Slovakia. However, Poland experienced highly asymmetric labour supply shocks over the sample period (as compared to Slovakia). It should be noted as well that the estimated correlation coefficients must be interpreted with caution due to high standard errors of estimation.

4

5   Adjustment dynamics under EMU: IRF analysis

The estimated model parameters (Table 4) are used to simulate impulse-response functions to asymmetric shocks of various types in the home (Polish or Slovak) economy. In order to keep the comparative perspective and focus on the mid-term adjustment process rather than short-term impact size, the magnitude of the shocks for both countries are set to 1. Also, the initial impact of the shocks could is differentiated to account for the differences in the parameters included in the **C** matrix, translating into matrix **N** in the model solution.[4] Dashed lines represent approximate 90-percent confidence intervals, obtained as 5th and 95th percentiles out of 5000 runs of IRF with different parameter sets drawn from a normal distribution with mean at FIML point estimates and estimated variance-covariance matrix.[5]

In every case, the response of common monetary policy to country-specific domestic shocks is negligible. As the nominal interest rate remains virtually unchanged, we concentrate on alternative adjustment channels.

Impulse-response functions after an asymmetric demand shock (Figure 3) confirm the general insight from the estimation results: the competitiveness channel in Slovakia seems to be more effective, as the response of most categories in consideration is flatter and they return more quickly to the steady state than in Poland. In particular, Poland needs a deeper response in terms of trade in order to restore the equilibrium. There is also more volatility in internal terms of trade, real wages, as well as tradable and nontradable inflation rates. Notably, the adjustment of output both in T and NT is slower in the Polish economy.

Note the most remarkable difference in impulse-response functions in tradable output of both economies. In Poland, a positive demand shock induces a sharper increase on impact and the deviation of output from the steady state remains positive over the first few quarters. During this time, the terms of trade appreciate considerably, but the boom is still supported by strong consumption habits and the real interest rate effect. On the contrary, Slovak output almost returns to equilibrium within a year after the shock. The response of the competitiveness channel is immediate and deep, confirming that in a highly open and exchange rate sensitive economy it can act very effectively as a replacement for the autonomous monetary policy. The Slovak tradable output stabilizes immediately after the changes in consumption and terms of trade die out, without any remarkable pattern of overshooting.

A similar picture emerges when we analyze the cost-push shocks in the tradable sector (Figure 4). In both countries in question, this enables producers to lower their tradable output prices. Higher Calvo probabilities and price indexation parameters in Poland lead to more volatile paths of tradable inflation and much deeper external terms of trade depreciation. However, agents expect prices to rise after the shock subsides which reduces the real interest

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[4]In other words, the demand shock has the same impact influence on consumption, the supply shocks – on real marginal costs in both sectors, and the labour supply shocks – on nominal wage dynamics.

[5]31 draws for Poland and 32 for Slovakia were rejected (0.6% of draws) due to violation of the Blanchard-Kahn conditions for the obtained parametrization.

Figure 3: Impulse-response functions for PL/SK after an asymmetric demand shock
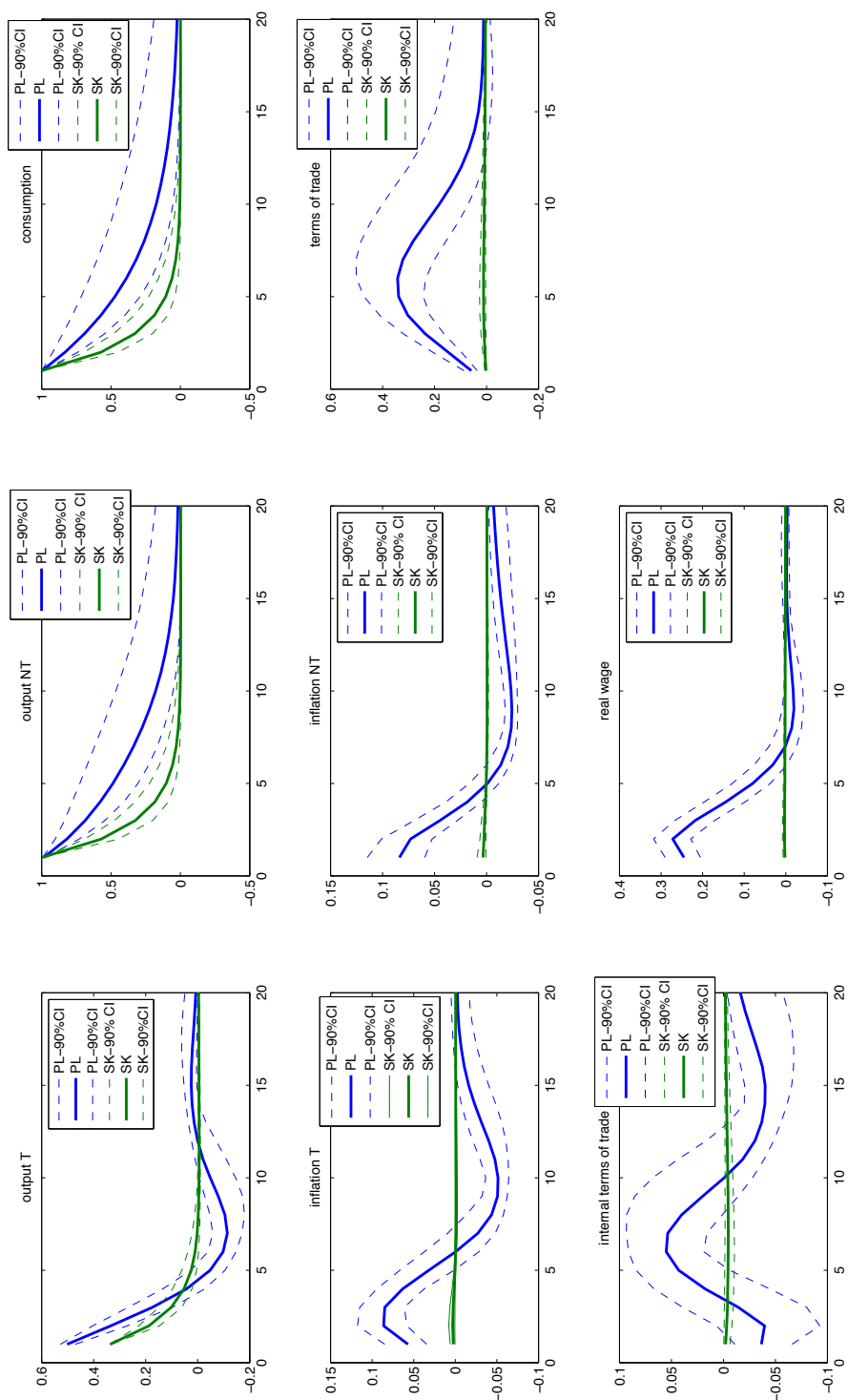
Source: author.

rates. This – along with rising real wages – fuels a boom in consumption. The strongly hump-shaped response of the Polish consumption reflects a strong real interest rate effect. It turns out that differences in size and openness of the economies dominate the insignificantly higher estimate of σ for Poland.

Higher consumption demand boosts output in both sectors. In the tradable sector, this adds to the boom resulting from the terms of trade depreciation. In the nontradable sector, the substitution effect associated with higher relative price of nontradables dominates. As a result, the adjustment is longer and the amplitude of consumption, nontradable output and nontradable inflation – higher. In spite of higher labour market flexibility in Poland, the real wages adjust more slowly due to a higher inertia in price dynamics in the nontradables. The slowdown in the nontradable sector has a higher amplitude, lasts longer and the recovery exhibits a pattern of overshooting. In a symmetric case of adverse supply shock in the tradables, this would suggest a prolonged boom in the nontradables, followed by a bust.
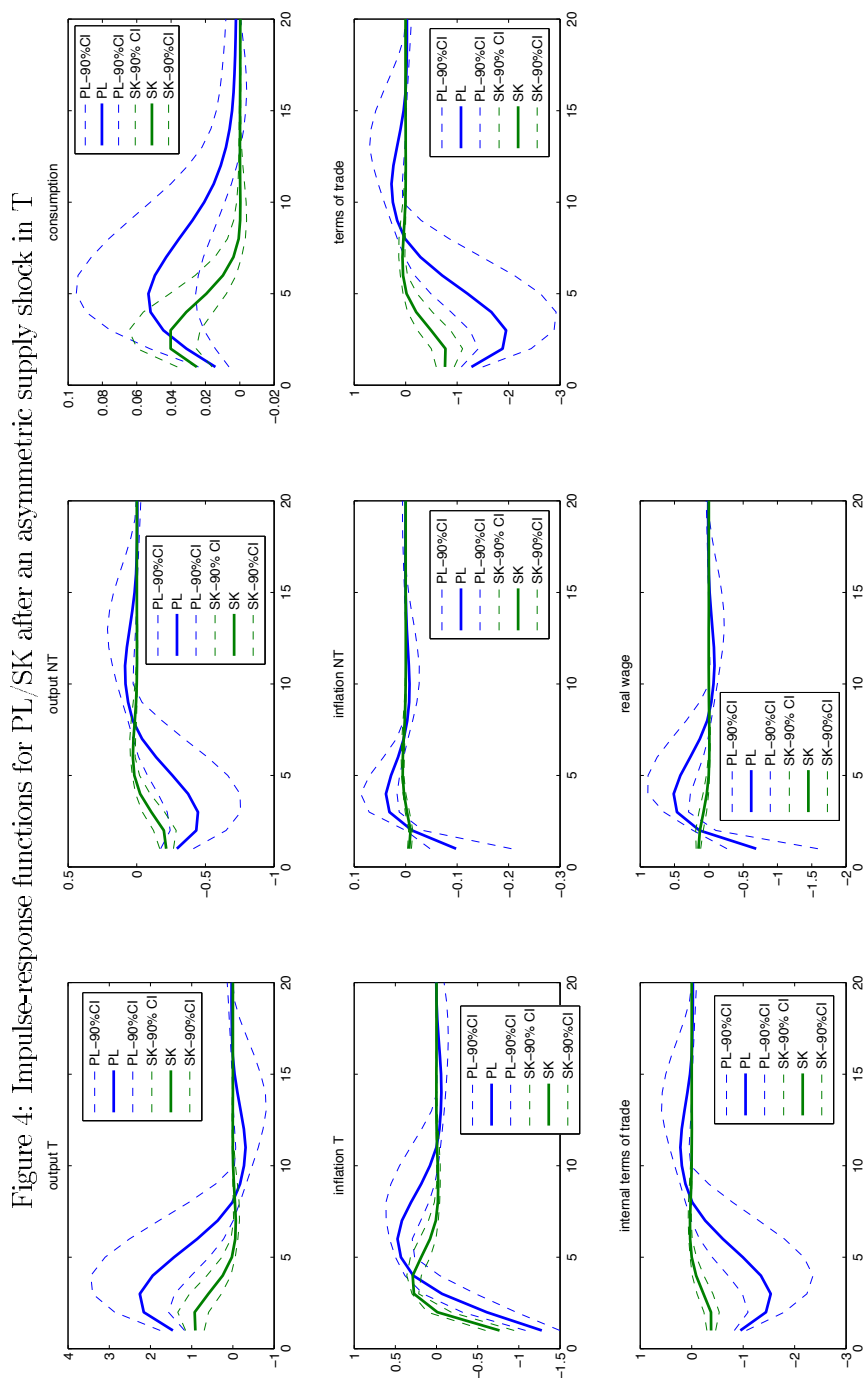
Figure 5 completes the picture with dynamic responses to supply shocks in the nontradable sector. They are directly addressed neither by the common monetary policy (focused on the foreign big economy) nor the competitiveness channel (focused on the tradable sector), so they might have particularly persistent effects. The response to this sort of shocks also differs between Poland and Slovakia, although to a lesser extent than after supply shocks to tradables.

Directly after the shock, both economies face a drop in nontradable inflation. Rising real wages increase the consumption. As expected, it responds weaker on impact, but more sluggishly in Poland. At the beginning, the substitution effect lowers the tradable output in both economies. However, the shock quickly spills over into the tradable sector via labour market. Its higher flexibility in Poland induces a fall in real wages and – in consequence – tradable inflation as well, followed by marked terms of trade depreciation. This results in a more volatile response in Polish tradable output and external terms of trade than in Slovakia.
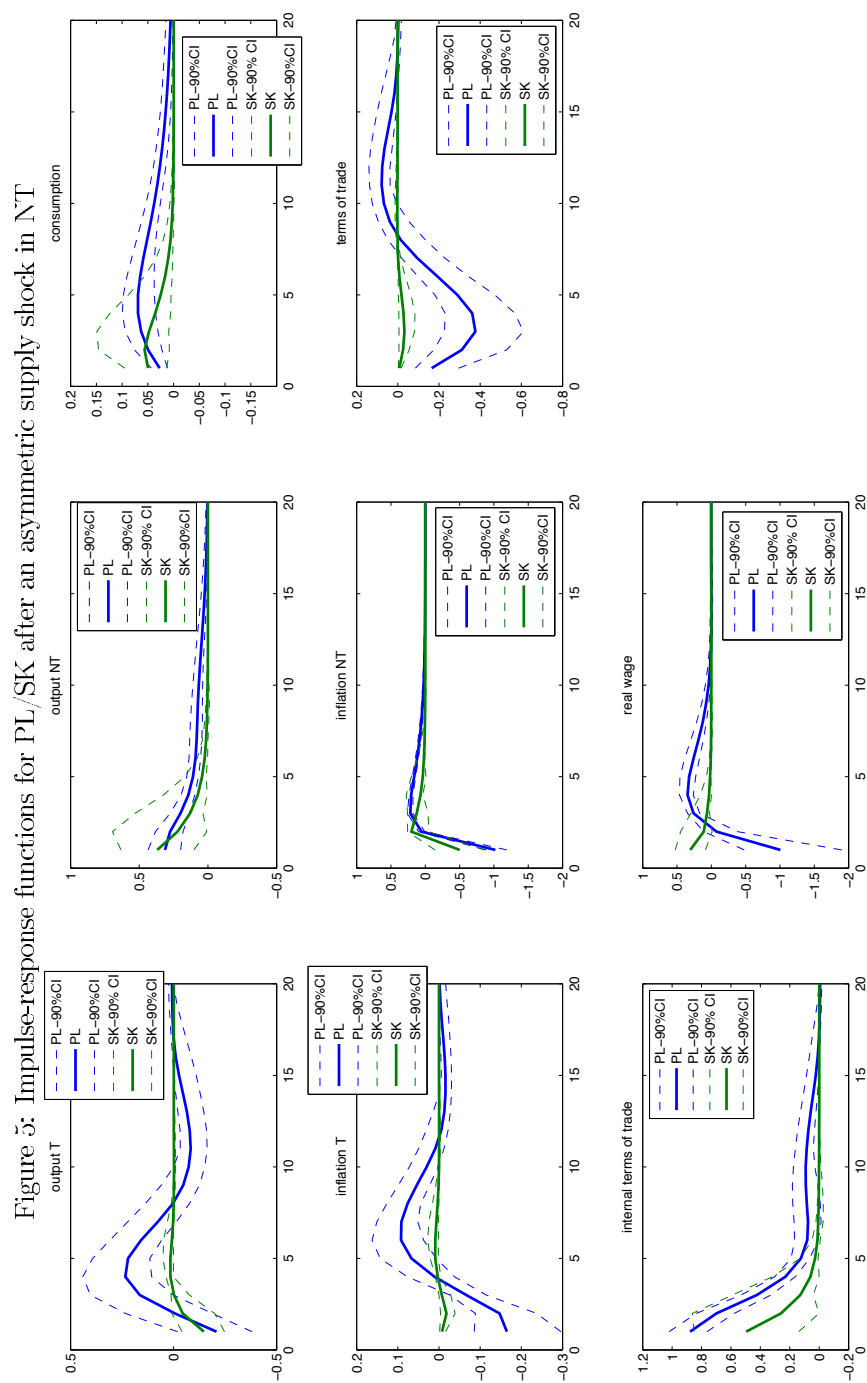
In the medium term, the prices of nontradable goods are gradually rising. Notably, more persistent inflation rate in the nontradables allow the agents in the Polish economy to form markedly positive inflation expectations, driving further the boom in consumption (coupled with stronger consumption habits). The terms of trade peak around 10 quarters years after the shock. At this moment, both consumption and output are already falling and the boom turns into a mild bust. Summing up, the Polish response to an asymmetric supply shock in the nontradable sector suffers from a lag, more marked procyclicality of the real interest rates and leads to somewhat more volatility in the macroeconomic variables than the Slovak one.

Finally, Poland exhibits a quicker and less volatile response to an asymmetric adverse labour supply shock (Figure 6). When the impact effect on the nominal (and hence real) wages is comparable, the differences in the adjustment phase result mainly from a higher flexibility of the Polish labour market. The pressure in the Slovak labour market manifests itself

in strongly growing prices of both tradable and – even more – nontradable products. This, along with falling output, allows the agents to expect a recession and deflation in the economy. Low inflation expectations translate into rising real interest rates and lead to additional drop in consumption. The slowdown in consumption (and output in both sectors) is deeper in Slovakia than in Poland. Bottom line, the procyclical amplitude induced by the real interest rate effect can be higher with stronger labour market rigidity.
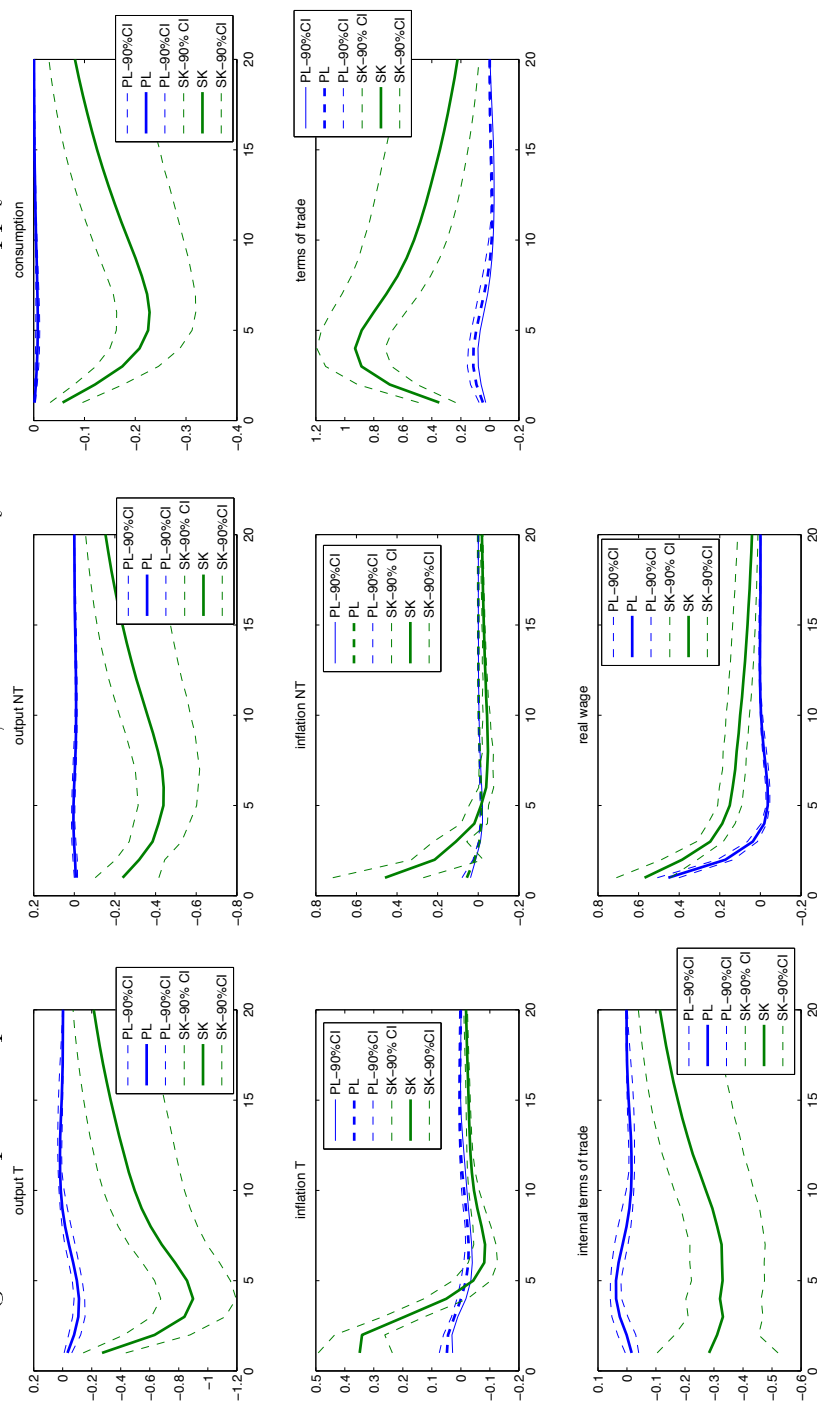

Figure 4: Impulse-response functions for PL/SK after an asymmetric supply shock in T

Source: author.

Figure 5: Impulse-response functions for PL/SK after an asymmetric supply shock in NT

Source: author.

Figure 6: Impulse-response functions for PL/SK after an asymmetric labour supply shock

Source: author.

6   MCI-ratio and adjustment to permanent interest rate shock

We use the DSGE model derived in Section 3 and estimated in Section 4 to address the issue of structural misalignment in monetary policy for both Central European economies in question. In particular, we are interested in the consequences of a permanent interest rate fall after the accession to the euro area. According to the estimates of Flaig and Wollmershäuser (2007) and Calmfors (2007), a number of lower-income economies have probably suffered from a structural stress in the initial years after the euro adoption.

Reconsider the region-specific Euler curve (32), containing i_t as an autonomous policy rate in the home economy. Assuming that the home economy is in its zero-inflation steady state, i_t equals ρ, i.e. the country-specific natural interest rate.

$$c_t = \frac{h}{1+h} c_{t-1} + \frac{1}{1+h} E_t c_{t+1} - \frac{1-h}{(1+h)\sigma} (i_t - E_t \pi_{t+1}) + \frac{1-h}{(1+h)\sigma} (\varepsilon_{d,t} - E_t \varepsilon_{d,t+1})$$

Let $\Delta i \equiv \rho - \rho^*$ denote an exogenous shift associated with the euro adoption and switch to the common monetary policy regime. This component can be modelled as a permanent shock:

$$c_t = \frac{h}{1+h} c_{t-1} + \frac{1}{1+h} E_t c_{t+1} - \frac{1-h}{(1+h)\sigma} (i_t - E_t \pi_{t+1}) + \frac{1-h}{(1+h)\sigma} [\varepsilon_{d,t} - E_t \varepsilon_{d,t+1} - \Delta i]$$

In the simulations, we set the fall in nominal interest rates to 1 p.p. in annual terms, which corresponds to $\Delta i = -0.25$ [p. p.] in our quarterly model. To keep the results comparable, we apply the same shock for the Polish and the Slovak economy. However, the magnitude of the results differs according to what natural interest rate disparity one might assume for each of these countries.

A critical point in this simulation is calibrating the persistence of this shock. It is highly implausible (and numerically more challenging) to assume that the shock would last forever. The catching-up process will probably tend to close the natural interest rate disparity, at least when we do not take into account other factors possibly involved. This implies the shock's serial correlation below unity. An exact value strongly depends on an arbitrary assumption of how long the process of technological convergence would last. In what follows, we assume the serial correlation in the demand shock $\rho_c = 0.962$, which corresponds to a scenario where 99% real convergence would materialize itself within 30 years. Sensitivity of the main result to this assumption has been tested in Table 6.

It needs perhaps to be stressed that this shock persistence affects the results of the simulations mainly via the channel of agents' expectations. As the adjustment to the new mid-term "steady state" is relatively quick (approximately 2-3 years), the effects of this shock dying out and reversal are not yet observable.

The results of the simulation are presented in Figure 7. In the upper panels, the response of main macroeconomic characteristics of the region are presented for Poland (left-hand) and Slovakia (right-hand). We can disentangle the realignment into several aspects:

1. **Permanent terms of trade appreciation.** When the economic activity is constantly boosted by the low nominal interest rate, the competitiveness channel must counteract this effect by reducing the external demand. This can only be achieved when the terms of trade appreciate. The size of this permanent appreciation is a function of many model parameters, but the estimated MCI-ratio plays here the most prominent role.

   An estimated appreciation of 5.1% in Poland and 3.4% in Slovakia in not surprising. On the one hand, the impact of the shock depends on the intertemporal elasticity of substitution, the main determinant of MCI-ratio's numerator. On the other hand, the required terms of trade appreciation that counterbalances this impulse is all the more pronounced, the less sensitive an economy is to external real appreciation.

   These numbers rest on the assumption that the expected period of closing the natural interest rate gap is 30 years. Taking into account an interval of 25 to 50 years, it might differ from 4.3% to 8.6% for Poland and from 2.7% to 6.3% for Slovakia.

2. **Opening (or deepening) a current account deficit.** The mid-term impact of the shock on consumption is positive, as opposed to the effect on output in the tradable sector. Coupled with a relatively weak growth of the nontradable output (see point 3), this leads to a persistent current account deficit. The access to the savings of the foreign big economy is necessary to keep the interest rate low. This effect is perfectly in line with the experience of catching-up economies within the euro area (such as Greece or Portugal; see e.g. Fagan and Gaspar, 2007). Importantly, our model does not contain any corrective mechanism for the current account or net foreign asset stock in the home economy. According to Blanchard and Giavazzi (2002), "up to a first order" this should not be seen as inherently problematic. Indeed, the questions about thresholds and mechanisms of such correction in a monetary union remain, to the best of our knowledge, without any consensus answer in the literature today. One possible implementation would be to introduce some form of the financial sector (with borrowing constraints) into the model.

3. **Vague impact on the nontradable sector.** Simulation results for both economies show that the permanent impulse would result in a weak depreciation of the internal terms of trade. This should lead to a moderate fall in demand for the nontradable goods, insufficient to prevent a growth of the nontradable output in line with rising consumption. In general, the outcome depends here on the elasticity of substitution between the tradable and nontradable goods in the consumption basket, as well as the relative size of the regions, sectors and the degree of openness.

4. **Probable drop in the tradable output.** There is a qualitative similarity between Poland and Slovakia in terms of the new tradable output level after the permanent

shock under consideration. While the Polish output of tradable goods fell quite strongly (almost 4%), the Slovak one fell by more than 2%. Like in the case of the nontradable sector, this outcome depends on how sensitive the economic activity is to the relative prices and domestic consumption demand, as well as to the size of terms of trade appreciation. In the case of Poland, the simulation shows a higher terms of trade appreciation and a lower growth in consumption after the economy adjusts to the permanent shock.

Figure 7: Permanent 1 p.p. shock in nominal interest rates
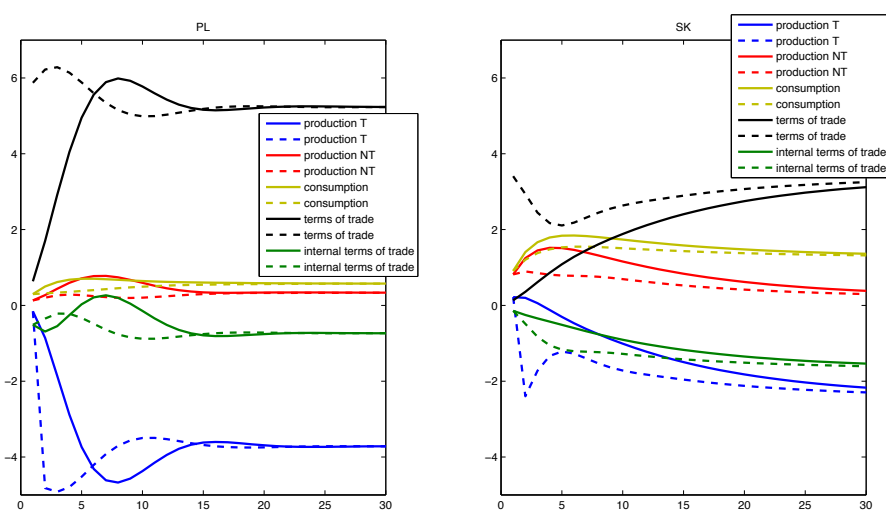
Source: author.

In Figure 7, an alternative adjustment scenario is considered (dotted lines). Instead of a fully market-based transition to the new mid-term equilibrium (solid lines), it is assumed that policymakers know how much the terms of trade will need to appreciate on a permanent basis. This shift is precisely applied in conjunction with the interest rate shock. It would be equivalent to setting the conversion rate of the national currency against the euro (or the central parity) at a stronger level than the pre-euro economic fundamentals (including the nominal interest rates that would prevail if euro adoption was not expected) would justify. The revaluations of the Slovak koruna in the ERM II might possibly be seen as a part of this process.

Table 6: Persistence of structural stress in monetary policy vs required terms of trade appreciation

time of natural rate disparity [years] expected by agents	25	**30**	35	40	45	50
corresponding ρ_c	0.955	**0.962**	0.968	0.972	0.975	0.977
permanent terms of trade appreciation − PL [%]	4.292	**5.130**	6.141	7.057	7.935	8.649
permanent terms of trade appreciation − SK [%]	2.720	**3.389**	4.212	4.969	5.703	6.304

Source: author.

For Slovakia, in the alternative scenario, all the variables on the adjustment path seem to be less volatile. It is then highly probable that a welfare analysis run with this model would indicate it as a preferable scenario. However, to confirm this in general, a much more sophisticated investigation would have to be repeated in a large-scale model with a more detailed sectoral breakdown, financial constraints and frictions, investment and multiple stages of production. The case seems to be even more difficult to assess for Poland, as consumption and tradable output exhibit some overshooting pattern under the alternative scenario. A detailed analysis of this issue would also need a more elaborate application of the permanent shock (i.e. a gradual fall in interest rates, as agents start to discount euro area entry in their expectations).

In general, both scenarios might have their advantages and disadvantages whose thorough consideration exceeds the depth of this analysis. The market-based scenario (dashed lines) would probably induce a prolongued period of higher inflation, which seems to be particularly dangerous for expectations in countries with a short history of low inflation. Here, on the contrary, rationality of expectations was assumed. This scenario might also be undesirable when the inflation persistence or price rigidities are high. On the other hand, the revaluation-based scenario (solid lines) could potentially hit the tradable sector severely, including advanced and innovative branches. In Slovakia (see Figure 7), a sharp drop of the tradable output was observed. This would put the producers of tradable goods under substantial time pressure. On the contrary, under gradual appreciation, they would have more time for the necessary restructuring processes.

6

7 Conclusion

In this paper, we attempted to compare Poland's and Slovakia's capacity to absorb macroeconomic asymmetric shocks in the euro area. We also considered a permanent fall in the nominal interest rates as a stimulus, shifting the economy to a new, "mid-term" steady state. For this purpose, we applied a New Keynesian DSGE model of a 2-region, 2-sector currency union, including various sources of cross-regional heterogeneity.

The estimates suggest that, when it comes to the competitiveness channel, Slovakia is better equipped to adjust after asymmetric demand and supply shocks than Poland. This difference mainly stems from the fact that Slovakia is a smaller and more open economy with a smaller impact of real interest rates procyclicality and lower persistence in the stock of consumption habits. On top of that, this economy is characterized by lower estimated Calvo probabilities and backward-looking components in the Phillips curves for both sectors. Consequently, the competitiveness channel – a mechanism of adjustment via external relative prices of domestic tradables – should work more efficiently.

However, the Polish economy ranks better in terms of estimated labour market flexibility. As labour market conditions change more swiftly and nominal wages lead to a quicker realignment in the labour market, the relative disadvantage of Poland – as compared to Slovakia – is at least partly offset. This is particularly visible when labour supply shocks are taken into consideration. In an economy with stronger labour market rigidity, this sort of shock is able to induce an incomparably higher macroeconomic volatility.

When the fall in nominal interest rates is persistent, the economy shifts to a different position. The main feature of this permanent adjustment is an appreciation of the external terms of trade. Its quantitative assessment strongly depends on the assumption how long the natural interest rate differential will last. With a 30-year long period of real convergence, the simulated terms of trade appreciation for Poland amounts to 5.1%, whereby for Slovakia 3.4%. If agents expect a longer (shorter) period of real convergence, the magnitude of mid-term appreciation would rise (fall). This difference is intuitive, given the fact that the Polish economy is less sensitive to external competitiveness developments than the Slovak one.

The latter finding suggests that the ultimate conversion rate could to some extent be considered as a tool faciliating the transition to the new equilibrium point. However, the impact of such measure on the tradable sector requires a more profound investigation. It further research, it would also be interesting to see the evolution of the estimated coefficients (and the results conditional upon them) over time, as well as to address some unresolved estimation issues via Bayesian methods.

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